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Steel Design Guide

Stability Design of Steel Buildings

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AMERICAN INSTITUTE OF STEEL CONSTRUCTION

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Preface

This Design Guide provides guidance in the application of the stability design provisions that were introduced in the 2005 AISC *Specification for Structural Steel Buildings* and the 13th Edition AISC *Steel Construction Manual*. Although some of the relevant section and equation numbers have changed in the 2010 AISC *Specification for Structural Steel Buildings* and the 14th Edition AISC *Steel Construction Manual*, the 2010 provisions for stability design are similar, being a refinement and simplification of the 2005 provisions. Thus, the guidance and recommendations given in this document apply equally to the 2010 AISC *Specification* and 14th Edition AISC *Manual.*

Although some jurisdictions in the United States are using a more current version of the *International Building Code*, the 2006 IBC is most common at the time of writing of this document. Because the 2006 IBC refers to the 2005 AISC *Specification*, those provisions are the basis of this document. To assist the reader, however, summaries are provided to highlight the refinements and simplifications made in the 2010 AISC *Specification* provisions. The changes for 2010 are indicated in “Update Notes” in shaded boxes analogous to the User Notes in the *Specification*; some of the changes in equation numbers and section references are indicated in bracketed statements in line with the text.

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Chapter 1
Introduction

1.1 PURPOSE OF THIS DESIGN GUIDE

With the 2005 AISC *Specification for Structural Steel Buildings* (AISC, 2005a), hereafter referred to as the AISC *Specification*, the state of the art was advanced to include three methods for stability design, including the introduction of a powerful new approach—the direct analysis method (DM). The DM is a practical alternative to the more traditional effective length method (ELM), which has been the basis of stability considerations in earlier editions of the AISC *Specification*, and continues to be permitted. In addition, the third method provided is a streamlined design procedure called the first-order analysis method (FOM), which is based upon the DM with a number of conservative simplifications.

The primary purpose of this Design Guide is to discuss the application of each of the aforementioned three methods and to introduce the DM to practicing engineers. The DM is permitted and referenced in Chapter C of the AISC *Specification*, and its procedural details are described in Appendix 7. As explained in Chapter C and in this Design Guide, the DM is required in cases where the second-order effects due to sidesway are significant.

> **Update Note:** The stability provisions of the 2010 AISC *Specification* (AISC, 2010) are technically very similar to those in the 2005 edition. Where there are technical changes, they are in the direction of being less conservative: A structure that conforms to the 2005 AISC *Specification* could reasonably be expected to be in conformance with the stability requirements of the 2010 edition as well.
>
> The provisions have, however, been substantially rearranged and reorganized for 2010 in the interest of greater transparency and clarity. The effects that must be considered in design for stability are spelled out and it is stated that any rational method that accounts for those effects, including the three prescribed methods, is permitted. The direct analysis method is presented in Chapter C as the primary method; the effective length and first-order analysis methods, and limitations on their use, are presented in Appendix 7. All three of the methods are identified explicitly by name (the ELM and FOM were not named in 2005).

Some of the attractive features of the DM include:

- There is no need to calculate *K* factors.
- The internal forces are represented more accurately at the ultimate limit state.
- The method applies in a logical and consistent manner for all types of steel frames, including braced frames, moment frames, and combined framing systems.

Other purposes of this Design Guide are as follows:

- Discuss the requirements for overall stability design in the 2005 AISC *Specification* as well as in the 2006 International Building Code (ICC, 2006) and the 2005 edition of ASCE/SEI 7, *Minimum Design Loads for Buildings and Other Structures* (ASCE, 2005), hereafter referred to as ASCE/SEI 7.
- Describe the traditional ELM and update designers on new conditions placed on its use.
- Introduce the new FOM and explain when this method can be advantageous.
- Discuss application of stability methods to seismic design.
- Highlight common pitfalls and errors in stability analysis and design.
- Provide an overview of basic principles of stability analysis and design for practical steel structures.
- Provide guidance on benchmarking of second-order analysis software.
- Illustrate how the DM can be applied to provide streamlined and efficient solutions for assessment of column stability bracing.

This Design Guide illustrates the application of the overall stability design requirements of the AISC *Specification* using representative examples taken from routine design office practice. Emphasis is placed on practical applications as opposed to theoretical derivations. The examples use wide-flange shapes predominantly for the members. However, the material presented can be applied to frames designed using other rolled shapes and hollow structural sections, as well as built-up sections.

This Design Guide does not address the specifics of the different methods of second-order frame analysis. An extensive list of references is provided for users needing additional background on the theoretical basis of the provisions. The *Guide to Stability Design Criteria for Metal Structures* (Ziemian, 2010) is referenced for detailed background and developments in a number of the primary and related topic areas.

1.2 HOW TO USE THIS DESIGN GUIDE

This guide describes and illustrates the application of the three methods of stability design contained in the AISC *Specification*. In addition, it addresses a number of other related topics important to the stability design of steel buildings, and provides references that will serve to give readers a more complete understanding.

Chapter 1 provides an overview and discussion of key general considerations. Chapters 2, 3 and 4 present each of the methods of stability analysis and design. Chapter 2 addresses the effective length method (ELM), Chapter 3 explains the direct analysis method (DM), and Chapter 4 discusses the first-order analysis method (FOM). Example analysis and design calculations are provided at the end of each of these chapters. Chapter 5 provides an overview of several important special topics pertinent to steel building stability design. Several appendices provide more detailed discussions.

This Design Guide can be used in a variety of ways as described in the following, depending on the reader's interests and intentions. For readers interested in quickly becoming proficient in performing stability design using any one of the three methods referenced in Chapter C of the AISC *Specification*:

1. Read Chapter 1 as an overview and proceed to Chapter 2 for the ELM, Chapter 3 for the DM, or Chapter 4 for the FOM.
2. Review the design examples worked for the desired method at the end of the corresponding chapter.
3. If seismic design is required, see Section 5.1, Application to Seismic Design.
4. See Appendix D for discussion of proper benchmarking of second-order analysis software.
5. Read Appendix C to learn about the modeling of out-of-plumbness in taller building structures when using the DM or ELM methods.
6. See Section 5.2 for discussion of common pitfalls in stability analysis and design.

It is not necessary to read all sections of this guide to immediately begin solving problems by any of the three methods. The overview chapter for each of the design methods and each of the corresponding design examples show all the steps required to solve a given problem.

For readers interested in an overview of stability design methods in general or in the theoretical background to any method should read Chapter 1 and Appendix A, followed by the chapter covering the specific method of interest and the various pertinent references. For guidance in benchmarking of second-order analysis programs, see Appendix D of this guide. For readers who want to learn about stability bracing design using second-order analysis, read Appendix 6 of the AISC *Specification* and Commentary and Appendix E of this design guide.

A summary of design recommendations is contained at the end of each chapter or major section where appropriate. This allows for a quick review of the salient points covered in the particular chapter or section.

1.3 OVERVIEW OF STABILITY ANALYSIS AND DESIGN METHODS

The 2006 *International Building Code* adopts various reference standards for the definition of load effects and requirements pertaining to specific construction materials. It references the ASCE/SEI 7 Standard, *Minimum Design Loads for Buildings and Other Structures*, for loading requirements, including dead, live, wind, seismic, snow and rain loads. For structural steel design, it references AISC standards, including ANSI/AISC 360-05, *Specification for Structural Steel Buildings* and ANSI/AISC 341-05, *Seismic Provisions for Structural Steel Buildings* (AISC, 2005c). Each of these documents contains requirements for stability and this Design Guide provides a synthesis of many of these requirements with an emphasis on the overall stability design of steel building frames.

Note that stability design provisions have been significantly updated in the 2005 AISC *Specification*. The design for overall frame stability is addressed in Chapter C, Stability Analysis and Design; Appendix 1, Inelastic Analysis and Design; Appendix 6, Stability Bracing for Columns and Beams; and Appendix 7, Direct Analysis Method. Design for stability of individual members and structural components is addressed in many of the other chapters throughout the AISC *Specification*.

Update Note: The 2010 AISC *Specification* defines "design" as the combination of analysis to determine the required strengths of components and the proportioning of components to have adequate available strength. To be consistent with this definition, Chapter C is now titled Design for Stability and Appendix 1 is titled Design by Inelastic Analysis. In addition, the direct analysis method is now in Chapter C, while Appendix 7, titled Alternative Methods of Design for Stability, presents the effective length method and the first-order analysis method.

Part 2 of the 13th Edition AISC *Steel Construction Manual* (AISC, 2005b), hereafter referred to as the AISC *Manual,* contains a brief discussion of requirements pertaining to overall frame stability, including a simplified application of the ELM. Table 2-1 in the AISC *Manual* summarizes the three available methods for stability analysis and design covered in the AISC *Specification* and this guide. An expanded form of this table is included here as Table 1-1 as a convenient reference for the designer.

Update Note: Note the slight difference in the definition of Y_i within the definition of the notional load equation for N_i throughout this Design Guide (based explicitly on the 2005 AISC *Specification*) versus the definition in the 2010 AISC *Specification*. Note that this difference does not have any effect on the value derived from the respective notional load equations. They both yield identical results.

For this Design Guide and as given in the 2005 AISC *Specification* Appendix 7, Section 7.3:

$$N_i = 0.002Y_i$$

where

N_i = notional load applied at level i, kips

Y_i = gravity load applied at level i from the LRFD load combinations or 1.6 times the ASD load combinations, as applicable, kips

For the 2010 AISC *Specification*:

$$N_i = 0.002\alpha Y_i \qquad \text{(2010 } \textit{Spec}\text{. Eq. C2-1)}$$

where

N_i = notional load applied at level i, kips

Y_i = gravity load applied at level i from the LRFD load combination or the ASD load combination, as applicable, kips

α = 1.0 (LRFD); α = 1.6 (ASD)

Update Note: Table 2-1 in the 13th Edition AISC *Manual* is Table 2-2 in the 14th Edition AISC *Manual* (AISC, 2011). Several section and appendix references in Table 1-1 (of this Design Guide) will be different when applied to the 2010 AISC *Specification*: The DM is in Chapter C, the ELM and FOM are in Appendix 7, and the "B_1-B_2" technique is in a new Appendix 8, Approximate Second-Order Analysis.

1.4 IMPLEMENTATION OF SECOND-ORDER ANALYSIS IN THE DESIGN PROCESS

The calculation of overall second-order effects applies to all types of frames: braced frames, moment frames and combined systems. Additionally, a second-order analysis must include all gravity load stabilized by the corresponding frame or frames, including loads on elements such as leaning columns and tilt-up walls. Traditionally speaking, the destabilizing effects from gravity columns and/or tilt-up walls have often been overlooked entirely, or only a part of the gravity load has been included; this can result in significant underestimation of the actual forces and displacements associated with the sidesway of the structure.

Part of the challenge faced by code writers in developing guidelines for the handling of overall stability effects in design is the wide range of approaches currently used by practicing engineers in performing the structural analysis of building frames. These methods may be as simple as amplification of first-order analysis results using planar frames and approximate hand or spreadsheet methods; or as advanced as three-dimensional analysis of the complete building structural system, including the lateral load resisting frames, leaning columns, and possibly even the floor framing. The wide range of differences in approaches stems from the rapid change in analysis methods used over time and prolific increases in more sophisticated computer software capable of modeling large structures with relative speed and economy.

Update Note: An important change in the 2010 AISC *Specification* is that it allows use of a P-Δ-only analysis (that is, one that neglects the influence of P-δ effects on the response of the structure) under certain conditions, specified in 2010 AISC *Specification* Section C2.1(2). The conditions that allow use of the P-Δ-only analysis will be found to apply to most buildings. This represents an important simplification of analysis requirements. (See the Update Note in Appendix D for more on this.)

Two common ways in which second-order analysis may be implemented in the design process are discussed in the following. These approaches are discussed briefly to contrast their differences, and to point out some of the challenges in designing for second-order effects.

1.4.1 Amplifier-Based Procedures

Amplifier-based procedures are methods of second-order analysis in which (1) the calculated internal forces caused by design loadings are first-order, and therefore, linear elastic, (2) amplification factors are determined based on the ratio of the strength load levels to certain idealized elastic buckling load levels, and (3) these amplification factors are applied to the calculated internal forces to account for second-order effects. There are many different ways to apply amplification factors to first-order analysis results, each with various ranges of applicability. One common method provided in AISC *Specification* Section C2.1b is known as the B_1-B_2 method.

Update Note: The B_1-B_2 method has been moved into a separate appendix (Appendix 8) in the 2010 AISC *Specification* to emphasize that it is simply an analysis technique, not a method of design for stability analogous to the DM, ELM and FOM.

Table 1-1. Summary of Main Provisions for Stability Analysis and Design

	Direct Analysis Method (DM)	Effective Length Method (ELM) (See Note 5)	First-Order Analysis Method (FOM)
AISC *Specification* Reference	Appendix 7	Section C2.2a	Section C2.2b
Limitations on the Use of the Method	None	$\Delta_{2nd}/\Delta_{1st}$ or $B_2 \le 1.5$	$\Delta_{2nd}/\Delta_{1st}$ or $B_2 \le 1.5$, $\alpha P_r/P_y \le 0.5$
Analysis Type	Second-order elastic (See Note 1)	Second-order elastic (See Note 1)	First-order elastic
Structure Geometry in the Analysis	Nominal (See Note 2)	Nominal	Nominal
Notional Loads in the Analysis (See Note 3)	$0.002Y_i$ Minimum if $\Delta_{2nd}/\Delta_{1st} \le 1.5$ Additive if $\Delta_{2nd}/\Delta_{1st} > 1.5$ (See Note 2)	$0.002Y_i$ Minimum (See Note 2)	$2.1(\Delta/L)Y_i \ge 0.0042Y_i/\alpha$ Additive (See Note 6)
Member Stiffnesses in the Analysis	Use $EA^* = 0.8EA$ Use $EI^* = 0.8\tau_b EI$ $\tau_b = 1.0$ for $\alpha P_r/P_y \le 0.5$ $\tau_b = 4[(\alpha P_r/P_y)(1 - \alpha P_r/P_y)]$ for $\alpha P_r/P_y > 0.5$ (See Note 4)	Use nominal *EA* and *EI*	Use nominal *EA* and *EI*
Design of Individual Members	Use Chapters E, F, G, H and I, as applicable	Use Chapters E, F, G, H and I, as applicable	Use Chapters E, F, G, H and I, as applicable
	Use $K = 1$ for calculating member strengths	Determine K for calculating member strengths from sidesway buckling analysis (Can use $K = 1$ for braced frames; can use $K = 1$ when $\Delta_{2nd}/\Delta_{1st} \le 1.1$)	Use $K = 1$ for calculating member strengths
	No further member stability considerations	No further member stability considerations	Apply amplification $B_1 = C_m/(1 - \alpha P_r/P_{e1}) \ge 1$ to beam-column moments

General Note: $\Delta_{2nd}/\Delta_{1st}$ is the ratio of second-order drift to first-order drift (this ratio can be taken to be equal to B_2 calculated as specified in AISC *Specification* Section C2.1b). The ratio Δ/L in the FOM is the maximum first-order story drift ratio for all stories in the building. Δ/L in the FOM is calculated using the LRFD or ASD required loads directly (i.e., with no increase of the ASD loads by $\alpha = 1.6$). All other terms should be calculated using the LRFD load combinations or using 1.6 times the ASD load combinations, i.e., $\alpha = 1.0$ for LRFD and $\alpha = 1.6$ for ASD. When using ASD with an explicit second-order analysis, the resulting internal forces are divided by $\alpha = 1.6$ prior to conducting member design checks. When using ASD with amplifier-based procedures, the 1.6 factor is embedded in the amplifier, permitting all the first-order analyses to be conducted at the $\alpha = 1.0$ load level but implicitly considering the amplification at the 1.6 level.

Note 1: Any method of second-order analysis that properly incorporates both P-Δ and P-δ effects is allowed, including procedures such as the amplified first-order analysis "B_1-B_2" method described in Section C2.1b.

Note 2: One is allowed to model the corresponding nominal initial imperfection directly in lieu of applying the $0.002Y_i$ minimum or additive notional loads.

Note 3: Notional loads are lateral loads applied at each level of the structure, either as minimum lateral loads in gravity-only load combinations or as lateral loads applied in addition to other lateral loads in all load combinations. These loads are equivalent to the destabilizing effects of a nominal out-of-plumbness.

Note 4: One can use $\tau_b = 1.0$ in all members if additional notional loads of $0.001Y_i$ are applied, additive with any lateral loads. Reduction of all flexural rigidities by $0.8\tau_b$ and all other elastic stiffnesses by 0.8 is recommended.

Note 5: A simplified version of the ELM is shown in Part 2 of the AISC *Manual*.

Note 6: The notional load given here for the first-order analysis method is correct in its application of α, although it is different from that presented in the AISC *Specification*.

One key attribute of amplifier-based procedures is that the structure can be analyzed separately for the various types of loading, using simple and efficient linear elastic analysis procedures. Subsequently, the results from these analyses can be combined using superposition and the B_1 and B_2 amplification factors are applied to the end results.

The values of the B_1 factors depend on the axial forces in the columns relative to estimated column nonsway buckling loads; this is referred to as the member effect. The values of the B_2 factors are influenced by the overall gravity load in the various levels of the structure compared to the estimated total gravity load at overall sidesway buckling of these levels; this is referred to as the structure or sway effect. See Appendix B in this Design Guide for an overview of the application of the B_1 and B_2 amplification factors.

Amplifier-based procedures are often used for preliminary analysis and design, and may involve significant approximations that may need to be improved upon in the final analysis and design. These procedures allow the gravity and lateral load analyses to be handled separately, which provides for simplicity and convenience in the design process. The gravity load analysis may be conducted by hand using simple moment coefficients or by computer software that analyzes all or a portion of the floor framing. Similarly, the method used for preliminary lateral load analysis can range from a simple portal type method to plane frame or 3D frame computer methods. The lateral load analysis is conducted using only lateral loads without any gravity loads.

Amplifier-based methods lend themselves to regular, orthogonal framing with defined levels and predictable load paths. Their correct application is less clear in cases such as complex buildings where the geometry or loads are not symmetrical, where the beams are not aligned with orthogonal *x*- and *y*-directions in plan, where several planar frames share in providing lateral stability to framing that does not lie within the plane of any of these frames, where significant torsional effects are encountered, and/or where the subdivision of the structure into "stories" or "levels" is not clear.

Note also that second-order effects can be significantly different for each code-prescribed load combination because of the different vertical loads for each combination. One typical simplification is the use of a single conservative amplifier that is applied to all the various load combinations.

1.4.2 Explicit Second-Order Analysis

In an explicit second-order analysis, the gravity and lateral loads are considered together in the same model and a separate second-order analysis is carried out for every load combination considered in the design. The geometry of the structure and the detailed distribution of the loads, stiffnesses and displacements throughout the structure are addressed explicitly in the calculation of the second-order effects. As such, this approach avoids simplifying assumptions from story-by-story or level-by-level idealization, even for complex framing arrangements and loadings.

Explicit second-order analysis may be performed using a complete three-dimensional (3D) model, or with a simplified two-dimensional model. If a complete 3D model is used, the idealization of the general spatial response of a 3D structure into *x* and *y* responses can be eliminated.

If a 2D model is used, it is common to incorporate a "dummy column" that is added to the lateral frame analysis as shown in Figure 1-1. Except for the beams that are part of the lateral load resisting frames, the floor system does not need to be included in the overall analysis model. The "dummy column" is a single member with a large enough value of AE/L to be called axially rigid, attached to a lateral load resisting frame at each floor with a rigid link that is pinned at each end. Its sole purpose is to model the effect of the gravity load applied to all the gravity columns stabilized by the particular frame.

Whether a dummy column or the actual framing is included in the model, the model must capture the effect of all of the gravity load stabilized by the framing that is modeled. The stiffness of the floor and/or roof diaphragms must be considered when deciding how to apportion the lateral loads and the leaning column effects. In many structures, even in cases where it is sufficient to assume that the diaphragms are rigid in the plane of the floor or roof, it can be challenging to account for second-order effects in a 2D model, particularly when the analysis includes torsional effects, quartering wind loads, unbalanced wind loads, and diaphragm stiffness effects.

Live load reduction is more difficult to apply using this approach, because code provisions for live load reduction are

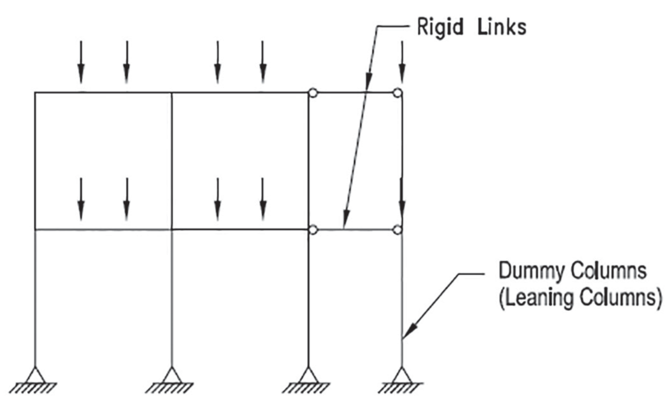


Fig. 1-1. Dummy column in 2D model.

based on the assumption that live load effects are determined separately from the other load effects for each structural member. Ziemian and McGuire (1992) provide a method of implementing live load reduction in explicit second-order analysis, by applying compensating joint loads.

1.4.3 Modeling Recommendations

Regardless of the approach taken, stability analysis and design of steel buildings should be based on a model that captures the essential behavior of the frame under vertical and lateral loads and accurately accounts for second-order effects, including both P-Δ and P-δ effects. Basic checks to verify that a specific approach or computer program is able to satisfy the requirement for accurately capturing these second-order effects can be found in Appendix D, Practical Benchmarking and Application of Second-order Analysis Software.

The second-order analysis implementation in many software packages considers only P-Δ effects. That is, only the effect of relative transverse displacements (Δ) at the end nodes of the frame elements is considered in the calculation of the second-order effects. The influence of the detailed element transverse displacements between the element nodal locations (i.e., the P-δ effects) is not considered. Some design programs do consider P-δ effects on members using the B_1 amplification factor during the member design process, and this accounts for the amplification of the moments between the member ends. However, the B_1 amplifier does not account for the increase in P-Δ effects due to reduction in the member sidesway stiffness due to the P-δ effects. When P-δ effects are significant, they may be captured by subdividing members into multiple elements. See Appendices A and D in this guide for further discussion.

As discussed in detail in Appendix A, Basic Principles of Stability, the second-order effects on the sidesway response can be measured as the ratio of second-order story drift to first-order story drift ($\Delta_{2nd}/\Delta_{1st}$). This ratio can be estimated using the AISC *Specification* sidesway amplification factor, B_2, which is defined by Equation C2-3 in Chapter C [Equation A-8-6 in Appendix 8 in the 2010 AISC *Specification*].

1.4.4 Nonlinearity of Second-Order Effects

Because second-order effects are a nonlinear problem, an accurate second-order analysis is contingent upon incorporating all the gravity loads stabilized by the lateral load resisting frames at the appropriate factored load level. That is, the second-order effect is not proportional to the gravity load but rather increases at a greater rate with larger gravity load. This can be observed by examining the rate of change of the B_2 amplification factor from the AISC *Specification* Equation C2-3 versus $\Sigma P/\Sigma P_{e2}$ as shown in Figure 1-2. Note in the figure the rapid change in B_2 as $\Sigma P/\Sigma P_{e2}$ increases. For instance, as $\Sigma P/\Sigma P_{e2}$ increases from 0.60 to 0.65 (an 8% increase), B_2 increases from 2.5 to 2.86 (a 14% increase). Thus, only a small error in the applied load, ΣP, or the sidesway stiffness can lead to a large change in the internal forces at this load level. It is for this reason that designers are encouraged to maintain relatively small second-order amplification levels—the authors recommend that the amplification generally should be limited to less than 1.5. See Chapter 5 for example limits on B_2 determined from the seismic P-Δ and drift limits of ASCE/SEI 7 Sections 12.8.7 and 12.12.

In addition, when performing a second-order analysis, the analysis must be conducted at the ultimate load level. This is accounted for by the factor α in the AISC *Specification*, which is applied as a multiplier to the load combinations ($\alpha = 1.0$ for LRFD load combinations and 1.6 for ASD load combinations). Because of the nonlinearity, a second-order analysis at ASD load combinations without α would underestimate the actual second-order effects that must be considered in the structure.

It may be convenient, and typically is conservative, to conduct a second-order analysis using the worst case of factored gravity loads. When considering service load combinations, if drift limits that reflect actual damage are to be determined, it is important to determine the second-order effects on the corresponding story drifts. If arbitrary drift limits, such as $H/400$, as have been used historically, are to be used, a first-order story drift calculation may be acceptable. LeMessurier (1976 and 1977) shows several example frames where neglecting second-order effects leads to a substantial underestimation of service load drifts.

1.4.5 Summary of Design Recommendations

Following is a summary of the steps required to perform a second-order analysis.

1. Model the structure with an approach that captures the essential first- and second-order behavior of the frame under the required loading combinations. When ASCE/SEI 7 wind load Method 1 does not apply, ASCE/SEI 7 requires wind load combinations (see Figure 6-9 in ASCE/SEI 7) that include eccentric and quartering wind loads, and both of these loadings require consideration of 3D effects on frame behavior. Buildings in Seismic Design Categories C, D, E and F require analysis for earthquake forces from a combination of two orthogonal directions or seismic forces from any direction (see ASCE/SEI 7 Section 12.5). Also, requirements for accidental torsional loading caused by a 5% displacement of the center of mass along with amplification for torsional irregularities (Type 1a, 1b) for structures in Seismic Design Categories C, D, E and F require consideration of 3D effects. In simple symmetrical buildings, 2D

modeling may still be possible. Otherwise, these requirements often dictate a 3D analysis of the structure.

2. Include all the gravity loads stabilized by the lateral load resisting system, including leaning column loads, load effects from tilt-up walls, etc., as part of the frame analysis to properly capture the second-order effects. In 2D models of structures where the floor diaphragms are effectively rigid, gravity loads from leaning columns can be included as concentrated load(s) at the floor levels on a "dummy column." The floor system (except beams that are part of the lateral load resisting system) does not need to be included in the overall analysis model.
3. For design using ASD, all second-order analyses must be performed at ultimate load levels ($\alpha = 1.6$) to properly capture the magnitude of the second-order effects.

1.5 THE CONCEPT OF NOTIONAL LOADS

The concept of notional loads is new to the AISC *Specification* and to most designers in the U.S. Notional loads are an integral part of stability design methods in Canada and Australia, and in countries using the Eurocode. The concept of notional loads is an integral part of all methods of analysis presented in the AISC *Specification*.

Notional loads are lateral loads applied at each level of the structure, either as minimum lateral loads in gravity-only load combinations or as lateral loads applied in addition to other lateral loads in all load combinations. These loads are intended to account for the destabilizing effects of initial geometric imperfections, such as a nominal out-of-plumbness, inelasticity, and second-order effects, or a combination of these, depending on the specifics of the method being implemented. Notional loads are defined in terms of the gravity loads applied at the same level in the structures at which the gravity loads are applied. The gravity load used to determine a notional load can be equal to or greater than the gravity load associated with a particular load combination being evaluated. Notional loads for the direct analysis method are defined in AISC *Specification* Appendix 7, Section 7.3 [2010 AISC *Specification* Section C2.2b] as follows:

$$N_i = 0.002Y_i \qquad (1\text{-}1)$$

where

N_i = notional lateral load applied at level i, kips

Y_i = gravity load from the LRFD load combination or 1.6 times the ASD load combination applied at level i, as applicable, kips

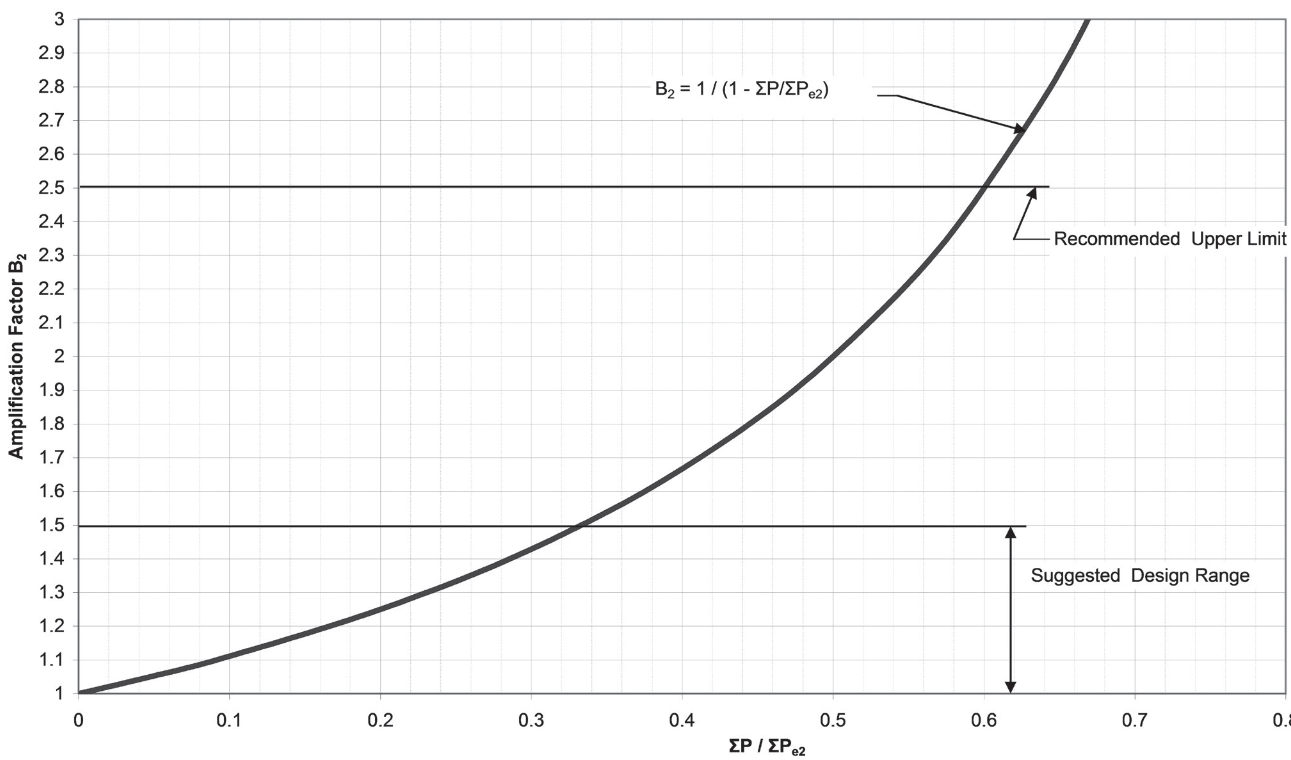


Fig. 1-2. Amplification factor, B_2.

Update Note: Note the slight difference in the definition of Y_i within the definition of the notional load equation for N_i throughout this Design Guide (based explicitly on the 2005 AISC *Specification*) versus the definition in the 2010 AISC *Specification*. Note that this difference does not have any effect on the value derived from the respective notional load equations. They both yield identical results.

For this Design Guide and as given in the 2005 AISC *Specification* Appendix 7, Section 7.3, the notional lateral load is defined in Equation 1-1 as just discussed.

For the 2010 AISC *Specification*:

$$N_i = 0.002\alpha Y_i \qquad \text{(2010 } \textit{Spec}\text{. Eq. C2-1)}$$

where

N_i = notional load applied at level i, kips

Y_i = gravity load applied at level i from the LRFD load combination or the ASD load combination, as applicable, kips

α = 1.0 (LRFD); α = 1.6 (ASD)

The notional load coefficient of 0.002 is based on an assumed initial story out-of-plumbness of 1/500 as specified in the AISC *Code of Standard Practice for Steel Buildings and Bridges* (AISC, 2005d). This notional load gives a reasonable estimate of the influence of a uniform out-of-plumbness of 1/500. An additional notional load, $N_i = 0.001Y_i$, may be used to account for the influence of residual stresses in the direct analysis method. Details of the application of notional loads in the direct analysis method are found in Chapter 3. Notional loads for the first-order analysis method are discussed in Chapter 4 and for the effective length method in Chapter 2.

Note that the notional load equation may be interpreted as a single notional load value to be applied at a given story level. However, in any building structure where the location of the notional load in plan is a consideration, separate notional loads proportional to the specific gravity load should be applied at each location where the gravity loads are applied to the structure.

1.6 THE INFLUENCE OF APPLIED VERTICAL LOADS ON STABILITY

Figure 1-2 showed the effect of vertical load on second-order effects as depicted by the AISC amplification factor, B_2. The larger the applied vertical load, ΣP, for a constant, ΣP_{e2}, the greater is B_2 and its effect on the stability of the building. Also, the effect on stability is nonlinear—B_2 increases more rapidly as the vertical load increases. Clearly, consideration must be given to the magnitude of the applied vertical load and its influence on the stability of the structure, which extends beyond the second-order amplification of the internal forces.

For the effective length method outlined in Chapter 2, the impact on the effective length factor, K, can be seen in the formulations shown as K_2 in AISC *Specification* Commentary Equations C-C2-5 and C-C2-8, which also contain the ΣP term. In the direct analysis method outlined in Chapter 3 and the first-order analysis method outlined in Chapter 4, a similar effect manifests itself in the ΣP term contained in the definition of Y_i used in the determination of notional loads or minimum lateral loads that are part of the stability design process for those methods.

ASCE/SEI 7 specifies the appropriate load combinations that are to be used for the design of buildings. While the standard is clear in its definition of the load combinations and appropriate combination factors to be used, special consideration is warranted when applying these load combinations for the purpose of stability calculations. Many of the applied vertical loads such as live load, L, and wind load, W, vary in time and in space. The magnitude of these loads must be evaluated because the magnitude ΣP has a major influence on stability effects. For live load, the ASCE/SEI 7 load standard considers the variation in load magnitude on various components such as beams, columns and foundations via the live load reduction concept. This concept is reasonable to use when applying stability concepts contained in this guide, because the live load reduction factor generally increases with increasing tributary or influence area. However, the magnitude of the wind load specified in the standard requires further consideration.

The ASCE standard attempts to adjust the vertical design wind pressure on roof surfaces affecting the vertical load, W, with the concept of effective wind area (refer to ASCE/SEI 7 Section 6.2, Definitions). The larger the effective wind area, the lower the design wind pressure based on a consideration of the spatial and temporal averaging of the wind pressure over a given tributary area of member. However, the effective wind area concept was developed for roof elements and members, such as roof fasteners, roof purlins and roof trusses (components and cladding). Stability of an entire building as reflected in the overall ΣP term is influenced by a potentially much larger tributary area than used by the standard for the smaller components such as fasteners, purlins and trusses.

Determination of Vertical Loads (ΣP) and Notional Loads (N_i) with Wind Loads on Roofs. For wind load combinations specified by ASCE/SEI 7 that act on the roofs of buildings, the question may arise as to whether upward vertical wind pressures on roof surfaces can be considered when calculating ΣP for evaluation of overall stability effects in any of the analysis and design methods, and when determining Y_i for calculation of N_i, where N_i is specified to be an additive lateral load. The following discussion is intended to provide guidance on this subject, as it is not specifically addressed in ASCE/SEI 7 or the AISC *Specification* and is a matter of engineering judgment.

Code-specified wind pressures for roof surfaces that load lateral force resisting systems, as contained in ASCE/SEI 7 Section 6.5.12.2.2, for example, are based on peak wind pressures obtained at some instant in time during a design wind storm, which could last over several hours or more. During this design storm event, roof pressures are expected to vary significantly from this peak value and, in some cases depending on roof slope, can even reverse in sign from negative (suction, acting away from the roof surface) to positive (pressure, acting towards the roof surface) values. Thus, it generally is unconservative to expect this peak design pressure to be applicable at all times over the entire roof surface during the design wind storm. This is particularly significant when considering a reduction in gravity loads due to roof wind pressure and its effect on ΣP and Y_i over a large roof surface where a large amount of gravity load is stabilized by a given lateral load system (refer to Examples 2.2, 3.2 and 4.2 for an extreme case where a very large roof area is stabilized by the lateral load resisting system). It would not be reasonable to assume that the roof wind pressure is at its peak value over the entire area of the roof at the same instant in time (see ASCE/SEI 7 Commentary Figure C6-6, which shows the variation of first-order wind effects with time). In buildings where wind uplift on the roof gives a significant reduction in the vertical loads stabilized by the lateral load resisting system and the sidesway amplification is large, it would be more reasonable to base the consideration of global stability on a reduced wind pressure such as the mean value expected during the design wind storm. This value could be reasonably assumed to be some fraction (say 40 to 50%) of the peak value specified in the code. Conservatively, the overall stability design could be based on ignoring the effect of vertical uplift wind pressures on the gravity load, ΣP, and Y_i. This can be accomplished without increasing the number of ASCE/SEI 7 load combinations by adding dummy columns to the structural analysis model to compensate for the net vertical uplift from the ASCE/SEI 7 wind loadings. A more accurate determination of the applicable roof pressure to use for stability design can be determined from an experienced wind engineering consultant and/or a wind tunnel test of the actual building when such a study is warranted.

1.7 INTRODUCTION TO THE DESIGN EXAMPLES

In this guide, two example problems are employed to demonstrate the application of the AISC stability design provisions. The first problem is a representative braced column line taken from a two-story warehouse building. The second example is a single-story building with a large number of interior gravity columns with moment connections only to the end columns in each bay in one direction and exterior wall bracing in the orthogonal direction. Each of the three AISC stability procedures is applied to these two problems.

In general, the problems have been simplified to shorten the calculations and to emphasize the AISC stability design provisions. Several representative members are designed and checked in each of the examples. Simplified loadings are used for many of the load cases and some load combinations are not considered. The designer is encouraged to study the appropriate chapters in ASCE/SEI 7 as they relate to accurate wind and snow loadings for actual building designs (items not considered in this guide include partial snow loading, drift snow loading, quartering wind loads, vertical wind load on roof surfaces, and multi-directional seismic loading). Where serviceability drift checks under wind load are specified, nominal dead, live and wind loads are used unless noted otherwise. The selection of service load combinations is a matter of engineering judgment. The designer is referred to ASCE/SEI 7 Appendix C and to AISC Design Guide 3, *Serviceability Design Considerations for Steel Buildings* (West et al., 2003) for information that can be used in selecting service loads.

In some cases in the design examples presented in this guide, the lateral load is analyzed separately in each 180° direction, even for symmetrical problems. This is done for convenience because of the automated member design routine used with the computer software employed for the analysis and design calculations. Generally, lateral loads acting in each positive and negative direction (i.e., 180° apart) must be considered unless the building is perfectly symmetrical in geometry, member sizes and loading.

In complex structures, it can be difficult to predict which load combinations will control the design of a given member in a frame. Thus, it can be advantageous for the designer to automate the design process so that all load combinations are automatically included. Benchmarking of the software is essential to ensure its correctness prior to applying it in an automated production analysis and design setting.

The solution to any building design problem should begin with a detailed consideration of the type and method of connecting the lateral load resisting frame members. Also, the decision of which building columns are to be treated as "leaning columns" versus columns that are part of the lateral load resisting system needs to be made early in the design process. It is wise for the designer to draw conceptual connection details to determine the flow of forces through the joints on the structure. How the members are connected can determine the type of member to be used in the lateral load resisting frame (wide flange, HSS, angle, rod, WT, etc.), the member orientation in space, and the size of the member selected. The fabricated and erected cost of structural steel buildings is heavily influenced by the type of connections used because of labor costs compared to material costs of the steel itself. Hence, each example problem in this guide begins with a brief discussion of the types of framing envisioned.

All of the example problems in this Design Guide are analyzed using a computer program that accurately accounts for second-order effects, including P-δ and P-Δ effects. All column and beam members contain three interior nodes (four member elements) to ensure that the P-δ effect is considered accurately for all the potential loadings (except for columns and braces in braced frames, where there are no loadings at intermediate locations along their member lengths and where the member end conditions are idealized as pins; in these cases no interior nodes are added). In general, for programs that account for P-Δ effects only, as many as six elements per member may be necessary to sufficiently capture the P-δ effects (refer to Appendix D for further discussion of this topic). It is difficult to provide recommendations for the number of elements required for specific programs without knowing the approach used by the specific computer software. The above level of refinement is not always necessary; however, many of the examples in this guide have high axial load relative to the Euler buckling load of some of their columns. Many software programs, even those that include some handling of P-δ effects in their element formulations, will require more than one element per member for the engineer to obtain beam-column interaction checks that are accurate to within 3 to 5% of that obtained using the converged solution in these cases.

Update Note: As explained in greater depth in an Update Note in Appendix D, for most buildings the 2010 *Specification* allows simpler analysis than used in these example problems.

Unless otherwise noted, flexural, shear and axial deformations are always considered in the analyses for the examples presented in this Design Guide. Centerline dimensions are used in all cases to estimate joint deformation effects and no additional flexibility of the joints is considered. In all the examples contained in this Design Guide, a separate second-order analysis is performed for each individual load combination considered. In practice, some software programs may base the second-order analysis for all load combinations on a single load combination, usually the one with the highest gravity loads. This is a conservative assumption for determining the overall second-order effects in the other load combinations and usually reduces analysis time and effort.

Readers seeking to duplicate the analysis results presented for the example problems are encouraged to verify that the software they are using accurately accounts for all the important second-order effects and that the second-order analysis for each load combination is undertaken as described in the foregoing. Refer to Appendix D for further discussion.

Design is an iterative process. Initial member sizes must be chosen based on experience and/or a preliminary analysis. For braced frame structures, the design is often controlled by strength as opposed to drift. Thus, for braced frames, the initial sizes can be based on preliminary calculations for strength limit states. For moment frames, the design is often controlled by stiffness or wind or seismic drift as opposed to strength. Thus, the initial sizes can be based on drift calculations (Cheong-Siat-Moy, 1976), or computer software can be used to test member sizes for the desired drift limit. The problem solutions shown in this guide begin with member sizes obtained from this process.

Chapter 2
Effective Length Method (ELM)—
Design by Second-Order Analysis

2.1 INTRODUCTION

The effective length method (ELM) is not specifically mentioned by name in the AISC *Specification* or the Commentary. It is referred to in Commentary Section C1.1 as the traditional approach. The specific requirements for the ELM are presented in AISC *Specification* Section C2.2a. The method applies primarily to moment frames where an effective length factor, K, (or equivalently the elastic buckling stress, F_e) must be determined for use in determining the available axial compressive strength of the member. The ELM is applicable to unbraced or braced frames. However, Section C2.2a(4) [2010 AISC *Specification* Appendix 7, Section 7.2.3] recommends that $K = 1.0$ be used for braced frames. The ELM becomes more problematic for use in combined systems where K must be based on a more general system buckling analysis of some type as opposed to the use of traditional K charts or formulas.

Update Note: The effective length method is identified by name in the 2010 AISC *Specification*; requirements for the ELM are presented in Section 7.2 in Appendix 7, Alternative Methods of Design for Stability.

The 2005 AISC *Specification* restricts the use of the ELM to cases where the ratio of second-order to first-order story drifts, which may be taken as B_2, is less than or equal to 1.5. The AISC *Specification* also requires the application of notional lateral loads, $N_i = 0.002Y_i$, in gravity-only load combinations, where Y_i is the vertical load at level i. These restrictions are placed on the ELM to reduce the potential of large errors in the internal forces and moments at the ultimate load level in some stability sensitive frames.

2.2 STEP-BY-STEP PROCEDURE

The step-by-step application of the ELM is described in detail below:

1. Develop a model of the building frame that captures all the essential aspects of the frame behavior and accounts for all wind and seismic loads and load directions. This method uses the nominal structural geometry and member properties as usual. There is another method that will be discussed later that uses adjusted properties and geometry.
2. Determine all gravity loads that are stabilized by the lateral load resisting system.
3. Determine the lateral loads corresponding to the wind and seismic requirements.
4. Determine the notional lateral loads, which are intended to account for the overall effects of out-of-plumb geometric imperfections, and apply them solely in the gravity-only load combinations in the ELM. These loads are calculated and applied as follows:

   (a) $N_i = 0.002Y_i$, the notional load at level i where Y_i = gravity load (dead, live, snow) at level i from the LRFD load combination being considered or 1.6 times the ASD load combination.

   Update Note: Note the slight difference in the definition of Y_i within the definition of the notional load equation for N_i throughout this Design Guide (based explicitly on the 2005 AISC *Specification*) versus the definition in the 2010 AISC *Specification*. Note that this difference does not have any effect on the value derived from the respective notional load equations. They both yield identical results.

   For this Design Guide and as given in the 2005 AISC *Specification* Appendix 7, Section 7.3:

   $$N_i = 0.002Y_i$$

   where
   N_i = notional load applied at level i, kips
   Y_i = gravity load applied at level i from the LRFD load combinations or 1.6 times the ASD load combinations, as applicable, kips

   For the 2010 AISC *Specification*:

   $$N_i = 0.002\alpha Y_i \qquad \text{(2010 } \textit{Spec.} \text{ Eq. C2-1)}$$

   where
   N_i = notional load applied at level i, kips
   Y_i = gravity load applied at level i from the LRFD load combination or the ASD load combination, as applicable, kips
   α = 1.0 (LRFD); α = 1.6 (ASD)

   (b) For gravity-only load combinations that cause a net sidesway due to nonsymmetry of the loads or geometry, the notional loads should be applied in the direction that increases the net sidesway. For structures with multiple stories or levels and in which the sidesway deformations are in different directions in different stories or levels, it is necessary to include a pair of load combinations separately considering the notional loads associated with an out-of-plumbness in each direction. Generally, one need *not* apply notional loads in a direction opposite from the sidesway to minimize the reduction in internal forces in certain components due to the sidesway. For gravity load combinations with no sidesway, it is necessary to include a pair of load combinations separately considering the notional loads in each $\pm$ direction to account for possible out-of-plumbness in each direction, unless there is symmetry of frame geometry, loading and member sizes.

(c) For gravity-only load combinations, apply N_i independently about each of two orthogonal building axes. These axes should be selected as approximate principal lateral stiffness directions for the overall building structure. (Note: "independently" means that the notional loads are applied only in one direction at a time, similar to the application of ASCE/SEI 7 wind or seismic lateral loads in one orthogonal-axis direction at a time). One need not consider any off-axis (i.e., diagonal) notional lateral loading relative to the approximate principal stiffness directions of the structure.

(d) For general structures, the notional loads may be applied at each location where gravity load is transferred to the structural columns. The load Y_i is the gravity load transferred to the columns at each of these locations.

5. Perform a second-order analysis for all applicable load combinations. Any second-order analysis method that properly considers both P-Δ and P-δ effects is permitted. Note that, unlike first-order analysis, superposition of basic load cases is not appropriate when a general second-order analysis is employed since the second-order effects are nonlinear. However, when the B_1-B_2 approach to second-order analysis is implemented, superposition of basic load cases is appropriate.

6. Design the various members and connections for the forces obtained from the above analysis according to the applicable provisions of the AISC *Specification*.

7. For the beam-columns in moment frames, apply the AISC *Specification* interaction Equations H1-1a and H1-1b (or where applicable, Equation H1-2 at the designer's option). The following is a brief synopsis of how this is done; note that it is not a complete treatise on use of the effective length method. Also note that the K factor determined assumes that destabilizing effects, such as due to gravity-only columns, have been considered. The AISC *Specification* Commentary provides many alternative approaches for doing so, and denotes these K factors as K_2. For clarity in this text, and as a reminder that effects such as these must be considered, the subscript 2 is used in this summary.

(a) Determine the effective length factor, K_2, (or alternatively P_{e2}) for each column. For tiered framing, it is recommended that any of the Equations C-C2-5, C-C2-6, C-C2-8, C-C2-9 or C-C2-10 in the AISC *Specification* Commentary [Equations C-A-7-5, C-A-7-6, C-A-7-8, C-A-7-9 or C-A-7-10 in the 2010 AISC *Specification* Commentary] may be used for this purpose.

(b) When second-order effects are small, such that $\Delta_{2nd}/\Delta_{1st}$ or $B_2 \leq 1.1$, K may be taken equal to 1.0 in the calculation of P_n (but not in the calculation of B_2).

(c) Adjustments to K_2 (or P_{e2}) for column inelasticity are permissible according to the procedure outlined in Chapter C of the AISC *Specification* Commentary Section C2.2b [Appendix 7, Section 7.2 in the 2010 AISC *Specification* Commentary].

(d) When using the sidesway-uninhibited alignment chart as the underlying tool for calculation of K_2, e.g., using the story buckling method of Equations C-C2-8 or C-C2-9 of the AISC *Specification* Commentary [Equations C-A-7-8 or C-A-7-9 of the 2010 AISC *Specification* Commentary], additional adjustments are typically necessary to account for less than ideal framing conditions other than those assumed in the development of the alignment chart. See pages 16.1-241 to 16.1-243 of the AISC *Specification* Commentary [pages 16.1-512 to 16.1-514 in the 2010 AISC *Specification* Commentary] for guidance. Examples of these calculations are provided at the end of this chapter.

(e) For more complex framing systems in which the definition of stories is not clear or in which frames in different planes or at different orientations in plan share in providing the lateral load resistance, an eigenvalue buckling analysis is permitted to determine member buckling loads or corresponding K values. However, overall system buckling analysis can result in unnecessarily high K values in members with low axial force that are not necessarily participating in the actual buckling mode. Examples of this problem can include upper story columns in multi-story frames or columns that are framed with relatively flexible partially restrained (PR) connections (ASCE, 1997; White and Hajjar, 1997; White et al., 2006).

8. Confirm for each level of the frame that second-order sidesway effects, measured by the ratio of the average second- to first-order story drifts ($\Delta_{2nd}/\Delta_{1st}$ or B_2), are less than or equal to 1.5 (based on a model with nominal member properties). Stiffen the structure as necessary to ensure that this requirement is satisfied or use the direct analysis method (DM) of AISC *Specification* Appendix 7 [2010 AISC *Specification* Chapter C].
9. Check the seismic drift limits according to ASCE/SEI 7 Section 12.12 and the maximum P-Δ effects as prescribed by ASCE/SEI 7 Section 12.8.7. Note that P-Δ effects are checked for a load factor no greater than 1.0 on all gravity design loads, P_x, in ASCE/SEI 7 Equation 12.8-16.
10. Check the wind drifts for service level wind loads. Note that this check is a serviceability check, not a code requirement. Also note that for moment frames, drift under selected wind or seismic load levels will typically control the design. Therefore, this check should be made first in the initial proportioning of member sizes for these frame types. In general, it is recommended that a second-order analysis be employed to accurately determine service load story drifts if these drifts are to be compared against actual drift damage limits for the cladding and partition types that are employed. First-order drift analysis may be used for other limiting conditions.

A simplified version of the ELM is described in Part 2 of the AISC *Manual*. This version of the ELM uses a six step process based entirely on the use of first-order analysis and the use of a specified first-order story drift limit. Second-order effects are determined by applying a tabulated B_2 value to the total member moments [Table 2-1 in the 14th Edition AISC *Manual*]. The effect of sidesway of the structure under gravity load, due to lack of symmetry of the geometry or of the load, is neglected. The tabulated B_2 values are based on a story stiffness approach, using Equations C2-3 and C2-6b in the AISC *Specification* [Equations A-8-6 and A-8-7 in the 2010 AISC *Specification*]. The B_2 table also indicates cases where $K = 1.0$ may be used in the design.

2.3 ADVANTAGES, DISADVANTAGES AND RESTRICTIONS ON USAGE

The advantages of the ELM are:

1. The ELM is less sensitive to the accuracy of the second-order analysis than the DM.
2. The ELM is well known to designers and is the traditional approach for the design of steel frames (except for the new notional load requirements and the limitations placed on the magnitude of the second-order effects in the 2005 AISC *Specification*).
3. The ELM requires less labor than the DM for simple cases where the calculation of the effective length factor is straightforward.
4. The ELM has been implemented in many existing software packages used by designers today.
5. The AISC *Manual* provides a simplified version of the ELM that is highly streamlined and provides adequate solutions for many frames.

The disadvantages of the ELM are:

1. The ELM does not account as accurately for internal forces as the DM. However, ELM designs are acceptable given the limits on the method specified within the AISC *Specification*.
2. The ELM requires the calculation of the effective length factor, K, or the corresponding column buckling load, P_e, which can be difficult and subject to error in many moment frame configurations.
3. The application of the method is limited to frames with smaller second-order effects ($\Delta_{2nd}/\Delta_{1st}$ or $B_2 \le 1.5$ based on nominal member properties) to avoid significant errors in the determination of internal forces.
4. The method is more difficult to apply and requires significant engineering judgment for some frame types, including combined braced and moment frames, portal frames with significant axial compression in the beams or rafters, frames where some of the columns are in tension due to uplift, and buildings in which a large part of the framing participates little in the buckling of a critical portion of the structure. The reader is referred to ASCE (1997), White and Hajjar (1997), and White et al. (2006) for additional discussion of these types of cases.
5. The method results in larger maximum and average errors relative to benchmark distributed plasticity solutions than the DM (Maleck and White, 2003).

The ELM is limited to cases where second-order effects ($\Delta_{2nd}/\Delta_{1st}$ or $B_2 \le 1.5$ based on a model with nominal member properties) are relatively small.

2.4 OBSERVATIONS ON FRAME BEHAVIOR—ELM

Figures 2-1 through 2-3 compare the ELM and DM results for several basic W10×60 cantilever columns with $F_y = 50$ ksi subjected to a vertical load P and a lateral load $0.01P$— both loads increasing proportionally from zero until the member capacities are reached. The cantilever columns shown in these figures are one variation of a more general frame stability model discussed in Appendix A. That is, these columns are, in effect, Model A of Figure A-2 with $P_g = 0$ and $P_m = P$. The bending is applied about the strong-axis of the members. A relatively stocky column ($L_c/r_x = 20$), an intermediate-length column ($L_c/r_x = 40$), and a more slender column ($L_c/r_x = 60$) are considered. For the ELM solution, a second-order analysis is conducted to determine the maximum moment, M_{max}, at the column bases using the nominal EI values and B_2 calculated from Equation A-12 in Appendix A of this Design Guide. The variations in the axial force P versus the moment M_{max} at the column bases under increasing load are shown in Figures 2-1 through 2-3. These curves are labeled as the "Force-point trace (ELM)." The ELM beam-column (P-M) strength interaction curves for these members are also shown in Figures 2-1 through 2-3. These curves are defined by Equations H1-1a and H1-1b of the AISC *Specification*. Note that for the ELM, these strength interaction curves are determined using $K = 2$ in the first term of Equations H1-1a and H1-1b. The members are assumed to be braced sufficiently in the out-of-plane direction such that their in-plane strengths govern. The member capacities can be expressed as the axial force, P, at the intersection of the force-point traces with the strength interaction curves. These member capacities are $P = 585$ kips for the shorter column, $P = 350$ kips for the intermediate-length column, and $P = 196$ kips for the more slender column.

In addition, Figures 2-1 through 2-3 show the force-point traces (P versus M_{max}) from an analysis by the DM along with the beam-column strength interaction curves for the DM. These curves are discussed further along with the ELM results in Chapter 3, where the application of the DM is addressed.

It should be noted that B_2 from Equations B-4 and B-7 is larger than 1.5 where $P > 1{,}110$ kips in Figure 2-1. Therefore, the ELM is applicable for the full range of the loading up to the strength limit at $P = 585$ kips for this problem according to the AISC *Specification*. However, for the case shown in Figure 2-2, $B_2 > 1.5$ for $P > 280$ kips. One should note that the base moment in the ELM analysis is approximately 30% smaller than that determined in the DM analysis at this load level. At the axial force strength limit of $P = 350$ kips, the ELM analysis underestimates the base moment determined from the DM analysis by approximately 40%. For the more slender case shown in Figure 2-3, $B_2 > 1.5$ for $P > 70$ kips. The base moment in the ELM analysis is again approximately 30% smaller than that determined from the DM analysis at this load level. At the axial force strength limit of $P = 196$ kips for this column, the ELM analysis underestimates the base moment determined from the DM analysis by approximately 60%. The reader is referred to White et al. (2006) for example comparisons of the DM analysis results to the results from refined distributed plasticity analysis solutions, where it is shown that the DM produces reasonably good estimates of the internal moments at the maximum strength limit. The sidesway amplification limit of $B_2 \le 1.5$ in the ELM is aimed at providing loose limits on the errors in the internal moments used for design as well as the internal moments transferred to other components such as foundations, beam-to-column connections, and beams in stability critical framing systems.

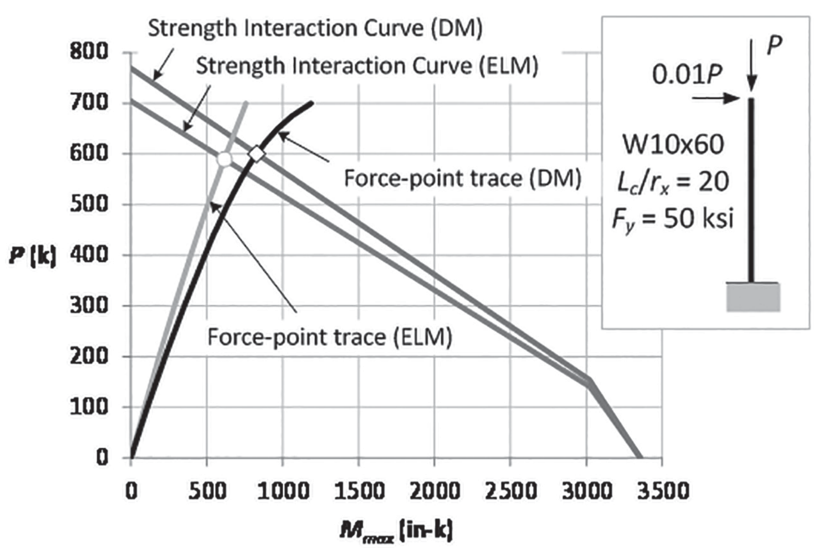


Fig. 2-1. Force-point traces and beam-column strength interaction curves, Model A (stocky column).

2.5 SUMMARY OF DESIGN RECOMMENDATIONS

Following is a summary of design recommendations for application of the ELM.

1. Consider using the effective length method (ELM) when second-order sidesway effects are known to be low (second-order story drift/first-order story drift less than or equal to 1.5, or $B_2 \leq 1.5$, in a model based on the nominal member properties) and when the calculation of effective length factors (K) is easy and straightforward.

2. This traditional method is easy to apply when the frame geometry is rectangular, bay widths and story heights are approximately equal, and there is little irregularity in the frame geometry and loading. That is, the ELM is relatively easy to apply to framing for which the effective length alignment charts of the AISC *Specification* Commentary are applicable.

3. In general, care must be taken in calculating the sidesway effective length factor K_2. It is recommended that the designer use the equations in the AISC *Specification* Commentary to Chapter C [Appendix 7 in the 2010

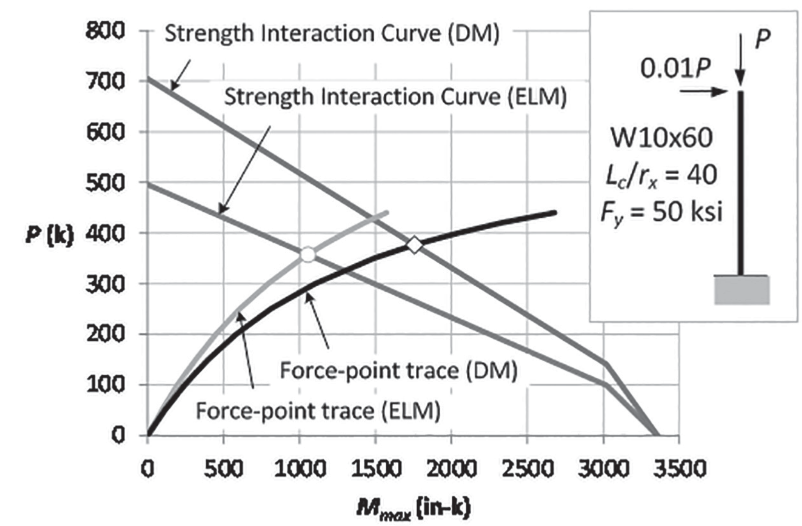


Fig. 2-2. Force-point traces and beam-column strength interaction curves, Model A (intermediate-length column).

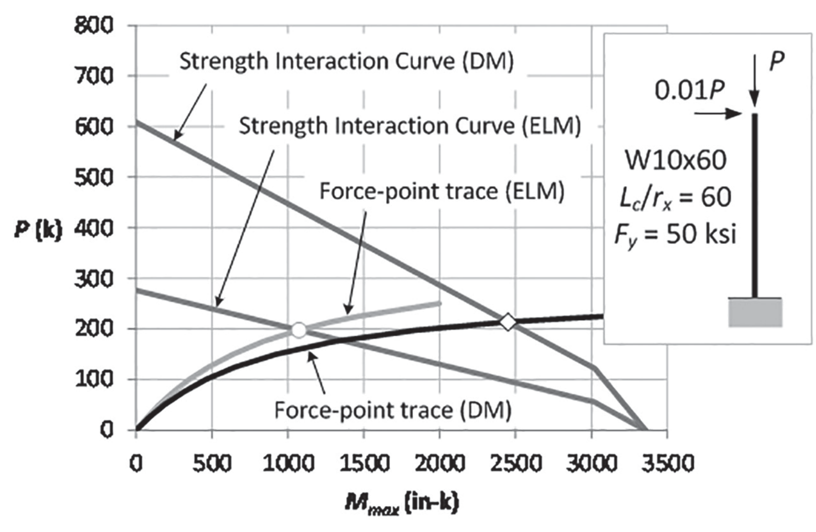


Fig. 2-3. Force-point traces and beam-column strength interaction curves, Model A (slender column).

AISC *Specification* Commentary]. It is also permissible and expedient for complex frames to perform an eigenvalue buckling analysis to determine K_2 values using analysis software. This approach has potential shortcomings and requires experience and judgment.

4. If the ratio of the second-order to first-order drift exceeds 1.5 for any story (i.e., if $B_2 > 1.5$ based on the nominal elastic stiffness of the structure), that story must be stiffened to achieve a value less than or equal to 1.5 or the DM must be used.

5. For braced frames, since K may be taken as 1.0 by definition, the ELM is the preferred method if $B_2 \leq 1.5$.

2.6 DESIGN EXAMPLES

See Section 1.7 for a background discussion relating to the solution of the example problems. The design examples are implemented using the 2005 AISC *Specification* and 13th Edition AISC *Manual.*

Example 2.1—Two-Story Warehouse, Typical Braced Frame Building

Given:

Size the braced frame columns, beams and rod bracing for a typical bay of the braced frame building shown in Figure 2-4. Consider dead, live and wind load combinations using ASCE/SEI 7 load combinations and design by ASD. Consider wind load in the plane of the braced frame only. Solve using the effective length method (ELM). All columns are braced out-of-plane at the floor and the roof. The lateral load resistance is provided by tension rod bracing only. All beam-to-column connections are simple “pinned” connections. Maintain interstory drift limit $\Delta/L \leq 1/100$ under nominal wind load and assume nominal out-of-plumbness of $\Delta_o/L = 0.002$. This is the AISC *Code of Standard Practice* maximum permitted out-of-plumbness. All steel is ASTM A992, except that the tension rods are ASTM A36 steel.

The loading is as follows:

Roof

Dead load, w_{RD}	= 1.0 kip/ft
Live load, w_{RL}	= 1.2 kip/ft
Estimated roof beam weight	= 0.076 kip/ft
Estimated roof interior column weight	= 0.065 kip/ft
Estimated roof end column weight	= 0.048 kip/ft
Wind load, W_R	= 10 kips

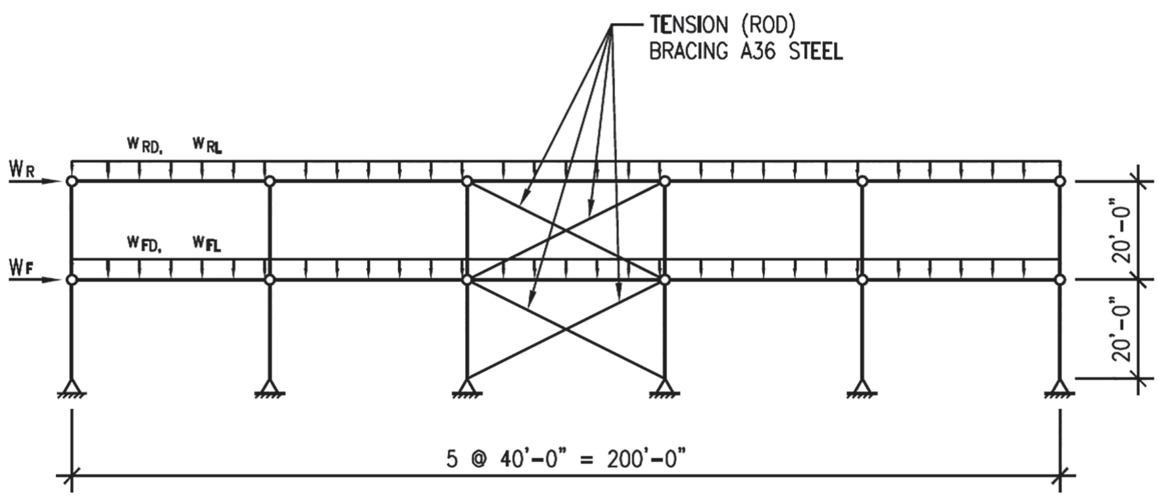


Fig. 2-4. Example 2.1 braced frame elevation.

Floor

Dead load, w_{FD}	= 2.4 kip/ft
Live load, w_{FL}	= 4.0 kip/ft
Estimated floor beam weight	= 0.149 kip/ft
Estimated floor interior column weight	= 0.065 kip/ft
Estimated floor end column weight	= 0.048 kip/ft
Wind load, W_F	= 20 kips

(Note: Wind load on roof surfaces as specified in ASCE/SEI 7 is not considered in this problem.)

Solution:

From AISC *Manual* Table 2-3, the material properties are as follows:

Beams and columns
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Bracing
ASTM A36
F_y = 36 ksi
F_u = 58 ksi

The analysis was performed using a general second-order elastic analysis program including both P-Δ and P-δ effects. See Figure 2-5 for the member labels used. Refer to Section 1.7 for a more detailed discussion of the analysis computer models utilized in this Design Guide. See Section 2.2 for a step-by-step description of the ELM.

Description of Framing

All lateral load resistance is provided by the tension only rod bracing. The tension rods are assumed to be pin connected using a standard clevis and pin (see AISC *Manual* Tables 15-3 and 15-7). Beams within the braced frame are bolted into the column flanges using double angles (see AISC *Manual* Figure 13-2(a)). A single gusset plate connecting the tension rod is shop welded to the beam flange and field bolted to the column flange (see AISC *Manual* Figure 13-2(a)). All other columns outside the braced frames are leaning columns with simple beam-to-column connections.

Design Approach

Design is an iterative process. Preliminary sizes should be chosen based on experience or a preliminary analysis. Braced frame structures are often controlled by strength as opposed to wind or seismic drift. Thus, for this problem, the sizes are estimated from a preliminary strength check and then used in the computer analysis. The member sizes used in the following analyses are those shown in Figure 2-6.

Analysis Load Combinations

The member design forces are obtained by analyzing the structure for 1.6 times ASD load combinations and then dividing the results by 1.6. It should be noted that the notional loads already include, by definition, the 1.6 amplification. The load combinations from ASCE/SEI 7 Section 2.4.1 used for the second-order analysis are given in Table 2-1.

If the B_1-B_2 approach is used to account for second-order effects, the analysis need not include the 1.6 multiplier on either the load combinations or in the determination of notional loads since this is already included in the B_1-B_2 calculation.

Note that since the structure loading and geometry are symmetric and symmetry of the frame is enforced in the member selection, the wind load is considered in only one direction.

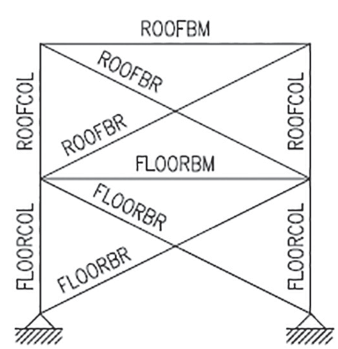


Fig. 2-5. Braced frame member labels.

Table 2-1. 1.6 Times ASD Load Combinations

Comb1 = $1.6(D) + N$
Comb2 = $1.6(D + L) + N$
Comb3 = $1.6(D + L_r) + N$
Comb4 = $1.6(D + 0.75L + 0.75L_r) + N$
Comb5 = $1.6(D + W)$
Comb6 = $1.6(D + 0.75W + 0.75L + 0.75L_r)$
Comb7 = $1.6(0.6D + W)$
D = dead load L = live load L_r = roof live load W = wind load N = minimum lateral load for gravity-only combinations = $0.002Y_i$ Y_i = gravity load from 1.6 times ASD load combinations applied at level i

Assume that drift from a second-order analysis is less than 1.5 times the drift from a first-order analysis at both levels of the structure so that the ELM applies. This assumption must be checked after the analysis is complete. The provisions in AISC *Specification* Section C2.2a apply.

Minimum Lateral Loads ($N = 0.002Y_i$)

These loads are to be applied together with gravity-only load combinations. The gravity loads are:

Roof

$$\Sigma D = (1.0\text{ kip/ft})(200\text{ ft}) + (0.076\text{ kip/ft})(200\text{ ft}) + (0.065\text{ kip/ft})(20\text{ ft})(4) + (0.048\text{ kip/ft})(20\text{ ft})(2)$$
$$= 222\text{ kips}$$
$$\Sigma L_r = (1.2\text{ kip/ft})(200\text{ ft})$$
$$= 240\text{ kips}$$

Floor

$$\Sigma D = (2.4\text{ kip/ft})(200\text{ ft}) + (0.149\text{ kip/ft})(200\text{ ft}) + (0.065\text{ kip/ft})(20\text{ ft})(4) + (0.048\text{ kip/ft})(20\text{ ft})(2)$$
$$= 517\text{ kips}$$
$$\Sigma L = (4.0\text{ kip/ft})(200\text{ ft})$$
$$= 800\text{ kips}$$

The notional load is determined from $N = 0.002Y_i$. N needs to be evaluated for each load combination. For example, at the roof level for Comb3:

$$N = 0.002Y_i \tag{1-1}$$
$$= 0.002(1.6D + 1.6L_r)$$

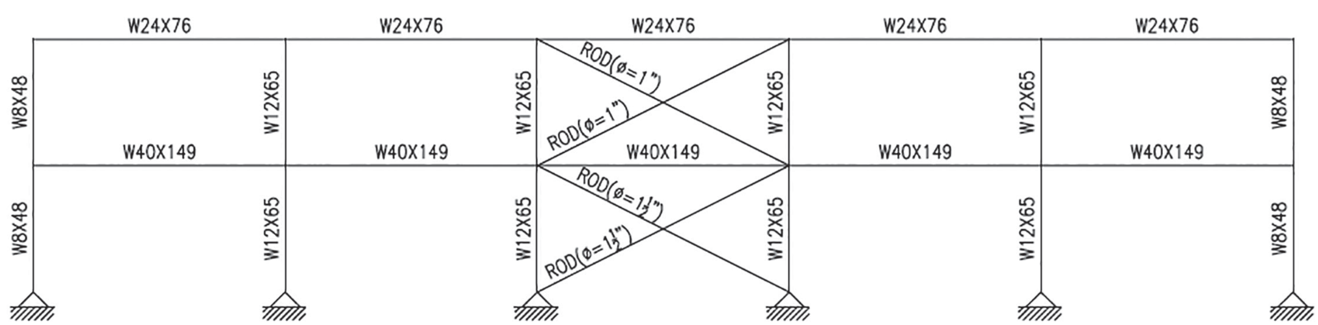


Fig. 2-6. Final member sizes using the effective length method.

Table 2-2. Notional Load, N_i, for each Gravity Load Combination, kips				
	Comb1	**Comb2**	**Comb3**	**Comb4**
ROOF	0.710	0.710	1.48	1.29
FLOOR	1.65	4.21	1.65	3.57

Table 2-3. Required Beam Flexural Strength (ASD)							
Member	**M_r, kip-in.**						
	Comb1	**Comb2**	**Comb3**	**Comb4**	**Comb5**	**Comb6**	**Comb7**
ROOFBM	2580	2580	**5470**	4750	2600	4770	1560
FLOORBM	6120	**15700**	6120	13300	6150	13400	3690

$$= 0.002[1.6(222 \text{ kips}) + 1.6(240 \text{ kips})]$$
$$= 1.48 \text{ kips}$$

Minimum lateral loads for Comb1 through Comb4 are listed in Table 2-2.

Design of Beams

From a second-order analysis, member required flexural strengths for each load combination are tabulated in Table 2-3. The required flexural strength from the controlling load combination is presented in bold. Note that the member flexural strengths in the table are for ASD and were obtained by dividing the analysis results by 1.6.

The beam flexural strengths shown in Table 2-3 are slightly larger than $wL^2/8$ due to magnification of the first-order moments by the axial compression in the beams. However, the beams are designed here for gravity load alone without considering the axial load from the lateral notional load. The amount of axial load in the beams depends on the assumed load path of the lateral forces, which are applied to the lateral load resisting frames (through collector beams and/or floor and roof diaphragm connections to the beams). Even if a conservative estimate of the axial force in these members is included, the influence on the beam-column interaction value for these beams is small. The beams are assumed fully braced by the floor and roof diaphragm.

Roof Beams

The required strength of the roof beams is:

$$M_r = 5{,}470 \text{ kip-in.}/12 \text{ in./ft}$$
$$= 456 \text{ kip-ft } (\text{Comb3})$$

Try a W24×76.

From Table 3-2 of the AISC *Manual*, for a fully braced beam, the allowable flexural strength is:

$$\frac{M_n}{\Omega_b} = \frac{M_{px}}{\Omega_b}$$
$$= 499 \text{ kip-ft} > M_r \quad \textbf{o.k.}$$

Floor Beams

The required strength of the floor beams is:

$$M_r = 15{,}700 \text{ kip-in.}/12 \text{ in./ft}$$
$$= 1{,}310 \text{ kip-ft } (\text{Comb2})$$

Table 2-4. Required Column Axial Strength							
Member	P_r, kip						
	Comb1	Comb2	Comb3	Comb4	Comb5	Comb6	Comb7
ROOFCOL	43.7	43.7	**91.7**	79.7	43.7	79.7	26.2
FLOORCOL	147	**307**	194	302	142	299	83.0

Try a W40×149.

From Table 3-2 of the AISC *Manual*, for a fully braced beam, the allowable flexural strength is:

$$\frac{M_n}{\Omega_b} = \frac{M_{px}}{\Omega_b}$$
$$= 1{,}490 \text{ kip-ft} > M_r \quad \textbf{o.k.}$$

Design of Columns in the Braced Frame

Member required axial compressive strengths for each load combination are tabulated in Table 2-4 with the controlling value in bold.

Roof Columns

The required strength of the roof columns is:

$P_r = 91.7$ kips (Comb3)
$K = 1.0$ for braced frame columns

Try a W8×31.

From AISC *Manual* Table 4-1 with $KL = 20$ ft, the allowable axial compressive strength is:

$$\frac{P_n}{\Omega_c} = 97.1 \text{ kips} > P_r \quad \textbf{o.k.}$$

Floor Columns

The required strength of the floor columns is:

$P_r = 307$ kips (Comb2)
$K = 1.0$ for braced frame columns

Try a W12×65.

From AISC *Manual* Table 4-1 with $KL = 20$ ft, the allowable axial compressive strength is:

$$\frac{P_n}{\Omega_c} = 360 \text{ kips} > P_r \quad \textbf{o.k.}$$

Design of Braces

Member required tensile strengths for each load combination are tabulated in Table 2-5 with the controlling value in bold. (Note: The small values shown for the gravity load only load combinations come from the minimum lateral loads, N):

Roof Braces

The required tensile strength of the roof braces is:

$P_r = 11.8$ kips (Comb5)

Table 2-5. Required Brace Axial Strength							
	P_r, kips						
Member	Comb1	Comb2	Comb3	Comb4	Comb5	Comb6	Comb7
ROOFBR	0.535	0.536	1.24	1.05	**11.8**	9.26	11.5
FLOORBR	1.89	4.58	2.62	4.50	**36.2**	30.0	35.0

Try a 1-in.-diameter rod (ASTM A36 steel).

The allowable tensile yielding strength is determined from AISC *Specification* Section D2(a) as follows:

$$
\begin{aligned}
\frac{P_n}{\Omega_t} &= \frac{F_y A_g}{\Omega_t} \\
&= \frac{(36\text{ ksi})(\pi/4)(1.00\text{ in.})^2}{1.67} \\
&= 16.9\text{ kips} > P_r \quad \textbf{o.k.}
\end{aligned}
$$

The allowable tensile rupture strength is determined from AISC *Specification* Section D2(b), assuming that the effective net area of the rod is $0.75A_g$, as follows:

$$
\begin{aligned}
\frac{P_n}{\Omega_t} &= \frac{F_u A_e}{\Omega_t} \\
&= \frac{(58\text{ ksi})(0.75)(\pi/4)(1.00\text{ in.})^2}{2.00} \\
&= 17.1\text{ kips} > P_r \quad \textbf{o.k.}
\end{aligned}
$$

Floor Braces

The required tensile strength of the floor braces is:

$P_r = 36.2$ kips (Comb5)

Try a 1½-in.-diameter rod (ASTM A36 steel)

The allowable tensile yielding strength is determined from AISC *Specification* Section D2(a) as follows:

$$
\begin{aligned}
\frac{P_n}{\Omega_t} &= \frac{F_y A_g}{\Omega_t} \\
&= \frac{(36\text{ ksi})(\pi/4)(1\tfrac{1}{2}\text{ in.})^2}{1.67} \\
&= 38.1\text{ kips} > P_r \quad \textbf{o.k.}
\end{aligned}
$$

The allowable tensile rupture strength is determined from AISC *Specification* Section D2(b), assuming that the effective net area of the rod is $0.75A_g$, as follows:

$$\frac{P_n}{\Omega_t} = \frac{F_u A_e}{\Omega_t}$$
$$= \frac{(58\text{ ksi})(0.75)(\pi/4)(1\tfrac{1}{2}\text{ in.})^2}{2.00}$$
$$= 38.4\text{ kips} > P_r \quad \textbf{o.k.}$$

Check Drift under Nominal Wind Load (Serviceability Wind Drift Check)

Drift is checked under a serviceability load combination (nominal dead + nominal live + nominal wind). The nominal wind load is specified in this problem to be used in the serviceability load combination and the interstory drift is limited based on $\Delta/L \leq 1/100$. The appropriate serviceability load combination is a matter of engineering judgment. The drift check is made using a second-order analysis as recommended in this Design Guide when a direct check against damage is to be considered. The resulting drift at the roof level is 0.913 in. and 0.539 in. at the floor level. The interstory drift is checked as follows:

Floor

$$\Delta/L = 0.539\text{ in.}/240\text{ in.}$$
$$= 0.00225 < 0.01\ (=\ 1/100) \quad \textbf{o.k.}$$

Roof

$$\Delta/L = (0.913\text{ in.} - 0.539\text{ in.})/240\text{ in.}$$
$$= 0.00156 < 0.01\ (=\ 1/100) \quad \textbf{o.k.}$$

Check Magnitude of Second-Order Effects after Finalizing the Design

Second-order effects are checked for the controlling load combination $1.6(D + 0.75W + 0.75L + 0.75L_r)$; however, all load combinations must be checked. A computer program is used to find the first-order and second-order deflections.

Roof

$$\Delta_{2nd}/\Delta_{1st} = (1.16\text{ in.} - 0.689\text{ in.})/(1.02\text{ in.} - 0.595\text{ in.})$$
$$= 1.11 \leq 1.5 \quad \textbf{o.k.}$$

Floor

$$\Delta_{2nd}/\Delta_{1st} = 0.689\text{ in.}/0.595\text{ in.}$$
$$= 1.16 \leq 1.5 \quad \textbf{o.k.}$$

Therefore, the initial assumption that drift from a second-order analysis is less than 1.5 times the drift from a first-order analysis at both levels of the structure is confirmed.

Note: It is common practice to use two-story columns in tiered buildings to minimize splice connection costs. Thus, in this problem, the first floor columns are extended to the roof to omit the cost of the extra splice.

Observations

1. Second-order effects in the braced frame columns and braces are small but not insignificant for this problem. This is typical for many low-rise braced frame structures.
2. Drift does not control the design for this frame. This is typical of many braced frame structures in low-rise buildings.
3. The design of the tension bracing is governed by the strength requirements associated with one of the lateral load combinations. This is expected in braced frame structures.

4. The destabilizing effect of leaning columns should always be included in the design of the lateral load resisting system, regardless of the type of framing. In this example, the influence of the leaning columns is included through the second-order analysis which included all of the leaning columns and their loads.

Example 2.2—Large One-Story Warehouse Building

Given:

Design the braced frames and moment frames in the warehouse building shown in Figure 2-7 for gravity, snow and wind loads as specified by ASCE/SEI 7 load combinations using LRFD. Use the effective length method (ELM). Maintain columns no larger than the W24 series and beams no larger than the W30 series. Assume out-of-plumbness, $\Delta_o/L = 0.002$, the maximum permitted by the AISC *Code of Standard Practice.* Design for interstory drift control, $\Delta/L = 1/100$, for nominal wind load. Use ASTM A992 steel for wide-flange shapes and ASTM A36 steel for tension X-bracing. Assume the roof deck provides a rigid diaphragm and that the outside walls are light metal panels that span vertically between the roof and the ground floor. (Note: This defines the wind load path to the lateral load resisting system.)

The loading is as follows:

Dead load, D = 25 psf (not including steel self-weight of roof beams, columns, braced and moment frames)
Snow load, S = 30 psf
Wind load, W = 20 psf

(Note: Wind load on roof surfaces as required by ASCE/SEI 7 is not addressed in this problem.)

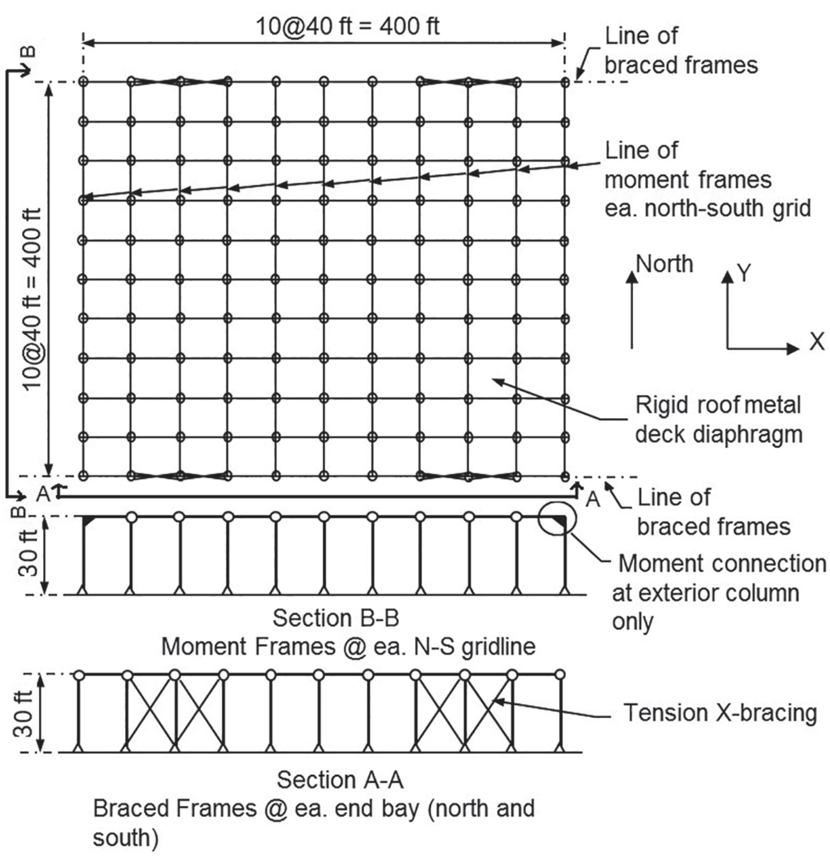


Fig. 2-7. Example 2.2 plan and sections.

Table 2-6. Preliminary Member Sizes	
Member	**Size**
Tension Brace (T1, T2)	1¼-in.-diameter rod
Moment Frame Beam (B1)	W24×131
Moment Frame Column (C1)	W24×117

Solution:

From AISC *Manual* Table 2-3, the material properties are as follows:

Beams and columns
ASTM A992
$F_y = 50$ ksi
$F_u = 65$ ksi

Bracing
ASTM A36
$F_y = 36$ ksi
$F_u = 58$ ksi

The analysis was performed using a general second-order elastic analysis program including both P-Δ and P-δ effects. Refer to Section 1.7 for a more detailed discussion of the analysis computer models utilized in this Design Guide. See Section 2.2 for a step-by-step description of the ELM.

Description of Framing

All lateral load resistance in the east-west direction is provided by tension-only X-bracing in the north and south end bays as specified in Section A-A of Figure 2-7. All lateral load resistance in the north-south direction is provided by moment frames on each north-south column line. A moment connection is provided between the exterior column and beam at the end bay of each north-south frame as shown in Section B-B of Figure 2-7. Assume that this moment connection is field welded with a complete-joint-penetration groove weld at the beam flange-to-column flange connection. A beam-to-column web connection is provided using a bolted single-plate connection (see AISC *Manual* Figure 12-4(a) for moment connection detail). The tension rod-to-gusset connections are assumed to be pinned connections using a standard clevis and pin (see AISC *Manual* Tables 15-3 and 15-7). Beams within the braced frame are bolted into the column flanges using double angles (see AISC *Manual* Figure 13-2(a)). A single gusset plate connecting the tension rod is shop welded to the beam flange and field bolted to the column flange (see AISC *Manual* Figure 13-2(a)). All other columns outside the braced frames and moment frame bays are leaning columns with simple beam-to-column connections.

Design Approach

Design is an iterative process. The first step is to estimate member sizes that are used in an analysis and then check for conformance to the strength requirements of the AISC *Specification* and serviceability drift limits that are established by the designer. In this type of building, braced frames are often controlled by strength limit states while moment frames are controlled by stiffness or drift. The selection of preliminary sizes is not shown but is obtained based on consideration of service load drift for the moment frames and strength for the braced frames. The preliminary sizes for the members defined in Figures 2-8 and 2-9 are given in Table 2-6.

Estimate of Framing Weight

Assume that roof purlins at 6 ft 8 in. center-to-center spacing in the east-west direction span to girders in the north-south direction located at column lines. Assume 7 psf self-weight for all steel framing, including purlins, columns, beams and braces. (Note: Use this weight for calculating the minimum lateral loads.)

The estimated dead load is:
Dead load = 25 psf + 7 psf = 32 psf

(Note: The weights specified include an allowance for all steel framing and exterior cladding.)

Analysis

Perform a second-order elastic analysis with a computer program that accurately includes both P-Δ and P-δ effects. The second-order effects are evaluated using LRFD load combinations. Refer to Section 1.7 for more details of the computer model analysis.

Before applying the minimum lateral loads as required for the ELM, verify the following:

1. Structure meets the interstory drift limit, $\Delta/L = 1/100$, under nominal wind load
2. $\Delta_{2nd}/\Delta_{1st} \le 1.5$ (AISC *Specification* Section C2.2)

The ELM can be used only if item 2 above is satisfied.

Serviceability Drift Limit

Wind drift for the moment frames in the north-south direction is evaluated first since drift often controls the sizes in moment frames. Wind drift is evaluated using the following serviceability load combination, where W is the specified nominal wind load:

$$1.0D + 0.5S + 1.0W$$

Roof deflections are shown below from a computer analysis for wind in the north-south direction:

$$\Delta_{1st} = 1.85 \text{ in.}$$
$$\Delta_{2nd} = 2.74 \text{ in.}$$
$$< 360 \text{ in.}/100 \text{ specified drift limit} = 3.60 \text{ in.} \quad \textbf{o.k.}$$

Note that, for the purpose of calculating wind drift, the nominal wind load, W, is used as required in the problem statement. Wind drift limits and the appropriate serviceability load combination are not specified in the building code and are a matter of engineering judgment. A second-order analysis is used in the wind drift calculation to more accurately determine the damage potential. Note that there is a 48% increase in wind drift from second-order effects as a result of the gravity load from the large number of leaning columns acting on the frame.

It is assumed that wind drift for the braced frames in the east-west direction will be small and the sizes controlled by strength.

LRFD Load Combinations

A complete list of all the load combinations considered from ASCE/SEI 7 Section 2.3 for both orthogonal (x-, y-) directions is shown in Table 2-7. Note that the ELM only requires a minimum lateral load to be included for the gravity-only load combinations. The load combinations shown include positive and negative directions for the lateral loads to allow automated design and eliminate the need for symmetry of loading and member sizes. For design that is performed manually, the number of load combinations can be reduced by half in this frame as long as symmetry of loading and member sizes is maintained in the analysis and design.

Check Magnitude of Second-Order Effects

Verify that $\Delta_{2nd}/\Delta_{1st} \le 1.5$ (AISC *Specification* Section C2.2). This check should be made with a model using the nominal properties of the members. For this structure, the check is carried out in the y-direction, the direction of the moment frames, because the second-order effects in this direction are expected to be greater than in the braced frame direction.

The results of the second-order analysis for selected load combinations are given in Table 2-8.

It is clear from Table 2-8 that the $\Delta_{2nd}/\Delta_{1st}$ limit of 1.5 is not satisfied with the initial member sizes. Therefore, second-order effects must be reduced if the effective length method is to be used. Alternatively, the direct analysis method could be used as shown in Example 3.2 in Chapter 3. In this example, the member sizes are increased to meet the 1.5 limit for application of the ELM and then the analysis is repeated. The new moment frame member sizes for the members defined in Figure 2-8 and Figure 2-9 that satisfy the $\Delta_{2nd}/\Delta_{1st} = 1.5$ limit are given in Table 2-9. Note that the braces are not changed since it is expected that the limit of 1.5 in this direction will be satisfied. These sections are used for the subsequent strength design checks.

Minimum Lateral Loads: N_x, N_y

Notional loads are applied as minimum lateral loads for each gravity load combination shown in Table 2-7. The total notional load for two typical load combinations is determined as follows:

Table 2-7. LRFD Load Combinations
Comb1 = 1.4(D) + N_x
Comb2 = 1.4(D) − N_x
Comb3 = 1.4(D) + N_y
Comb4 = 1.4(D) − N_y
Comb5 = 1.2(D) + 1.6(S) + N_x
Comb6 = 1.2(D) + 1.6(S) − N_x
Comb7 = 1.2(D) + 1.6(S) + N_y
Comb8 = 1.2(D) + 1.6(S) − N_y
Comb9 = 1.2(D) + 0.5(S) + 1.6(W_x)
Comb10 = 1.2(D) + 0.5(S) − 1.6(W_x)
Comb11 = 1.2(D) + 0.5(S) + 1.6(W_y)
Comb12 = 1.2(D) + 0.5(S) − 1.6(W_y)
Comb13 = 1.2(D) + 1.6(S) + 0.8(W_x)
Comb14 = 1.2(D) + 1.6(S) − 0.8(W_x)
Comb15 = 1.2(D) + 1.6(S) + 0.8(W_y)
Comb16 = 1.2(D) + 1.6(S) − 0.8(W_y)
Comb17 = 0.9(D) + 1.6W_x
Comb18 = 0.9(D) − 1.6W_x
Comb19 = 0.9(D) + 1.6W_y
Comb20 = 0.9(D) − 1.6W_y
x, y = direction of forces in plan x = east-west braced frame direction y = north-south moment frame direction D = nominal dead load S = nominal snow load W_x, W_y = nominal wind load in x-, y-direction N_x, N_y = minimum lateral load in x-, y-direction Note: Because this structure is not sensitive to quartering winds, those load combinations are not included in this summary for brevity.

Table 2-8. Second-order Displacement Check			
	Δ_{1st} (***y*****-direction)**	Δ_{2nd} (***y*****-direction)**	
Load Combination	**in.**	**in.**	$\Delta_{2nd}/\Delta_{1st}$
Comb3	0.221	0.320	1.45
Comb4	0.221	0.320	1.45
Comb7	0.427	1.05	2.47
Comb8	0.427	1.05	2.47
Comb11	2.96	4.69	1.58
Comb12	2.96	4.69	1.58
Comb15	1.48	3.66	2.47
Comb16	1.48	3.66	2.47
Comb19	2.96	3.70	1.25
Comb20	2.96	3.70	1.25

Table 2-9. Effective Length Method Final Sizes	
Member	**Size**
Tension Brace (T1, T2)	1¼-in.-diameter rod
Moment Frame Beam (B1)	W30×173
Moment Frame Column (C1)	W24×176

Comb5: $1.2D + 1.6S + N_x$

$$
\begin{aligned}
D &= (32\text{ psf})(400\text{ ft})(400\text{ ft})/(1{,}000\text{ lb / kip}) \\
&= 5{,}120\text{ kips} \\
S &= (30\text{ psf})(400\text{ ft})(400\text{ ft})/(1{,}000\text{ lb / kip}) \\
&= 4{,}800\text{ kips} \\
N_x &= 0.002\left[1.2(5{,}120\text{ kips}) + 1.6(4{,}800\text{ kips})\right] \\
&= 27.6\text{ kips}
\end{aligned}
$$

Comb7: $1.2D + 1.6S + N_y$

$$
\begin{aligned}
D &= (32\text{ psf})(400\text{ ft})(400\text{ ft})/(1{,}000\text{ lb / kip}) \\
&= 5{,}120\text{ kips} \\
S &= (30\text{ psf})(400\text{ ft})(400\text{ ft})/(1{,}000\text{ lb / kip}) \\
&= 4{,}800\text{ kips} \\
N_y &= 0.002\left[1.2(5{,}120\text{ kips}) + 1.6(4{,}800\text{ kips})\right] \\
&= 27.6\text{ kips}
\end{aligned}
$$

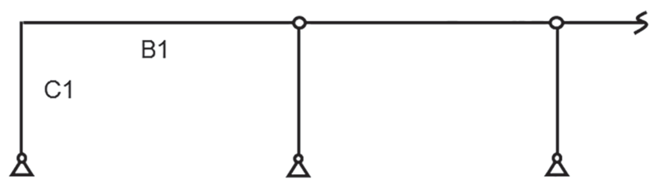


Fig. 2-8. North-south moment frame.

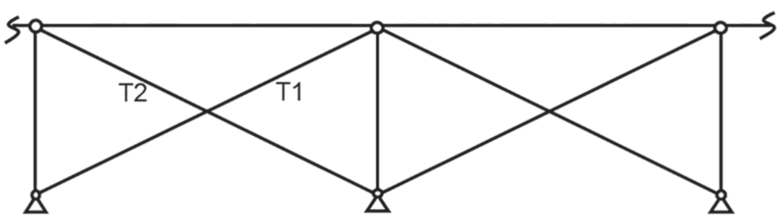


Fig. 2-9. East-west braced frame.

Table 2-10. Effective Length Method Member Forces—North-South Moment Frame					
Combination		C1		B1	
		P_r		P_r	
		$M_{r\text{-}i}$	$M_{r\text{-}j}$	$M_{r\text{-}i}$	$M_{r\text{-}m}$
Comb3	**P_r, kips**	-40		0	
	M_r, kip-in.	0	1800	-1800	3400
Comb4	**P_r, kips**	-41		0	
	M_r, kip-in.	0	2360	-2360	3120
Comb7	**P_r, kips**	-76		0	
	M_r, kip-in.	0	3340	-3340	6620
Comb8	**P_r, kips**	-79		0	
	M_r, kip-in.	0	4680	-4680	5960
Comb11	**P_r, kips**	-40		0	
	M_r, kip-in.	0	-1450	1450	5850
Comb12	**P_r, kips**	-56		0	
	M_r, kip-in.	0	6400	-6400	1930
Comb15	**P_r, kips**	-73		**0**	
	M_r, kip-in.	0	1690	**-1690**	**7450**
Comb16	**P_r, kips**	**-82**		0	
	M_r, kip-in.	**0**	**6330**	-6330	5130
Comb19	**P_r, kips**	-18		0	
	M_r, kip-in.	0	-2180	2180	3860
Comb20	**P_r, kips**	-33		0	
	M_r, kip-in.	0	4860	-4860	337

Member Forces for Strength Design

Tables 2-10 and 2-11 provide the member forces and moments determined from the second-order analysis for the appropriate load combinations.

The moment shown for the column includes M_r at end i and end j of the member. Note that column moments at the base are zero in the table because of the assumption of a pinned end at the base. The moment shown for the beam includes M_r at end i and at midspan m (end j moments are zero). The axial force in the beams is relatively small and is neglected.

The moment frame with the final member sizes as given in Table 2-9 is checked for the second-order analysis limit, $\Delta_{2nd}/\Delta_{1st} \leq$ 1.5. The results of this check are shown in Table 2-12. Note that the $\Delta_{2nd}/\Delta_{1st}$ limit is satisfied for all load combinations.

Representative Member Strength Design Checks

Member checks are given in the following for representative members using the final member sizes determined such that $B_2 \leq 1.5$ in all the load combinations for the moment frames, and based on satisfaction of the strength requirements for the braced frames.

North-South Moment Frame, Typical Interior Frame, Column C1

The governing load combination is Comb16 from Table 2-10, where $P_r = 82$ kips (compression) and $M_r = 6{,}330$ kip-in.

From AISC *Manual* Table 1-1, the W24×176 is not slender for compression and its properties are as follows:

$A_g = 51.7 \text{ in.}^2$
$I_x = 5{,}680 \text{ in.}^4$

Table 2-11. Effective Length Method Member Forces—East-West Braced Frame		
Combination	**T1, kips**	**T2, kips**
Comb1	2.40	0.00
Comb2	0.00	2.40
Comb5	4.99	0.00
Comb6	0.00	4.99
Comb9	**32.3**	0.00
Comb10	0.00	**32.3**
Comb13	17.1	0.00
Comb14	0.00	17.1
Comb17	31.2	0.00
Comb18	0.00	31.2

r_x = 10.5 in.
r_y = 3.04 in.

Determine the design compressive strength of the column. The effective length factor, K, about the x-axis is associated with lateral story buckling and, therefore, is determined using AISC *Specification* Commentary Equation C-C2-5.

$$\begin{aligned}\Sigma P_r \text{ all columns} &= (400\text{ ft})(400\text{ ft})\left[(1.2)(32\text{ psf})+(1.6)(30\text{ psf})\right]/(1{,}000\text{ lb/kip})\\ &= 13{,}800\text{ kips}\\ \Sigma P_r \text{ leaning columns} &= 13{,}800\text{ kips}-1{,}550\text{ kips}\\ &= 12{,}250\text{ kips}\end{aligned}$$

where 1,550 kips is the portion of gravity load on the moment frame columns as determined from a frame analysis.

$\Sigma H = 96.0$ kips with $\Delta_H = 0.806$ in. for first-order analysis of Comb16 (Table 2-12)

$$\begin{aligned}H &= (96.0\text{ kips})/(22\text{ columns})\\ &= 4.36\text{ kips}\end{aligned}$$

$$\begin{aligned}R_L &= \frac{\Sigma P_r \textit{ leaning columns}}{\Sigma P_r \textit{ all columns}}\\ &= \frac{12{,}250\text{ kips}}{13{,}800\text{ kips}}\\ &= 0.888\end{aligned}$$

The effective length factor in the x-direction is:

$$K_2 = \sqrt{\frac{\Sigma P_r}{(0.85+0.15R_L)P_r}\left(\frac{\pi^2 EI}{L^2}\right)\left(\frac{\Delta_H}{\Sigma HL}\right)} \geq \sqrt{\frac{\pi^2 EI}{L^2}\left(\frac{\Delta_H}{1.7HL}\right)} \qquad (\textit{Spec}.\text{ Comm. Eq. C-C2-5})$$

$$\begin{aligned}&= \sqrt{\frac{13{,}800\text{ kips}}{\left[0.85+0.15(0.888)\right](82\text{ kips})}\left[\frac{(\pi^2)(29{,}000\text{ ksi})(5{,}680\text{ in.}^4)}{(360\text{ in.})^2}\right]\left[\frac{0.806\text{ in.}}{(96\text{ kips})(360\text{ in.})}\right]}\\ &= 7.08\end{aligned}$$

Table 2-12. Second-Order Displacement Check in North-South Direction			
Load Combination	Δ_{1st} in.	Δ_{2nd} in.	$\Delta_{2nd}/\Delta_{1st}$
Comb3	0.120	0.145	1.21
Comb4	0.120	0.145	1.21
Comb7	0.232	0.344	1.48
Comb8	0.232	0.344	1.48
Comb11	1.61	2.02	1.25
Comb12	1.61	2.02	1.25
Comb15	0.806	1.19	1.48
Comb16	0.806	1.19	1.48
Comb19	1.61	1.81	1.12
Comb20	1.61	1.81	1.12

$$\begin{aligned}\sqrt{\frac{\pi^2 EI}{L^2}\left(\frac{\Delta_H}{1.7HL}\right)} &= \sqrt{\frac{(\pi^2)(29{,}000\text{ ksi})(5{,}680\text{ in.}^4)}{(360\text{ in.})^2}\left[\frac{0.806\text{ in.}}{1.7(4.36\text{ kips})(360\text{ in.})}\right]}\\ &= 1.95\end{aligned}$$

Therefore, $K_x = K_2 = 7.08$. The effective length is:

$$\begin{aligned}K_xL_x &= 7.08(360\text{ in.})\\ &= 2{,}550\text{ in.}\end{aligned}$$

The slenderness ratio is:

$$\begin{aligned}K_xL_x/r_x &= 2{,}550\text{ in.}/10.5\text{ in.}\\ &= 243\end{aligned}$$

The effective length factor about the y-axis is taken as $K_y = 1.0$ because the column is not contributing to the lateral stability in the y-direction.

$$\begin{aligned}K_yL_y/r_y &= 1.0(360\text{ in.})/(3.04\text{ in.})\\ &= 118\end{aligned}$$

The x-axis slenderness ratio controls; therefore, the design compressive strength is determined as follows from AISC *Specification* Section E3:

$$\begin{aligned}F_e &= \frac{\pi^2 E}{(KL/r_x)^2} \qquad \text{(\textit{Spec}. Eq. E3-4)}\\ &= \frac{(\pi^2)(29{,}000\text{ ksi})}{(243)^2}\\ &= 4.85\text{ ksi}\end{aligned}$$

$$\frac{F_y}{F_e} = \frac{50 \text{ ksi}}{4.85 \text{ ksi}}$$
$$= 10.3 > 2.25$$

Therefore, the following equation applies:

$$F_{cr} = 0.877F_e \qquad \text{(\textit{Spec}. Eq. E3-3)}$$
$$= 0.877(4.85 \text{ ksi})$$
$$= 4.25 \text{ ksi}$$

The nominal compressive strength is:

$$P_n = F_{cr}A_g \qquad \text{(\textit{Spec}. Eq. E3-1)}$$
$$= (4.25 \text{ ksi})(51.7 \text{ in.}^2)$$
$$= 220 \text{ kips}$$

The design compressive strength as defined in AISC *Specification* Section E1 is:

$$\phi_c P_n = 0.90(220 \text{ kips})$$
$$= 198 \text{ kips} > 82.0 \text{ kips} \quad \textbf{o.k.}$$

Determine the design flexural strength of the column. From AISC *Manual* Table 3-2, for a W24×176:

L_p = 10.7 ft
L_r = 37.4 ft
L_b = 30 ft
$\phi_b M_p$ = 1,920 kip-ft
BF = 27.6 kips

The required flexural strength at end i is $M_{ri} = 0$, and the required flexural strength at end j is M_{rj} = 6,330 kip-in.

Because L_p < 30 ft < L_r, use the following equation and the variables from AISC *Manual* Table 3-2 to interpolate between the available strength at L_p and the available strength at L_r. From AISC *Manual* Table 3-1, C_b = 1.67 (note that there is a linear moment diagram between the support and the brace). Therefore, the design flexural strength is:

$$\phi_b M_n = C_b\left[\phi_b M_{px} - BF\left(L_b - L_p\right)\right] \le \phi_b M_{px}$$
$$= 1.67\left[1{,}920 \text{ kip-ft} - (27.6 \text{ kips})(30.0 \text{ ft} - 10.7 \text{ ft})\right]$$
$$= 2{,}320 \text{ kip-ft}$$

Because 2,320 kip-ft > $\phi_b M_{px}$, then $\phi_b M_n = \phi_b M_{px}$ =1,920 kip-ft.

$$\phi M_n = (1{,}920 \text{ kip-ft})(12 \text{ in./ft})$$
$$= 23{,}000 \text{ kip-in.} > 6{,}330 \text{ kip-in.} \quad \textbf{o.k.}$$

Check the interaction of compression and flexure using AISC *Specification* Section H1:

$$\frac{P_r}{P_c} = \frac{82.0 \text{ kips}}{198 \text{ kips}}$$
$$= 0.414 > 0.2$$

Therefore use AISC *Specification* Equation H1-1a:

$$\frac{P_r}{P_c}+\frac{8}{9}\left(\frac{M_{rx}}{M_{cx}}+\frac{M_{ry}}{M_{cy}}\right)\le 1.0 \qquad \text{(\textit{Spec}. Eq. H1-1a)}$$

$$0.414+\frac{8}{9}\left(\frac{6{,}330\text{ kip-in.}}{23{,}000\text{ kip-in.}}+0\right)=0.659\le 1.0 \quad \textbf{o.k.}$$

North-South Moment Frame, Beam B1

The governing load combination for member B1 is Comb15 from Table 2-10 where the midspan moment is $M_{r\text{-}m}$ = 7,450 kip-in.

From AISC *Manual* Table 3-2, for a W30×173:

L_p = 12.1 ft
L_r = 35.5 ft
L_b = 6.67 ft (purlin spacing) < L_p = 12.1 ft
$\phi_b M_{px}$ = 2,280 kip-ft

Therefore, $\phi_b M_n = \phi_b M_p$ = 2,280 kip-ft, according to AISC *Specification* Section F2.

$\phi_b M_n$ = 2,280 kip-ft (12 in./ft)

= 27,400 kip-in. > M_r = 7,450 kip-in. **o.k.**

East-West Braced Frame, Tension Only Member, T1

The required compressive strength of the 1¼-in.-diameter rod is:

P_r = 32.3 kips for Comb9 from Table 2-11

The design tensile yielding strength is:

$$\begin{aligned}\phi_t P_n &= \phi_t F_y A_g \qquad \text{(from \textit{Spec}. Eq. D2-1)}\\ &= 0.90(36\text{ ksi})\left[\pi(1\tfrac{1}{4}\text{ in.})^2/4\right]\\ &= 39.8\text{ kips} > 32.3\text{ kips} \quad \textbf{o.k.}\end{aligned}$$

The design tensile rupture strength is (assume the effective net tension area is $0.75A_g$; this must be confirmed once connections are completed):

$$\begin{aligned}\phi_t P_n &= \phi_t F_u A_e \qquad \text{(from \textit{Spec}. Eq. D2-2)}\\ &= 0.75(58\text{ ksi})\left[(0.75)(\pi)(1\tfrac{1}{4}\text{ in.})^2/4\right]\\ &= 40\text{ kips} > 32.3\text{ kips} \quad \textbf{o.k.}\end{aligned}$$

Check for Serviceability Wind Drift and $\Delta_{2nd}/\Delta_{1st}$ ≤ 1.5 for Braced Frame E-W Direction

A check of the computer output shows wind drift and second-order effects are small for the braced frame. Roof deflections are shown in Table 2-13 for wind in the east-west direction for the various load combinations, which shows that the limitation for use of the ELM, $\Delta_{1st}/\Delta_{2nd}$ ≤ 1.5, is satisfied. Using load combination, 1.0D + 0.5S +1.0W, check that the required drift limit is satisfied, as follows:

Δ_{1st} = 0.408 in.
Δ_{2nd} = 0.439 in. < 360 in./100 = 3.60 in. **o.k.**

Table 2-13. Second-Order Displacement Check in East-West Direction			
Load Combination	Δ_{1st}, in.	Δ_{2nd}, in.	$\Delta_{2nd}/\Delta_{1st}$
Comb1	0.0582	0.0623	1.07
Comb2	0.0582	0.0623	1.07
Comb5	0.112	0.129	1.15
Comb6	0.112	0.129	1.15
Comb9	0.648	0.703	1.08
Comb10	0.648	0.703	1.08
Comb13	0.338	0.388	1.15
Comb14	0.338	0.388	1.15
Comb17	0.642	0.671	1.05
Comb18	0.642	0.671	1.05

Observations

1. The ELM provides a conservative design for this problem compared to the DM because of the $\Delta_{2nd}/\Delta_{1st} \leq 1.5$ limitation.
2. There were no seismic requirements for this problem, but the lateral load system in the north-south moment frame direction may not be satisfactory in higher seismic zones because of the magnitude of the second-order effects. This would need to be checked according to ASCE/SEI 7 Section 12.8.7.
3. The moment frame design is strongly influenced by the large leaning column load and the fact that there are relatively few moment connections.
4. The braced frame design is controlled by strength while the moment frame design is controlled by stiffness (necessary to satisfy the $B_2 \leq 1.5$ requirement). This is common in many buildings of this type.

Chapter 3
Direct Analysis Method (DM)

3.1 INTRODUCTION

With the rapid increase in the computational power and widespread use of the personal computer, code writers have sought to expand the approaches available for stability design of steel structures. The effective length method (ELM) was first introduced in the 1961 AISC *Specification* (AISC, 1961) and it has served the profession for over 45 years. However, new techniques have been developed in recent years that provide improved design procedures by taking advantage of the power of the personal computer and second-order analysis software. Many of these techniques involve the use of notional loads, and various methods of this sort have been developed and are popular in countries around the world (Canada, Australia and European countries using the Eurocodes). Beginning in late 1999, an AISC-SSRC Task Committee on stability sought to develop a new approach for stability design of steel structures with the goal of taking advantage of computer approaches for analysis. In addition, it was desired to

- eliminate the need for calculation of the column effective length factor, K, in the design process; a source of potential error and confusion for practicing engineers designing complex steel structures, and
- develop a stability design approach that applies, in the same logical and consistent way, to all types of structures, including braced frames, moment frames, and combined framing systems.

In 2002, the work was continued by AISC Task Committee 10 (TC10). The result of this extensive effort was the new direct analysis method (DM) contained in the 2005 AISC *Specification* Appendix 7. This method has its roots in notional load based analysis and design methods. However, modifications have been incorporated relative to the notional load methods to improve the accuracy and application to the broad range of steel structures found in practice. It is anticipated that this new method will become the preferred method in future editions of the AISC *Specification*. A summary of the development of this new method can be found in AISC-SSRC (2003).

Update Note: The direct analysis method is presented in Chapter C, Design for Stability, in the 2010 AISC *Specification*; other methods have been moved to Appendix 7, Alternative Methods of Design for Stability.

3.2 STEP-BY-STEP PROCEDURE

The DM can be used to design all types of building frames, including moment frames, braced frames, combined systems of braced and moment frames, and other hybrid and combined systems such as shear walls and moment frames. The method is referenced in Chapter C and is detailed in AISC *Specification* Appendix 7 [2010 AISC *Specification* Chapter C]. The method applies for all ranges of second-order effects without restriction. An accurate second-order analysis of the frame is required as specified in Chapter C. The method may be used for design by LRFD or ASD. Following is a detailed procedure for applying the DM:

1. Develop a model of the building frame that captures all the essential aspects of frame behavior. Remember to account for three-dimensional aspects of the loading for wind and seismic loads as prescribed by the applicable building code.
   (a) Use reduced member properties $EI^* = 0.8\tau_b EI$ and $EA^* = 0.8EA$ for all members which contribute to the stability of the frame. It is suggested that all steel member properties contributing to the elastic stiffness be multiplied by 0.8 with the exception of member flexural rigidities, which should be multiplied by $0.8\tau_b$. This includes connections, panel zones, diaphragms, column bases, and member shear stiffnesses. Uniform application of this stiffness reduction to all the structural components, including leaning columns, is most consistent with the fundamental basis of the DM and avoids possible anomalies in the analysis results such as false differential column axial deformations.
   (b) Apply the τ_b reduction as follows:

   $$\tau_b = 1.0 \text{ when } \alpha P_r/P_y \le 0.5$$
   $$\tau_b = 4\left(\alpha P_r/P_y\right)\left[1-\left(\alpha P_r/P_y\right)\right] \text{ when } \alpha P_r/P_y > 0.5$$

   where
   $\alpha = 1.0$ (LRFD); $\alpha = 1.6$ (ASD)

   Note: In lieu of applying $\tau_b < 1.0$, it is permissible to apply additional additive notional loads, N_i, of $0.001Y_i$ at all levels.
2. Determine all gravity loads that are stabilized by the lateral load resisting system.
3. Determine the lateral loads corresponding to the wind and seismic load requirements.

4. Determine the notional lateral loads, which are intended to account for the overall effects of out-of-plumb geometric imperfections, and apply them either as minimum lateral loads in the gravity-only load combinations or as additive lateral loads for all load combinations based on the following rules:

   (a) $N_i = 0.002Y_i$; Notional load at level i where Y_i = gravity load at level i from each LRFD load combination being considered or 1.6 times the ASD load combination.

   > **Update Note:** Note the slight difference in the definition of Y_i within the definition of the notional load equation for N_i throughout this Design Guide (based explicitly on the 2005 AISC *Specification*) versus the definition in the 2010 AISC *Specification*. Note that this difference does not have any effect on the value derived from the respective notional load equations. They both yield identical results.
   >
   > For this Design Guide and as given in the 2005 AISC *Specification* Appendix 7, Section 7.3:
   >
   > $$N_i = 0.002Y_i$$
   >
   > where
   >
   > N_i = notional load applied at level i, kips
   >
   > Y_i = gravity load applied at level i from the LRFD load combinations or 1.6 times the ASD load combinations, as applicable, kips
   >
   > For the 2010 AISC *Specification*:
   >
   > $$N_i = 0.002\alpha Y_i \qquad \text{(2010 } \textit{Spec.} \text{ Eq. C2-1)}$$
   >
   > where
   >
   > N_i = notional load applied at level i, kips
   >
   > Y_i = gravity load applied at level i from the LRFD load combination or the ASD load combination, as applicable, kips
   >
   > α = 1.0 (LRFD); α = 1.6 (ASD)
   >
   > Similarly, an additional lateral load may be used where $\alpha P_r/P_y > 0.5$ and $\tau_b = 1.0$. In this Design Guide and in the 2005 AISC *Specification* Appendix 7, Section 7.3, this additional lateral load is $0.001Y_i$, and in the 2010 AISC *Specification* Section C2.3, it is $0.001\alpha Y_i$.

   (b) The notional loads, N_i, may be applied as a minimum lateral load solely in the gravity-only load combinations in cases where the second-order effects as measured by the ratio of the average second-order to first-order story drifts ($\Delta_{2nd}/\Delta_{1st}$ or B_2 based on the nominal member properties) are ≤ 1.5 (or equivalently, $\Delta_{2nd}/\Delta_{1st}$ ≤ 1.71 for the DM model based on the reduced properties). Otherwise, the notional loads, N_i, must be applied as an additive lateral load to all load combinations.

   For wind load combinations on buildings where N_i must be applied additively, the wind pressures on the roof may be considered in calculating Y_i. This can reduce the total downward vertical load substantially for some low-rise building structures, but should be used with caution (see Section 1.6).

   (c) The notional loads should be applied in a direction that increases the overall destabilizing effect on the frame for the load combination being considered.

   - For gravity-only load combinations that cause a net sidesway due to nonsymmetry of the loads or geometry, the notional loads should be applied in the direction that increases the net sidesway. For structures with multiple stories or levels and in which the sidesway deformations are in different directions in different stories or levels, it is necessary to include a pair of load combinations separately considering the notional loads associated with an out-of-plumbness in each direction. Generally, one need not apply notional loads in a direction opposite from the sidesway to minimize the reduction in any internal forces due to the sidesway. For gravity load combinations with no sidesway, it is necessary to include a pair of load combinations separately considering the notional loads in each ± direction, unless there is symmetry of frame geometry, loading and member sizes.
   - For load combinations where the notional loads are combined with lateral loads, the notional loads should be applied only in the direction that adds to the effect of the lateral loads. One need not apply notional loads in a direction opposite from the lateral loads to minimize the reduction in internal forces in certain components due to the lateral load.

   (d) Subject to items (b) and (c), the notional lateral loads, N_i, should be applied independently about each of two orthogonal building axes. These axes should be selected as approximate principal lateral stiffness directions for the overall building structure. Note: "independently" means that the notional loads are applied only in one direction at a time. For structures where N_i must be applied additively, and when considering combination load effects such as required by ASCE/SEI 7 for wind and seismic loading, the notional loads should be included as part of each independent lateral loading. For instance, as part of the diagonal wind loading requirement in Figure 6-9 of ASCE/SEI 7, 75% of the respective notional loads should be applied simultaneously with the wind load along each principal axis direction. No torsional notional load is required to mimic the code eccentric torsion wind and seismic loadings.

(e) For general structural analysis, the notional loads may be applied at each location where gravity load is transferred to the structural columns. The load Y_i is the gravity load transferred to the columns at each of these locations.

Note that the total base shear due to out-of-plumbness effects is actually zero. Thus, the externally applied notional loads lead to small fictitious base shears in the structure. However, the notional lateral loads are derived from the equivalent story P-Δ shears associated with an initial nominal out-of-plumbness of the structure. Therefore, the total shear from the notional loads at the base of the frame is actually balanced by a corresponding story P-Δ shear at the base. The shear forces due to the notional loads tend to be small compared to other shear requirements. See the further discussion about the sensitivity of the response of representative structures to the notional loads later in this chapter.

5. Perform a second-order analysis for all the applicable load combinations (ASCE/SEI 7 Section 2.3.2—LRFD or Section 2.4.1—ASD). Any second-order analysis method that properly considers both P-Δ and P-δ effects is permitted. Note that, unlike first-order analysis, superposition of basic load cases is not appropriate when a general second-order analysis is employed because the second-order effects are nonlinear (see Section 1.4). However, if the B_1-B_2 approach is used, superposition is permitted.

6. Design the various members and connections for the forces obtained from the analysis according to the applicable provisions of the AISC *Specification*.

7. For all the beam-column members, apply the interaction Equations H1-1a and H1-1b (or where applicable, Equation H1-2) using an effective length factor, $K = 1.0$.

8. If the additive notional lateral loads of $0.002Y_i$ are *not* included in the lateral load combinations, as permitted by Step 4(b), confirm for each level of the frame that the second-order sidesway effects as measured by the ratio of the average second- to first-order story drifts ($\Delta_{2nd}/\Delta_{1st}$ or B_2) are less than or equal to 1.71 (based on a model using the reduced member stiffness properties). If this limit is violated, then the notional lateral loads must be applied additively with the lateral loads in the lateral load combinations. If the notional lateral loadings are applied additively in all the load combinations, this check may be skipped.

9. Check the seismic drift limits using nominal member properties according to ASCE/SEI 7 Section 12.12 and maximum P-Δ effects as prescribed by ASCE/SEI 7 Section 12.8.7. Note that P-Δ effects are checked for a load factor no more than 1.0 on all gravity design loads, P_x, in ASCE/SEI 7 Equation 12.8-16.

10. Check the wind drifts using nominal member properties for service level wind loads (mean recurrence interval for wind selected at designer's option). Note that this check is a serviceability check, not a code requirement. Also note that for moment frames, drift under wind and/or seismic load levels will typically control the design. Therefore, this check should be made first in the initial proportioning of member sizes for these frame types. In general, it is recommended that a second-order analysis should be employed to accurately determine service load story drifts if these drifts are to be compared against drift damage limits for the cladding and partition types employed in the building. In addition, note that the service drift analyses should not include any of the stiffness reductions or notional lateral loads associated with the DM strength analysis and design procedures.

The AISC *Specification* permits the designer to set up the frame model incorporating out-of-plumbness explicitly in the analysis without the need for applying notional lateral loads. This approach may be desirable in some cases because it avoids several potential problems such as:

1. Determining how to apportion notional loads within the floor plan in buildings containing flexible diaphragms.

2. Determining how to apply notional loads in buildings with sloping floors and roofs where the beams are not horizontal or in structures where axial loads are applied at intermediate positions along the length of a member.

3. Determining how to apply notional loads in other non-rectangular building geometries.

4. Avoiding fictitious story shears and foundation lateral loads due to externally applied notional loads. (Note: These effects are usually small.)

5. Determining notional loads that are equivalent to the appropriate geometric imperfections in general stability bracing problems, or in other problems such as analysis of general structures (space frames, arches, domes, etc.)

Explicit modeling of out-of-plumbness is easier to automate in computer-based design. In time, it is expected that software developers will incorporate routines so that the out-of-plumb geometry can be easily defined. However, unless automated methods of specifying out-of-plumbness are available in analysis software, it is often easier to apply notional loads. The software implementation needs to be able to apply the out-of-plumbness in the different directions of the drifts due to the applied loads in different load combinations. The reader is referred to Appendix C for further discussion of modeling geometric imperfections for taller building structures.

The designer is cautioned about the proper use of member properties in the design process. Reduced stiffness properties should not be used in serviceability checks or in post-processor design programs that check member sizes for code conformance. The nominal stiffnesses (e.g., unreduced *EI* and *EA*) are always used in the AISC *Specification* resistance equations. For the analysis, a uniform reduction of all elastic contributions to the stability by 0.8 is recommended. An additional reduction of *EI* by τ_b is required for heavily loaded beam-columns that participate in the lateral load resistance (or alternately, the use of an additional out-of-plumb effect of 0.001 is permitted in frames containing such members).

The AISC *Specification* states that the DM can be used for the design of lateral load resisting systems (braced frames, moment frames and combined systems). However, the DM is a versatile tool that is applicable for solving all types of structural problems such as long-span trusses and bracing systems, where an understanding of the behavior at the ultimate load level, including the effects of out-of-plumbness, is desired. In the DM, out-of-plumbness effects modeled directly or estimated by the use of notional loads, along with reduced member stiffnesses account for softening of the structure at the ultimate load level.

3.3 ADVANTAGES, DISADVANTAGES AND RESTRICTIONS ON USAGE

The advantages of the DM are:

1. The direct analysis method (DM) is applicable to all frame types including braced, moment and combined frames.

2. The DM permits all columns to be designed using an effective length factor, K = 1.0, thus avoiding many of the complexities and uncertainties in the proper calculation of K. This is a major benefit to the designer.

3. The method focuses the designer's attention on providing sufficient sidesway stiffness for the full structural system.

4. As with the ELM, the method focuses the designer's attention on the destabilizing effects of out-of-plumbness of frames by requiring the application of notional lateral loads or an explicit nominal out-of-plumbness.

5. The method focuses attention on the "softening" of the lateral load resisting frame at the ultimate limit state by requiring a reduction in member properties. It also directly highlights inelastic effects on column and frame stiffness by the application of τ_b in the analysis.

6. The DM provides more accurate estimates of the internal forces in the structure. The influence of geometric imperfections and stability effects is included in the calculation of the internal forces in the beams, beam-columns and their connections, whereas the ELM does not. This can be particularly important for beams and connections that support relatively light gravity loads but provide rotational restraint to the ends of columns. Also, these more accurate estimates can be particularly important for checking the strength of beam-columns subjected predominantly to uniaxial bending that are relatively weak in the direction out-of-the-plane of bending. Nonetheless, within its limits, the ELM is an acceptable method.

7. The method can be applied for inelastic analysis and design as well as for elastic analysis and design (see Appendix 1 in the AISC *Specification* and White et al. (2006)). The method can also be applied to composite and mixed framing systems.

The disadvantages of the DM are:

1. The DM requires the additional step of modifying member properties (EI^*, EA^*) in the computer model input. Care must be taken to always use the correct nominal properties in serviceability checks or in checking member sizes for code conformance.

2. Application of τ_b is an extra iterative step in the design process, although this step can be eliminated by increasing the notional loads by $0.001Y_i$, additive for each load combination. However, this additional notional lateral load impacts all elements in the lateral load resisting system, not just the highly loaded members.

3. Application of notional lateral loads is an additional step that designers are not accustomed to. However, for many practical building frames, the notional loads need be considered only as a minimum lateral load in the gravity-only load combinations.

4. The DM is more sensitive to the accuracy of the second-order analysis than the ELM. For structures that are governed, or nearly governed, by elastic sidesway buckling, the nonlinear variation of the internal forces and moments as the stability limit is approached results in rapidly increasing member unity check values, i.e., M_r/M_c for bending members, P_r/P_c for compression members, or the interaction equation value for beam-columns. As a result, the designer may mistakenly believe that members have ample reserve capacity due to a small calculated value for the unity checks when in reality these checks may reach or exceed 1.0 in the critical

Table 3-1. Interaction Equation Values								
	Effective Length Method				Direct Analysis Method			
L_c/r	P_u, kips	$P_u/\phi_c P_n$	M_u, kip-in.	$M_u/\phi_b M_n$	P_u, kips	$P_u/\phi_c P_n$	M_u, kip-in.	$M_u/\phi_b M_n$
20	587	0.83	620	0.19	598	0.78	848	0.25
40	352	0.71	1090	0.33	363	0.51	1820	0.54
60	195	0.71	1100	0.33	210	0.34	2480	0.74

members with only a small increase in the applied load on the structure.

5. The DM is based on assumed out-of-plumbness values that are estimated and largely uncertain in today's modern fabrication and erection procedures (the actual out-of-plumbness is often expected to be smaller in typical steel frame structures). However, many frames are not sensitive to the out-of-plumbness values used. Also, the nominal out-of-plumbness of 0.002, which is based on the column erection tolerances in the AISC *Code of Standard Practice*, produces results that are consistent with the accounting for geometric imperfections that is now included implicitly in design by the ELM.

The AISC *Specification* does not place any restrictions on the use of the DM.

3.4 OBSERVATIONS ON FRAME BEHAVIOR—DM VERSUS ELM

In this section, the cantilever beam-columns considered previously in Section 2.4 are studied in further detail. The results of the second-order analysis and design checks of these members by the DM are discussed and contrasted with the application of the ELM demonstrated in the previous chapter. As noted in Section 2.4, these beam-column models are a version of the more general frame stability Model A of Appendix A (Figure A-2 with $P_g = 0$). The cantilever columns, W10×60 members bent about their strong axis, are designed to carry an axial load, P, and lateral load, H, where $H = 0.01P$. The lateral load is applied in proportion to the axial load. The columns are considered fully braced out-of-plane. A relatively stocky column ($L_c/r_x = 20$), an intermediate length column ($L_c/r_x = 40$), and a slender column ($L_c/r_x = 60$) are examined.

For application of the DM, the second-order base moments are calculated using Equation A-12 in Appendix A of this Design Guide, but with the reduced member flexural rigidity, $0.8\tau_b EI$, as required by the method. The corresponding force-point traces and the beam-column strength interaction curves are shown in Figures 2-1 through 2-3. Note that for the DM, the beam-column strength curves are determined using $K = 1$ in the first term of Equations H1-1a and H1-1b.

The member available strengths determined by the DM may be compared with those determined from the ELM discussed in Chapter 2 by considering the intersection of the force-point traces with the corresponding strength interaction curves. Table 3-1 shows the LRFD values of $P = P_u$ and $M = M_u$ as well as the ratios $P_u/\phi_c P_n$ and $M_u/\phi_b M_n$ assuming the required strengths P_u and M_u fall approximately on the strength interaction curves for each of the methods. Based on Figures 2-1 to 2-3 and Table 3-1, the following observations can be made:

1. In each case, the maximum internal moments, M_u, for the DM at the member available strength are significantly larger than for the ELM, increasingly so as the column slenderness increases. Correspondingly, the P_u values are slightly larger for the DM.
2. The $P_u/\phi_c P_n$ values at the strength limit do not change appreciably with increasing slenderness for the ELM, while the same ratios for the DM reduce significantly with increasing slenderness at the same time that the moment ratios $M_u/\phi_b M_n$ increase. The $P_u/\phi_c P_n$ values are smaller at the strength limit for the DM since $\phi_c P_n$ is based on $K = 1.0$ regardless of column slenderness.
3. The ELM is less sensitive to the accuracy of the second-order analysis, relying more on the axial load ratio of the first term in the interaction equations. Unfortunately, the method relies heavily on the accuracy of the effective length factor, K_2, in the first term of the beam-column strength interaction equations. Determination of K_2 can be a challenge for complex frames.
4. The DM eliminates the need to calculate K factors and provides better estimates of the true internal moments in the structure. This is important for determining girder moments and connection forces for members restraining the columns as well as for checking beam-column out-of-plane strengths in members loaded predominantly in major-axis bending and axial compression.
5. The ratios between the axial and moment terms of the interaction equations varies significantly between the two methods, but the overall member capacities predicted by the two methods, represented by the values P_u reported in the table, are approximately the same. The ELM relies

more on the axial load ratio term of the beam-column strength interaction equations to ensure stability while the DM typically relies more on the moment ratio term.

6. As the lateral and vertical loads are increased proportionally on a frame to ultimate, the DM interaction equation values can increase dramatically as the strength limit is approached for more slender or more stability critical structures. For example, the interaction values are highly nonlinear near the ultimate load for the W10×60 with $L/r_x = 60$, where the second-order internal moments increase rapidly (see Figure 2-3). Fortunately, extreme nonlinearity of this sort occurs only in frames and load combinations in which (a) the structure is subjected solely or almost entirely to gravity loads and (b) the design of the frame is governed by this gravity-only load combination and the frame fails essentially by elastic sidesway buckling.

A comparison is made in Figures 3-1 and 3-2 between the DM and the traditional ELM according to the 1999 AISC *Specification* provisions, which did not have a notional load requirement. A W10×60 cantilever column ($L_c/r_x = 60$, $L = 22$ ft) bent about the strong axis is loaded with an external vertical load only, increasing from zero to its maximum capacity as defined by AISC. Prior to the 2005 AISC *Specification*, this column would have been designed only for the vertical load with an effective length factor $K = 2$. The corresponding force-point trace is defined by the curve OA in Figure 3-1. Note that under the AISC *Specification* prior to 2005, the calculated moment would have been zero at the base and the moment term in the interaction equations would have been zero. This same column, designed using the DM in the 2005 AISC *Specification*, has a notional load of $0.002P$ applied in the design. This results in a calculated second-order moment at the base and along the column length. The relationship between the internal moment (using Equation A-12) and the axial force under the proportionally increasing applied loads is defined by curve OB in Figure 3-1. The column design check is conducted using $K = 1$ in the DM and the resulting design has a significant moment component in the interaction equation. A plot of the interaction equation values with increasing load is shown for the two methods in Figure 3-2. Note the linear nature of the interaction equation values in the former design approach and the nonlinear nature of the interaction equation values with the DM. This comes about because of the nonlinear increase in the second-order base moment resulting from the axial load times the sidesway deflection. As the failure load is approached, the column

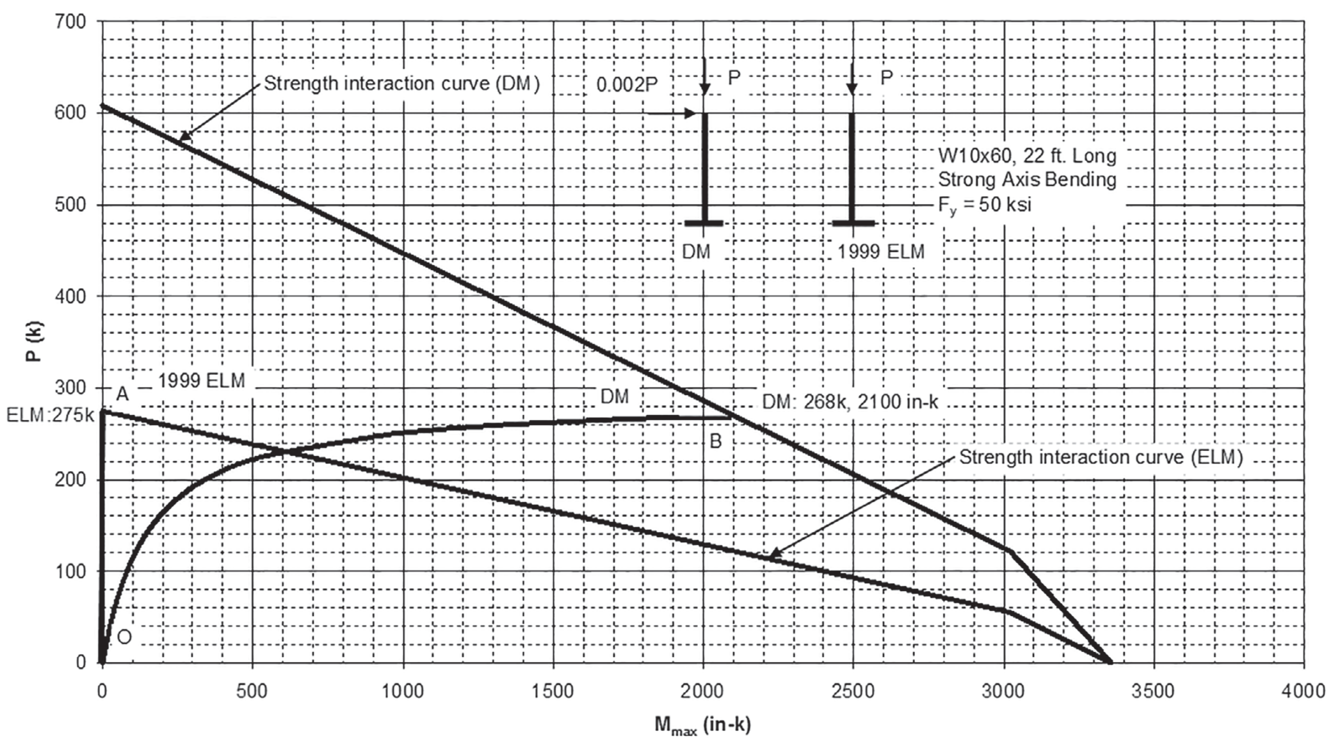


Fig. 3-1. Direct analysis method versus 1999 AISC Specification
effective length method, cantilever column with axial load application to failure.

stiffness reduces and the base moment increases dramatically because of the second-order effects. Conversely, there is zero calculated moment in the traditional ELM. Clearly, the DM model gives a more realistic representation of the physical response. Note that the new ELM, as presented in the AISC *Specification*, requires a notional load and base moment in this design check. Validation studies (Maleck and White, 2003; Martinez-Garcia, 2002; Martinez-Garcia and Ziemian, 2006; White et al., 2006) performed in developing and comparing the two methods have shown that either method (the DM or the revised ELM in the AISC *Specification*) is sufficiently accurate for design.

3.5 EFFECT OF VARYING NOTIONAL LOADS

The AISC *Specification* requires a notional load magnitude of $0.002Y_i$ but provides latitude for use of an alternative magnitude based on a tighter out-of-plumbness tolerance where the designer feels that it is justified. Until more data becomes available on the actual plumbness with which steel structures are built, it is suggested that the designer use the $0.002Y_i$ value in most problems to account for geometric imperfection effects. The influence of varying the amount of notional load on a simple cantilever beam-column is presented in Figure 3-3. Three different notional lateral loads ($0.001Y_i$, $0.002Y_i$ and $0.003Y_i$) are placed on a W10×60 cantilever beam-column with $L_c/r_x = 20$. This member is also assumed to provide the lateral support for a leaning column that has a load equal to the load on the column, P, and to resist an applied lateral load of $H = 0.01P$. The beam-column strength interaction curve (AISC *Specification* Equations H1-1a and H1-1b) and the force-point traces by the DM using the three different notional load values are illustrated in the figure. For this example, the axial load available strength changes (up or down) by approximately 20 kips (3.7% of the strength determined using the base $N_i = 0.002Y_i$) for each $0.001Y_i$ change (up or down) in notional load, while at the same time the column internal moment changes by about 10%. In many practical frames, a change in the notional load of $0.001Y_i$ has little effect on the design.

3.6 SUMMARY OF DESIGN RECOMMENDATIONS

Following is a summary of design recommendations for application of the DM.

1. Consider using the direct analysis method (DM) for all moment frame and combined framing systems as the

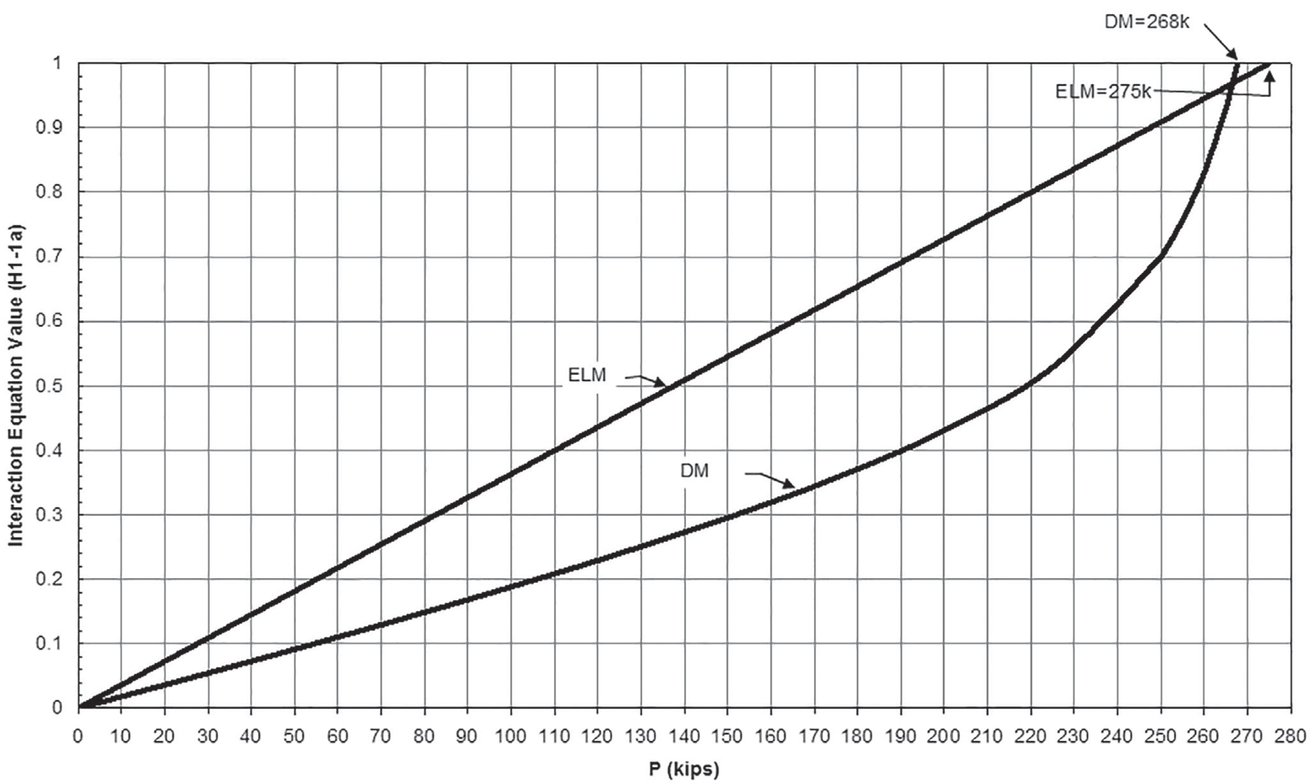


Fig. 3-2. Interaction equation, cantilever column with axial load application to failure, direct analysis method versus 1999 AISC Specification *effective length method.*

preferred design method for steel buildings. Stability sensitive braced frames also may benefit from using the DM, though K is already taken equal to 1 in the ELM in this case.

2. The DM also can be applied to the solution of other problems (e.g., long-span trusses and bracing systems) where an understanding of the structural behavior at the ultimate limit state is desired.

3. Use the $0.002Y_i$ notional load unless special effort is made to ensure that the completed structure is built with tighter tolerances than normally expected.

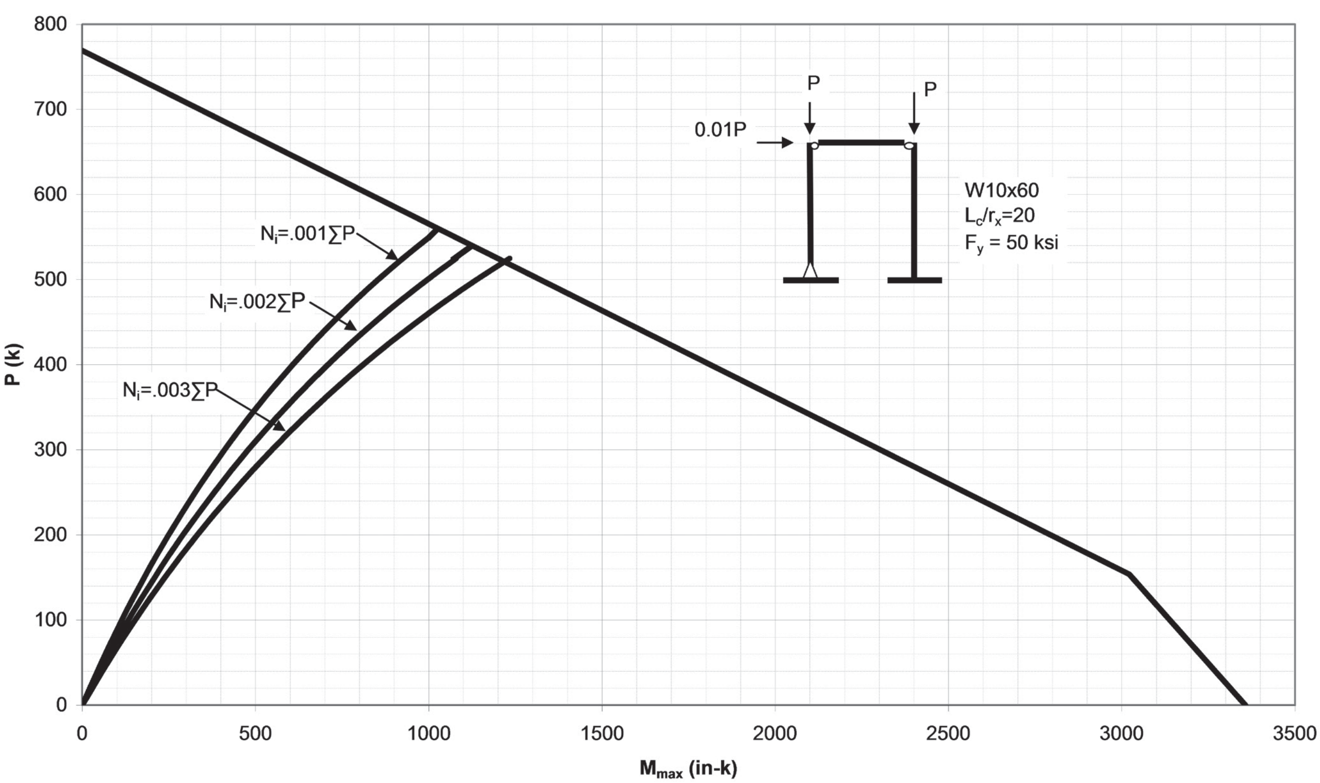


Fig. 3-3. Effect of varying notional load.

3.7 DESIGN EXAMPLES

See Section 1.7 for a background discussion of the solution to the example problems. The design examples are implemented using the 2005 AISC *Specification* and 13th Edition AISC *Manual.*

Example 3.1—Two-Story Warehouse, Typical Braced Frame Building

Given:

Size the braced frame columns, beams and rod bracing for a typical bay of the braced frame building shown in Figure 3-4. Consider dead, live and wind load combinations using ASCE/SEI 7 load combinations and design by ASD. Consider wind load in the plane of the braced frame only. Solve using the direct analysis method (DM). All columns are braced out-of-plane at the floor and the roof. The lateral load resistance is provided by tension rod bracing only. All beam-to-column connections are simple "pinned" connections. Maintain interstory drift limit $\Delta/L \leq 1/100$ under nominal wind load. Assume nominal out-of-plumbness of Δ_o/L = 0.002. This is the AISC *Code of Standard Practice* maximum permitted out-of-plumbness. All steel is ASTM A992, except that the tension rods are ASTM A36 steel.

The loading is as follows:

Roof

Dead load, w_{RD}	= 1.0 kip/ft
Live load, w_{RL}	= 1.2 kip/ft
Estimated roof beam weight	= 0.076 kip/ft
Estimated roof interior column weight	= 0.065 kip/ft
Estimated roof end column weight	= 0.048 kip/ft
Wind load, W_R	= 10 kips

Floor

Dead load, w_{FD}	= 2.4 kip/ft
Live load, w_{FL}	= 4.0 kip/ft
Estimated floor beam weight	= 0.149 kip/ft
Estimated floor interior column weight	= 0.065 kip/ft
Estimated floor end column weight	= 0.048 kip/ft
Wind load, W_F	= 20 kips

(Note: Wind load on roof surfaces as specified in ASCE/SEI 7 is not considered in this problem.)

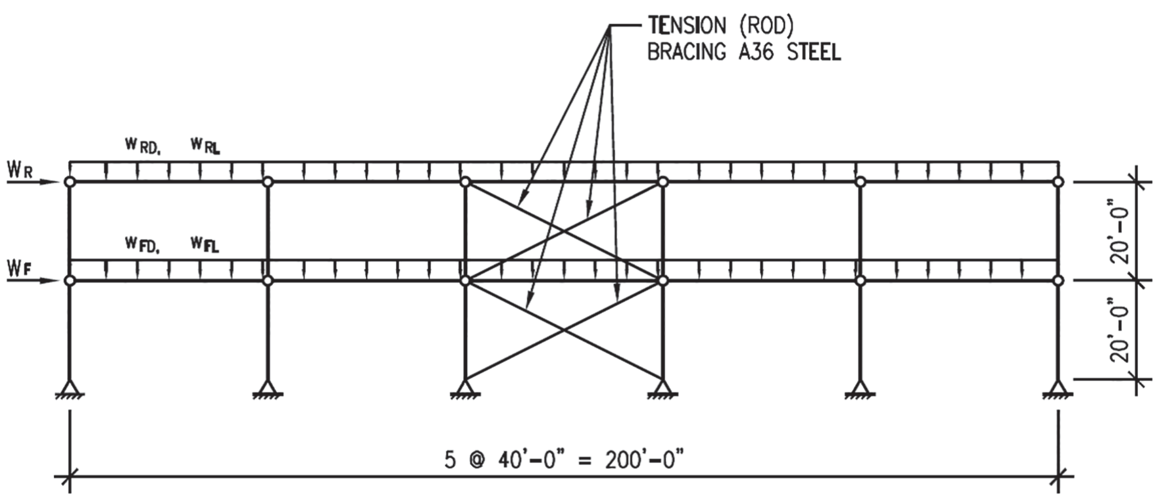


Fig. 3-4. Example 3.1 braced frame elevation.

Table 3-2. 1.6 Times ASD Load Combinations
Comb1 = 1.6(D) + N
Comb2 = 1.6(D + L) + N
Comb3 = 1.6(D + L_r) +N
Comb4 = 1.6(D + 0.75L + 0.75L_r) +N
Comb5 = 1.6(D + W)
Comb6 = 1.6(D + 0.75W + 0.75L + 0.75L_r)
Comb7 = 1.6(0.6D + W)
D = dead load L = live load L_r = roof live load W = wind load N = minimum lateral load for gravity-only combinations = 0.002Y_i Y_i = gravity load from 1.6 times ASD load combinations applied at level i

Solution:

From AISC *Manual* Table 2-3, the material properties are as follows:

Beams and columns
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Bracing
ASTM A36
F_y = 36 ksi
F_u = 58 ksi

The analysis was performed using a general second-order elastic analysis program including both P-Δ and P-δ effects. See Figure 3-5 for the member labels used. Refer to Section 1.7 for a more detailed discussion of the analysis computer models utilized in this Design Guide. Refer to Section 3.2 for a step-by-step description of the DM.

Description of Framing

All lateral load resistance is provided by the tension only rod bracing. The tension rods are assumed to be pin connected using a standard clevis and pin (see AISC *Manual* Tables 15-3 and 15-7). Beams within the braced frame are bolted into the column flanges using double angles (see AISC *Manual* Figure 13-2(a)). A single gusset plate connecting the tension rod is shop welded to the beam flange and field bolted to the column flange (see AISC *Manual* Figure 13-2(a)). All other columns outside the braced frames are leaning columns with simple beam-to-column connections.

Design Approach

Design is an iterative process. Preliminary sizes should be chosen based on experience or a preliminary analysis. Braced frame structures are often controlled by strength as opposed to wind or seismic drift. Thus, for this problem, the sizes are estimated from a preliminary strength check and then used in the computer analysis. The member sizes used in the analyses are those shown in Figure 3-6.

Analysis Load Combinations

The member design forces are obtained by analyzing the structure for 1.6 times ASD load combinations and then dividing the results by 1.6. It should be noted that the notional loads already include, by definition, the 1.6 amplification. The load combinations from ASCE/SEI 7 Section 2.4.1 used for the second-order analysis are given in Table 3-2:

If the B_1-B_2 approach is used to account for second-order effects, the analysis need not include the 1.6 multiplier on either the load combinations or in the determination of notional loads since this is already included in the B_1-B_2 calculation.

Table 3-3. Notional Load, N_i, for Each Gravity Load Combination, kips				
Level	**Comb1**	**Comb2**	**Comb3**	**Comb4**
Roof	0.710	0.710	1.48	1.29
Floor	1.65	4.21	1.65	3.57

Note that since the structure loading and geometry are symmetric and symmetry of the frame is enforced in the member selection, the wind load is considered in only one direction.

Notional Loads, N_i

The assumed out-of-plumbness is 0.002. The magnitude of notional loads to be applied is $N_i = 0.002Y_i$. Assume that drift from a second-order analysis is less than 1.5 times the drift from a first-order analysis at both levels of the structure; therefore, provisions of AISC *Specification* Appendix 7, Section 7.3(2) apply. [Note that this limit is 1.7 times the drift from a first-order analysis in the 2010 AISC *Specification*, which takes into account the stiffness adjustments.] This assumption must be checked after the analysis is complete. Based on the above assumption, the notional loads are to be applied solely in the gravity-only load combinations. The gravity loads are:

Roof

$$\Sigma D = (1.0 \text{ kip/ft})(200 \text{ ft}) + (0.076 \text{ kip/ft})(200 \text{ ft}) + (0.065 \text{ kip/ft})(20 \text{ ft})(4) + (0.048 \text{ kip/ft})(20 \text{ ft})(2)$$
$$= 222 \text{ kips}$$
$$\Sigma L_r = (1.2 \text{ kip/ft})(200 \text{ ft})$$
$$= 240 \text{ kips}$$

Floor

$$\Sigma D = (2.4 \text{ kip/ft})(200 \text{ ft}) + (0.149 \text{ kip/ft})(200 \text{ ft}) + (0.065 \text{ kip/ft})(20 \text{ ft})(4) + (0.048 \text{ kip/ft})(20 \text{ ft})(2)$$
$$= 517 \text{ kips}$$
$$\Sigma L = (4.0 \text{ kip/ft})(200 \text{ ft})$$
$$= 800 \text{ kips}$$

The notional load is determined from $N_i = 0.002Y_i$. N_i needs to be evaluated for each load combination. For example, at the floor level for Comb2:

$$N_i = 0.002\,Y_i$$
$$= 0.002\,(1.6D + 1.6L)$$
$$= 0.002\,[1.6(517 \text{ kips}) + 1.6(800 \text{ kips})]$$
$$= 4.21 \text{ kips}$$

Minimum lateral loads for Comb1 through Comb4 are listed in Table 3-3.

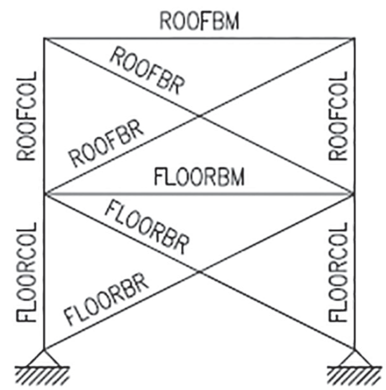


Fig. 3-5. Braced frame member labels.

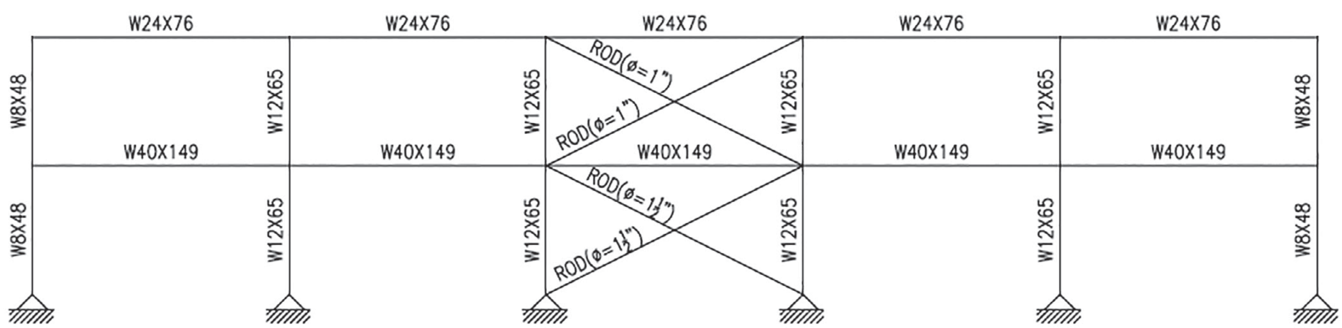


Figure 3-6. Final member sizes using the direct analysis method.

Table 3-4. Required Beam Flexural Strength (ASD)							
Member	**M_r, kip-in.**						
	Comb1	**Comb2**	**Comb3**	**Comb4**	**Comb5**	**Comb6**	**Comb7**
ROOFBM	2580	2580	**5470**	4750	2600	4770	1560
FLOORBM	6120	**15700**	6120	13300	6150	13400	3690

Member Properties Used in the Analysis Model

The axial stiffness (EA) of the beams, braces and columns that are part of the braced frame is modified to $EA^* = 0.8EA$ for analysis in the computer model as required by AISC *Specification* Appendix 7, Section 7.3(4).

Design of Beams

From a second-order analysis, member required flexural strengths for each load combination are given in Table 3-4. The required flexural strength from the controlling load combination is presented in bold.

Note that the member required flexural strengths in Table 3-4 are for ASD and were obtained by dividing the analysis results by 1.6.

The beam flexural strengths shown in Table 3-4 are slightly larger than $wL^2/8$ due to magnification of the first-order moments by the axial compression in the beams. However, the beams are designed here for gravity load alone without considering the axial load from the lateral load cases. The amount of axial load in the beams depends on the assumed load path of the lateral forces, which are applied to the lateral load resisting frames (through collector beams and/or floor and roof diaphragm connections to the beams). Even if a conservative estimate of the axial force in these members is included, the influence on the beam-column interaction value is small. The beams are assumed fully braced by the floor and roof diaphragm.

Roof Beams

The required strength of the roof beams is:

$$\begin{aligned} M_r &= 5{,}470 \text{ kip-in.}/12 \text{ in./ft} \\ &= 456 \text{ kip-ft } (\text{Comb3}) \end{aligned}$$

Try a W24×76.

From Table 3-2 of the AISC *Manual*, for a fully braced beam, the allowable flexural strength is:

$$\begin{aligned} \frac{M_n}{\Omega_b} &= \frac{M_{px}}{\Omega_b} \\ &= 499 \text{ kip-ft} > M_r \quad \textbf{o.k.} \end{aligned}$$

Floor Beams

The required strength of the floor beams is:

$$\begin{aligned} M_r &= 15{,}700 \text{ kip-in.}/12 \text{ in./ft} \\ &= 1{,}310 \text{ kip-ft } (\text{Comb2}) \end{aligned}$$

Try a W40×149.

From Table 3-2 of the AISC *Manual*, for a fully braced beam, the allowable flexural strength is:

Table 3-5. Required Column Axial Strength							
Member	P_r, kips						
	Comb1	Comb2	Comb3	Comb4	Comb5	Comb6	Comb7
ROOFCOL	43.9	43.9	**92.3**	80.4	49.0	84.9	31.4
FLOORCOL	148	309	197	306	169	**321**	109

Table 3-6. Required Brace Axial Strength							
Member	P_r, kips						
	Comb1	Comb2	Comb3	Comb4	Comb5	Comb6	Comb7
ROOFBR	0.547	0.548	1.30	1.10	**11.9**	9.51	11.6
FLOORBR	1.96	4.93	2.75	4.85	**36.9**	31.5	35.4

$$\frac{M_n}{\Omega_b} = \frac{M_{px}}{\Omega_b}$$
$$= 1{,}490 \text{ kip-ft} > M_r \quad \textbf{o.k.}$$

Design of Columns in the Braced Frame

Member required axial compressive strengths for each load combination are given in Table 3-5 with the controlling value in bold.

Roof Columns

The required strength of the roof columns is:

$P_r = 92.3$ kips (Comb3)
$K = 1.0$ for braced frame columns

Try a W8×31.

From AISC *Manual* Table 4-1 with $KL = 20$ ft, the allowable axial compressive strength is:

$$\frac{P_n}{\Omega_c} = 97.1 \text{ kips} > P_r \quad \textbf{o.k.}$$

Floor Columns

The required strength of the floor columns is:

$P_r = 321$ kips (Comb6)
$K = 1.0$ for braced frame columns

Try a W12×65.

From AISC *Manual* Table 4-1 with $KL = 20$ ft, the allowable axial compressive strength is:

$$\frac{P_n}{\Omega_c} = 360 \text{ kips} > P_r \quad \textbf{o.k.}$$

Design of Braces

Member required tensile strengths for each load combination are given in Table 3-6 with the controlling value in bold. (Note: The small values shown for the gravity load only load combinations come from the minimum lateral loads, N.)

Roof Braces

The required tensile strength of the roof braces is:

$P_r = 11.9$ kips (Comb5)

Try a 1-in.-diameter rod (ASTM A36 steel).

The allowable tensile yielding strength is determined from AISC *Specification* Section D2(a) as follows:

$$\frac{P_n}{\Omega_t} = \frac{F_y A_g}{\Omega_t}$$
$$= \frac{(36\text{ ksi})(\pi/4)(1.00\text{ in.})^2}{1.67}$$
$$= 16.9\text{ kips} > P_r \quad \textbf{o.k.}$$

The allowable tensile rupture strength is determined from AISC *Specification* Section D2(b), assuming that the effective net area of the rod is $0.75A_g$, as follows:

$$\frac{P_n}{\Omega_t} = \frac{F_u A_e}{\Omega_t}$$
$$= \frac{(58\text{ ksi})(0.75)(\pi/4)(1.00\text{ in.})^2}{2.00}$$
$$= 17.1\text{ kips} > P_r \quad \textbf{o.k.}$$

Floor Braces

The required tensile strength of the floor braces is:

$P_r = 36.9$ kips (Comb5)

Try a 1½-in.-diameter rod (ASTM A36 steel).

The allowable tensile yielding strength is determined from AISC *Specification* Section D2(a) as follows:

$$\frac{P_n}{\Omega_t} = \frac{F_y A_g}{\Omega_t}$$
$$= \frac{(36\text{ ksi})(\pi/4)(1\tfrac{1}{2}\text{ in.})^2}{1.67}$$
$$= 38.1\text{ kips} > P_r \quad \textbf{o.k.}$$

The allowable tensile rupture strength is determined from AISC *Specification* Section D2(b), assuming that the effective net area of the rod is $0.75A_g$, as follows:

$$\frac{P_n}{\Omega_t} = \frac{F_u A_e}{\Omega_t}$$
$$= \frac{(58\text{ ksi})(0.75)(\pi/4)(1\tfrac{1}{2}\text{ in.})^2}{2.00}$$
$$= 38.4\text{ kips} > P_r \quad \textbf{o.k.}$$

Check Drift under Nominal Wind Load (Serviceability Wind Drift Check)

Drift is checked under a serviceability load combination (Nominal Dead + Nominal Live + Nominal Wind). The nominal wind load is specified in this problem to be used in the serviceability load combination and the story drift is limited based on $\Delta/L \le 1/100$. The appropriate serviceability load combination is a matter of engineering judgment. The drift check is made using a second-order analysis as recommended in this Design Guide when a direct check against damage is to be considered. The resulting drift at the roof level is 0.915 in. and at the floor level is 0.539 in.

Floor

$$\begin{aligned}\Delta/L &= 0.539\text{ in.}/240\text{ in.}\\ &= 0.00225 < 0.01\ \left(=1/100\right) \quad \textbf{o.k.}\end{aligned}$$

Roof

$$\begin{aligned}\Delta/L &= \left(0.915\text{ in.} - 0.539\text{ in.}\right)/240\text{ in.}\\ &= 0.00157 < 0.01\ \left(=1/100\right) \quad \textbf{o.k.}\end{aligned}$$

Check Magnitude of Second-Order Effects After Finalizing the Design

Second-order effects are checked in the following for the load combination $1.6(D + 0.75W + 0.75L + 0.75L_r)$; however, all load combinations must be checked. A computer program is used to find the first-order and second-order deflections.

Roof

$$\begin{aligned}\Delta_{2nd}/\Delta_{1st} &= \left(1.16\text{ in.} - 0.689\text{ in.}\right)/\left(1.02\text{ in.} - 0.595\text{ in.}\right)\\ &= 1.11 \le 1.5 \quad \textbf{o.k.}\end{aligned}$$

Floor

$$\begin{aligned}\Delta_{2nd}/\Delta_{1st} &= 0.689\text{ in.}/0.595\text{ in.}\\ &= 1.16 \le 1.5 \quad \textbf{o.k.}\end{aligned}$$

Therefore, the initial assumption that drift from a second-order analysis is less than 1.5 times the drift from a first-order analysis at both levels of the structure is confirmed.

Note that unreduced axial stiffness (EA) of the members is used for this check. (Alternatively, this check can be made using the reduced stiffness model but changing the limit from 1.5 to 1.71.)

Note: It is common practice to use two-story columns in tiered buildings to minimize splice connection costs. Thus, in this problem, the first floor columns are extended to the roof to omit the cost of the column splice.

Observations

1. Second-order effects in the columns and braces are small but not insignificant for this problem. This is typical for many low-rise braced frame structures.
2. Drift does not control the design for this frame. This is typical of many braced frame structures in low-rise buildings.
3. The direct analysis method and the effective length method produce identical member sizes for this problem. See Example 2.1 for the solution to this problem using the ELM.
4. A comparison between the direct analysis method and the effective length method shows that, for this example, the controlling load combinations change.

5. For a braced frame, the direct analysis method increases the drift and thus, the second-order forces when compared to the effective length method. As a result, there may be less benefit or a penalty for braced frames.
6. The design of the tension bracing is governed by the strength requirements associated with one of the lateral load combinations. This is expected in braced frame structures.
7. The destabilizing effect of leaning columns should always be included in the design of the lateral load resisting system, regardless of the type of framing. In this example, the influence of the leaning columns is included through the second-order analysis, which included all of the leaning columns and their loads.

Example 3.2—Large One-Story Warehouse Building

Given:

Design the braced frames and moment frames in the warehouse building shown in Figure 3-7 for gravity, snow and wind loads as specified by ASCE/SEI 7 load combinations using LRFD. Use the direct analysis method (DM). Maintain columns no larger than the W24 series and beams no larger than the W30 series. Assume out-of-plumbness, $\Delta_o/L = 0.002$, the maximum permitted by the AISC *Code of Standard Practice.* Design for interstory drift control, $\Delta/L = 1/100$, for nominal wind load. Use ASTM A992 steel for wide-flange shapes and ASTM A36 steel for tension X-bracing. Assume the roof deck provides a rigid diaphragm and that the outside walls are light metal panels that span vertically between the roof and the ground floor. (Note: This defines the wind load path to the lateral load resisting system.)

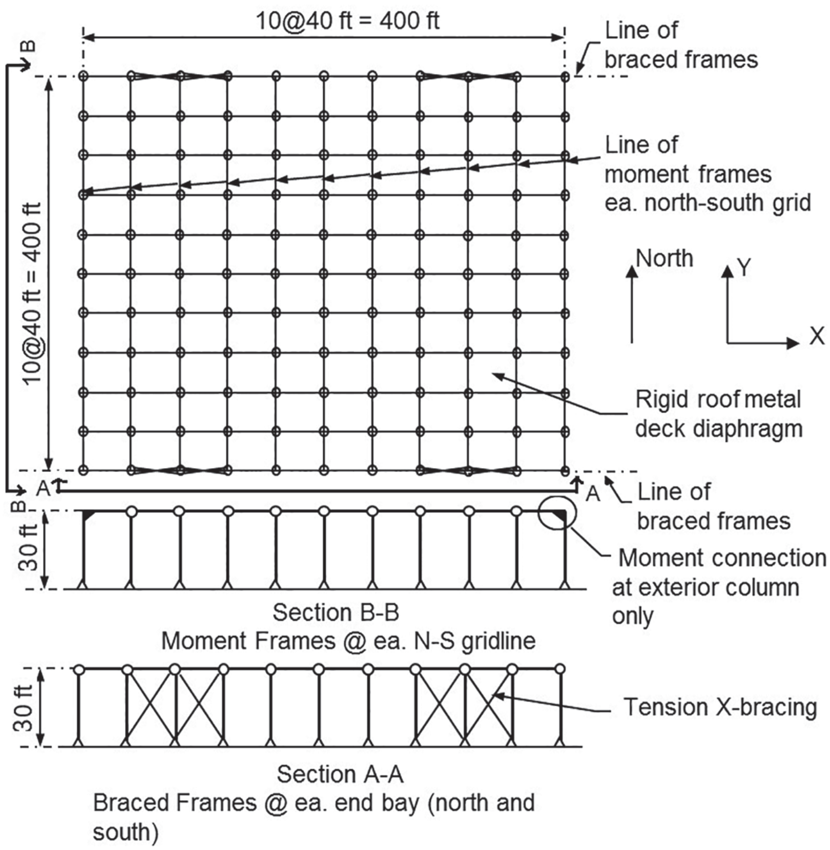


Fig. 3-7. Example 3.2 plan and sections.

Table 3-7. Preliminary Member Sizes	
Member	**Size**
Tension Brace (T1, T2)	1¼-in.-diameter rod
Moment Frame Beam (B1)	W24×131
Moment Frame Column (C1)	W24×117

The loading is as follows:

Dead Load, D = 25 psf (not including steel self-weight of roof beams, columns, braced and moment frames)
Snow Load, S = 30 psf
Wind Load, W = 20 psf

(Note: Wind load on roof surfaces as required by ASCE/SEI 7 is not addressed in this problem.)

Solution:

From AISC *Manual* Table 2-3, the material properties are as follows:

Beams and columns
ASTM A992
$F_y = 50$ ksi
$F_u = 65$ ksi

Bracing
ASTM A36
$F_y = 36$ ksi
$F_u = 58$ ksi

The analysis was performed using a general second-order elastic analysis program including both P-Δ and P-δ effects. Refer to Section 1.7 for a more detailed discussion of the analysis computer models utilized in this Design Guide. See Section 3.2 for a step-by-step description of the DM.

Description of Framing

All lateral load resistance in the east-west direction is provided by tension only X-bracing in the north and south end bays as shown in Section A-A of Figure 3-7. All lateral load resistance in the north-south direction is provided by moment frames on each north-south column line. A moment connection is provided between the exterior column and beam at the end bay of each north-south frame as specified in Section B-B of Figure 3-7. Assume that this moment connection is field welded with a complete-joint-penetration weld at the beam flange-to-column flange connection. A beam-to-column web connection is provided using a bolted single-plate connection (see AISC *Manual* Figure 12-4(a) for moment connection detail). The tension rod-to-gusset connections are assumed to be pinned connections using a standard clevis and pin (see AISC *Manual* Tables 15-3 and 15-7). Beams within the braced frame are oriented with the web vertical and are bolted into the column flanges using double angles (see AISC *Manual* Figure 13-2(a)). A single gusset plate connecting the tension rod is shop welded to the beam flange and field bolted to the column flange (see AISC *Manual* Figure 13-2(a)). All other columns outside the braced frames and moment frame bays are leaning columns with simple beam-to-column connections.

Design Approach

Design is an iterative process. The first step is to estimate member sizes that are used in an analysis and then check for conformance to the strength requirements of the AISC *Specification* and serviceability drift limits that are established by the designer. In this type of building, braced frames are often controlled by strength limit states while moment frames are controlled by stiffness. The selection of preliminary sizes is not shown but is obtained based on consideration of stiffness or service drift for the moment frames and strength for the braced frames. The preliminary sizes for the members defined in Figures 3-8 and 3-9 are given in Table 3-7.

Estimate of Framing Weight

Assume that roof purlins at 6 ft 8 in. center-to-center spacing in the east-west direction span to girders in the north-south direction located at column lines. Assume 7 psf framing self-weight for all steel framing, including purlins, columns, beams and braces. (Note: Use this weight for calculating the minimum lateral loads.)

The estimated dead load is:

Dead load = 25 psf + 7 psf = 32 psf

(Note: The weights specified include an allowance for all steel framing and exterior cladding.)

Analysis

Perform a second-order elastic analysis with a computer program that accurately includes both P-Δ and P-δ effects. The second-order effects are evaluated using LRFD load combinations. Refer to Section 1.7 for more details of the computer model analysis.

Before applying the minimum or notional lateral loads as required for the DM, verify the following:

1. Structure meets the interstory drift limit, $\Delta/L = 1/100$, under nominal wind load.
2. Determine whether $\Delta_{2nd}/\Delta_{1st} \leq 1.5$. This is used to determine whether notional lateral loads are applied as minimum lateral loads to gravity-only load combinations or are applied in all load combinations (AISC *Specification* Appendix 7.3).

These checks will be included in the following design calculations.

Serviceability Drift Limit

Wind drift for the moment frames in the north-south direction is evaluated first since drift often controls the sizes in moment frames. Wind drift is evaluated using the following serviceability load combination, where W is the specified nominal wind load:

$$1.0D + 0.5S + 1.0W$$

The first- and second-order roof deflections from a computer analysis using the preliminary sizes given in Table 3-7 for wind in the north-south direction are:

$\Delta_{1st} = 1.85$ in.
$\Delta_{2nd} = 2.74$ in.
< 360 in./100 specified drift limit = 3.60 in. **o.k.**

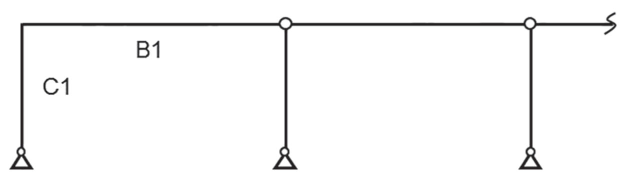


Fig. 3-8. North-south moment frame.

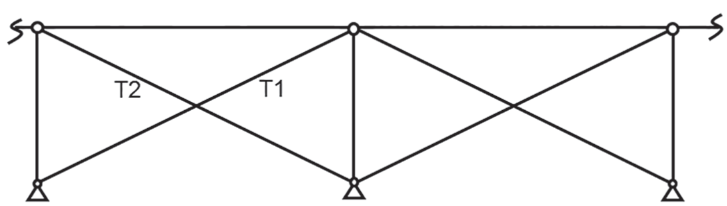


Fig. 3-9. East-west braced frame.

Table 3-8. LRFD Load Combinations
Comb1 = $1.4D + N_x$
Comb2 = $1.4D - N_x$
Comb3 = $1.4D + N_y$
Comb4 = $1.4D - N_y$
Comb5 = $1.2D + 1.6S + N_x$
Comb6 = $1.2D + 1.6S - N_x$
Comb7 = $1.2D + 1.6S + N_y$
Comb8 = $1.2D + 1.6S - N_y$
Comb9 = $1.2D + 0.5S + 1.6W_x$
Comb10 = $1.2D + 0.5S - 1.6W_x$
Comb11 = $1.2D + 0.5S + 1.6W_y + N_y$
Comb12 = $1.2D + 0.5S - 1.6W_y - N_y$
Comb13 = $1.2D + 1.6S + 0.8W_x$
Comb14 = $1.2D + 1.6S - 0.8W_x$
Comb15 = $1.2D + 1.6S + 0.8W_y + N_y$
Comb16 = $1.2D + 1.6S - 0.8W_y - N_y$
Comb17 = $0.9D + 1.6W_x$
Comb18 = $0.9D - 1.6W_x$
Comb19 = $0.9D + 1.6W_y + N_y$
Comb20 = $0.9D - 1.6W_y - N_y$
x, y = direction of forces in plan x = east-west braced frame direction y = north-south moment frame direction D = nominal dead load S = nominal snow load W_x, W_y = nominal wind load in x-, y-direction N_x, N_y = notional load in x-, y-direction Note: Because this structure is not sensitive to quartering winds, those load combinations are not included in this summary for brevity.

Note that, for the purpose of calculating wind drift, the nominal wind load, W, is used as required in the problem statement. Wind drift limits and the appropriate serviceability load combination are not specified in the building code and are a matter of engineering judgment. A second-order analysis is used in the wind drift calculation to more accurately determine the damage potential. Note that there is a 48% increase in wind drift from second-order effects as a result of the gravity load from the large number of leaning columns acting on the frame.

It is assumed that wind drift for the braced frames in the east-west direction will be small and the sizes controlled by strength.

LRFD Load Combinations

A complete list of all the load combinations considered from ASCE/SEI 7 Section 2.3 for both orthogonal (x, y) directions is shown in Table 3-8. It will be demonstrated later that the $\Delta_{2nd}/\Delta_{1st}$ is more than 1.5 in the north-south moment frame direction and less than 1.5 in east-west braced frame direction. Therefore, the notional loads must be applied to all load combinations in the north-south direction, and to the gravity-only load combinations in the east-west direction. Accordingly, the structure is analyzed for the load combinations in Table 3-8. The load combinations shown include positive and negative values on lateral

Table 3-9. Direct Analysis Method Final Sizes	
Member	**Size**
Tension Brace (T1, T2)	1¼- in.- diameter rod
Moment Frame Beam (B1)	W24×146
Moment Frame Column (C1)	W24×117

loads to allow automated design and eliminate the need for symmetry of loading and member sizes. For design that is performed manually, the number of load combinations can be reduced by half as long as symmetry of member size selection is maintained in the analysis and design.

Member Property Modifiers to be Used in the Analysis for Strength Limit State

Axial stiffness: $EA^* = 0.8EA$
Flexural stiffness: $EI^* = 0.8EI$ for analysis
τ_b is assumed and confirmed later as 1.0

Notional Loads: N_x, N_y

Notional loads are applied as required for each load combination shown in Table 3-8. Calculations are shown below to determine the total notional load for two typical load combinations:

Comb5: $1.2D + 1.6S + N_x$

$$\begin{aligned} D &= (32 \text{ psf})(400 \text{ ft})(400 \text{ ft})/1{,}000 \text{ lb / kip} \\ &= 5{,}120 \text{ kips} \\ S &= (30 \text{ psf})(400 \text{ ft})(400 \text{ ft})/1{,}000 \text{ lb / kip} \\ &= 4{,}800 \text{ kips} \\ N_x &= 0.002\left[(1.2\ )(5{,}120 \text{ kips}) + (1.6)(4{,}800 \text{ kips})\right] \\ &= 27.6 \text{ kips} \end{aligned}$$

Comb7: $1.2D + 1.6S + N_y$

$$\begin{aligned} D &= (32 \text{ psf})(400 \text{ ft})(400 \text{ ft})/1{,}000 \text{ lb / kip} \\ &= 5{,}120 \text{ kips} \\ S &= (30 \text{ psf})(400 \text{ ft})(400 \text{ ft})/1{,}000 \text{ lb / kip} \\ &= 4{,}800 \text{ kips} \\ N_y &= 0.002\left[(1.2)(5{,}120 \text{ kips}) + (1.6)(4{,}800 \text{ kips})\right] \\ &= 27.6 \text{ kips} \end{aligned}$$

Member Forces for Strength Design

Several analysis iterations are performed with increased member sizes above the preliminary design sizes shown earlier. The final sizes are shown in Table 3-9. The subsequent strength design checks are shown for these sizes.

The moment shown for the column includes M_r at end i and end j of the member. Note that column moments at the base are zero in the table because of the assumption of a pinned end at the base. The moment shown for the beam includes M_r at end i and at midspan m (end j moments are zero). The axial force in the beams is relatively small and is neglected.

Table 3-10. Second-Order Displacement Check for Application of Notional Load in East-West Direction			
Load Combination	Δ_{1st}, in.	Δ_{2nd}, in.	$\Delta_{2nd}/\Delta_{1st}$
Comb1	0.0582	0.0623	1.07
Comb2	0.0582	0.0623	1.07
Comb5	0.112	0.129	1.15
Comb6	0.112	0.129	1.15
Comb9	0.648	0.703	1.08
Comb10	0.648	0.703	1.08
Comb13	0.338	0.388	1.15
Comb14	0.338	0.388	1.15
Comb17	0.642	0.671	1.05
Comb18	0.642	0.671	1.05

Table 3-11. Second-Order Displacement Check for Application of Notional Load in North-South Direction			
Load Combination	Δ_{1st}, in.	Δ_{2nd}, in.	$\Delta_{2nd}/\Delta_{1st}$
Comb3	0.206	0.289	1.40
Comb4	0.206	0.289	1.40
Comb7	0.397	0.890	2.24
Comb8	0.397	0.890	2.24
Comb11	3.00	4.56	1.52
Comb12	3.00	4.56	1.52
Comb15	1.77	3.98	2.25
Comb16	1.77	3.98	2.25
Comb19	2.89	3.54	1.22
Comb20	2.89	3.54	1.22

Check for $\Delta_{2nd}/\Delta_{1st}$ Drift in East-West Direction

The $\Delta_{2nd}/\Delta_{1st}$ quantity is evaluated using the nominal properties of the lateral load resisting members and is shown in Table 3-10. The purpose of this check is to see if the notional loads need to be added to all load combinations in the east-west direction. It was assumed that $\Delta_{2nd}/\Delta_{1st} < 1.5$ so that the notional loads were only needed in the gravity only load combinations. The values presented in Table 3-10 confirm this assumption. This check could also be made using the reduced stiffness model if the second-order limit was changed from 1.5 to 1.71.

Check for $\Delta_{2nd}/\Delta_{1st}$ Drift in North-South Direction

In the north-south moment frame direction, the $\Delta_{2nd}/\Delta_{1st}$ drift ratios are greater than 1.5 as shown in Table 3-11. Any single value greater than 1.5 means that the notional load must be added to all load combinations in that direction. Therefore, the assumption of applying the notional loads additively to these load combinations was correct.

Representative Member Strength Design Checks

The final member sizes given in Table 3-9 are checked in the following.

North-South Moment Frame, Typical Interior Frame, Column C1

Table 3-12. Direct Analysis Method Member Forces—North-South Moment Frame					
Combination		C1		B1	
		P_r, kips		P_r, kips	
		$M_{r\text{-}i}$	$M_{r\text{-}j}$	$M_{r\text{-}i}$	$M_{r\text{-}m}$
Comb3	P_r, kips	-39.6		0	
	M_r, kip-in.	0	1810	-1810	3400
Comb4	P_r, kips	-41.1		0	
	M_r, kip-in.	0	2550	-2550	3030
Comb7	P_r, kips	-74.8		0	
	M_r, kip-in.	0	2720	-2720	6940
Comb8	P_r, kips	-80.9		0	
	M_r, kip-in.	0	5660	-5660	5460
Comb11	P_r, kips	-35.5		0	
	M_r, kip-in.	0	-3440	3440	6840
Comb12	P_r, kips	-60.7		0	
	M_r, kip-in.	0	8610	-8610	822
Comb15	P_r, kips	-64.1		0	
	M_r, kip-in.	0	-2420	2420	**9490**
Comb16	P_r, kips	**-91.6**		0	
	M_r, kip-in.	0	**10800**	-10800	2910
Comb19	P_r, kips	-16.9		0	
	M_r, kip-in.	0	-2930	2930	4220
Comb20	P_r, kips	-35.0		0	
	M_r, kip-in.	0	5720	-5720	-92.8

The governing load combination is Comb16 from Table 3-12, where $P_r = 91.6$ kips (compression) and $M_r = 10{,}800$ kip-in.

From AISC *Manual* Table 1-1, the properties of the W24×117 are as follows:

W24×117

$A = 34.4 \text{ in.}^2$

$r_y = 2.94$ in.

$h/t_w = 39.2$

$b_f/2t_f = 7.53$

For the DM, $K_2 = K = 1$, and the effective length is:

$$KL = (30 \text{ ft})(12 \text{ in./ft})$$
$$= 360 \text{ in.}$$

AISC *Manual* Table 1-1 indicates that the W24×117 is slender for compression when $F_y = 50$ ksi; therefore, the design compressive strength is determined as follows from AISC *Specification* Section E7. The y-axis slenderness ratio, KL/r_y, controls the strength ($KL/r_y > KL/r_x$).

$$\frac{KL}{r_y} = \frac{360 \text{ in.}}{2.94 \text{ in.}}$$
$$= 122$$

The elastic critical buckling stress is:

$$F_e = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2} \qquad \text{(\textit{Spec}. Eq. E3-4)}$$

$$= \frac{\pi^2 (29{,}000 \text{ ksi})}{(122)^2}$$

$$= 19.2 \text{ ksi}$$

With $Q = 1$, and $F_e < 0.44F_y = 22$ ksi, the flexural buckling stress is:

$$F_{cr} = 0.877F_e \qquad \text{(\textit{Spec}. Eq. E3-3)}$$

$$= 0.877(19.2 \text{ ksi})$$

$$= 16.8 \text{ ksi}$$

From AISC *Specification* Section E7:

$Q = Q_s Q_a$ for members with slender elements

Determine Q_a from AISC *Specification* Section E7.2. Determine whether $\frac{h}{t_w} = 39.2 \geq 1.49\sqrt{\frac{E}{F_{cr}}}$:

$$1.49\sqrt{\frac{E}{F_{cr}}} = 1.49\sqrt{\frac{29{,}000 \text{ ksi}}{16.8 \text{ ksi}}}$$

$$= 61.9$$

Because $\frac{h}{t_w} = 39.2 < 61.9$, then $Q_a = 1.0$.

Determine Q_s from AISC *Specification* Section E7.1. Determine whether $\frac{b_f}{2t_f} \leq 0.56\sqrt{\frac{E}{F_y}}$:

$$0.56\sqrt{\frac{E}{F_y}} = 0.56\sqrt{\frac{29{,}000 \text{ ksi}}{50 \text{ ksi}}}$$

$$= 13.5$$

Because $\frac{b_f}{2t_f} = 7.53 \leq 13.5$, then $Q_s = 1.0$ and $Q = Q_s Q_a = 1.0$.

The nominal compressive strength is:

$$P_n = F_{cr} A_g \qquad \text{(\textit{Spec}. Eq. E3-1)}$$

$$= (16.8 \text{ ksi})(34.3 \text{ in.}^2)$$

$$= 578 \text{ kips}$$

The design compressive strength as defined in AISC *Specification* Section E1 is:

$$\phi_c P_n = 0.90(578 \text{ kips})$$

$$= 520 \text{ kips} > 91.6 \text{ ksi}$$

Table 3-13. Direct Analysis Method Member Forces—East-West Braced Frame		
Combination	**T1, kips**	**T2, kips**
Comb1	2.47	0
Comb2	0	2.47
Comb5	5.28	0
Comb6	0	5.28
Comb9	33.2	0
Comb10	0	33.2
Comb13	17.9	0
Comb14	0	17.9
Comb17	31.5	0
Comb18	0	31.5

Determine the design flexural strength of the column. From AISC *Manual* Table 3-2, for a W24×146:

L_p = 10.4 ft
L_r = 30.4 ft
L_b = 30 ft
$\phi_b M_{px}$ = 1,040 kip-ft
BF = 17.1 kips

The required flexural strength at end i is $M_{ri} = 0$, and the required flexural strength at end j is M_{rj} = 10,800 kip-in.

Because L_p < 30 ft < L_r, use the following equation taken from the AISC *Manual* and the variables from AISC *Manual* Table 3-2 to interpolate between the available strength at L_p and the available strength at L_r. From AISC *Manual* Table 3-1 and AISC *Specification* Equation F1-1, C_b = 1.67 (note that there is a linear moment diagram between the support and the brace point). Therefore, the design flexural strength is determined as follows:

$$\begin{aligned}\phi_b M_n &= C_b\left[\phi_b M_{px} - BF\left(L_b - L_p\right)\right] \le \phi_b M_{px}\\ &= 1.67\left[1{,}040 \text{ kip-ft} - (17.1 \text{ kips})(30.0 \text{ ft} - 10.4 \text{ ft})\right]\\ &= 1{,}180 \text{ kip-ft}\end{aligned}$$

Because 1,180 kip-ft > $\phi_b M_{px}$, then $\phi_b M_n = \phi_b M_{px}$ = 1,040 kip-ft.

$\phi_b M_n$ = (1,040 kip-ft)(12 in./ft)
= 12,500 kip-in. > 10,800 kip-in. **o.k.**

Check the interaction of compression and flexure using AISC *Specification* Section H1:

$$\begin{aligned}\frac{P_r}{P_c} &= \frac{91.6 \text{ kips}}{520 \text{ kips}}\\ &= 0.176 < 0.2\end{aligned}$$

Therefore use AISC *Specification* Equation H1-1b:

$$\frac{P_r}{2P_c} + \left(\frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}}\right) \le 1.0 \qquad \text{(\textit{Spec}. Eq. H1-1b)}$$

$$\frac{0.176}{2} + \left(\frac{10{,}800 \text{ kip-in.}}{12{,}500 \text{ kip-in.}} + 0\right) = 0.952 \le 1.0 \qquad \textbf{o.k.}$$

Note: By inspection $P_r/(AF_y) < 0.5$ (AISC *Specification* Appendix 7, Section 7.3(3)). The assumption that $\tau_b = 1$ is o.k.

North-South Moment Frame, Beam B1

The governing load combination for member B1 is Comb15 from Table 3-12 where the midspan moment is $M_{r\text{-}m} = 9{,}490$ kip-in.

From AISC *Manual* Table 3-2, for a W24×146:

L_p = 10.6 ft
L_r = 33.7 ft
L_b = 6.67 ft (purlin spacing) < L_p = 10.6 ft
$\phi_b M_{px}$ = 1,570 kip-ft

Because $L_b < L_p$, $\phi_b M_n = \phi_b M_p = 1{,}570$ kip-ft, according to AISC *Specification* Section F2. (Note: For $L_b = 6.67$ ft, a W30×99 with $L_p = 7.42$ ft > 6.67 ft would work. However, the more flexible beam would increase the drift and second-order effects, so keep the larger size beam shown.)

$$\begin{aligned}\phi_b M_n &= (1{,}570 \text{ kip-ft})(12 \text{ in./ft})\\ &= 18{,}800 \text{ kip-in.} > M_r = 9{,}490 \text{ kip-in.} \quad \textbf{o.k.}\end{aligned}$$

East-West Braced Frame, Tension Only Member, T1

The required tensile strength of the 1¼-in.-diameter rod is:

$P_r = 33.2$ kips for Comb9 from Table 3-13

The design tensile yielding strength is:

$$\phi_t P_n = \phi_t F_y A_g \qquad \text{(from } \textit{Spec}\text{. Eq. D2-1)}$$

$$\begin{aligned}&= 0.90(36 \text{ ksi})\left[\pi\left(1\tfrac{1}{4} \text{ in.}\right)^2/4\right]\\ &= 39.8 \text{ kips} > 33.2 \text{ kips} \quad \textbf{o.k.}\end{aligned}$$

The design tensile rupture strength is (assume the effective net tension area is $0.75A_g$; this must be confirmed once connections are completed):

$$\phi_t P_n = \phi_t F_u A_e \qquad \text{(from } \textit{Spec}\text{. Eq. D2-2)}$$

$$\begin{aligned}&= 0.75(58 \text{ ksi})\left[(0.75)(\pi)\left(1\tfrac{1}{4} \text{ in.}\right)^2/4\right]\\ &= 40 \text{ kips} > 33.2 \text{ kips} \quad \textbf{o.k.}\end{aligned}$$

Observations

1. The DM provides a more economical design for the moment frame portion of this problem compared to the ELM because there is no $\Delta_{2nd}/\Delta_{1st} \le 1.5$ limitation.
2. There were no seismic requirements for this problem, but the lateral load system in the north-south moment frame direction may not be satisfactory in higher seismic zones because of the magnitude of the second-order effects. This would need to be checked according to ASCE/SEI 7 Section 12.8.7.
3. The moment frame design is strongly influenced by the large leaning column load and the fact that there are relatively few moment connections.
4. The braced frame design is controlled by strength, which is common for braced frames. Because there is no second-order stiffness $\Delta_{2nd}/\Delta_{1st} \le 1.5$ limit for the DM, the moment frame design was controlled by the strength check as well.

Chapter 4
First-Order Analysis Method (FOM)

4.1 INTRODUCTION

The AISC *Specification* Section C2.2b [2010 AISC *Specification* Appendix 7, Section 7.3] gives a conservative simplified procedure for stability design of steel frames where the analysis can be done by a first-order analysis, either using a computer program or hand methods. For the first-order analysis method (FOM) to apply, B_2 must be less than or equal to 1.5 and αP_r must be less than or equal to $0.5P_y$ for all the members whose flexural stiffnesses contribute to the lateral stability. Similar to the DM, the effective length factor may be taken equal to 1.0 in this method. The method requires the application of an additional notional lateral load at each level i in all load combinations, as follows:

$$N_i = 2.1\left(\Delta/L\right)Y_i \geq 0.0042\left(Y_i/\alpha\right) \quad \text{(from } Spec. \text{ Eq. C2-8)}$$

where

Y_i = gravity load from the LRFD load combination or 1.6 times the ASD load combination applied at level i, kips

Δ/L = maximum first-order interstory drift ratio for all stories in the structure due to design loads (the ASD design loads without the 1.6 multiplication factor for design by ASD, or the LRFD loads for design by LRFD), without the notional loads, N_i, applied.

Update Note: In the 2010 AISC *Specification*, the first-order analysis method, identified by name (it is not so identified in the 2005 edition), is presented in Appendix 7, Section 7.3, Alternative Methods of Design for Stability. Equation C2-8 of the 2005 AISC *Specification* is presented in a different form as Equation A-7-2 in the 2010 AISC *Specification.*

If the designer chooses to use ASD, then Y_i is based on 1.6 times the ASD loads and the right hand side of Equation C2-8 ($0.0042Y_i$) should be divided by $\alpha = 1.6$ (since Y_i is defined as the load at 1.6 times the ASD load combination). The reader should note that this is a correction to the 2005 AISC *Specification*. As published in the 2005 AISC *Specification*, the right hand side of Equation C2-8 sets the lower limit on the notional load at an artificially high level for design by ASD. In addition, the user note incorrectly directs the use of α in the determination of the first-order interstory drift. These corrections are made in the presentation of this Design Guide. Refer to Appendix B for background information on development of this method.

4.2 STEP-BY-STEP PROCEDURE

The FOM may be used for design by LRFD or ASD. Following is a detailed procedure for applying the FOM:

1. Develop a model of the building frame that captures all the essential aspects of first-order elastic frame behavior. Remember to account for three-dimensional (3D) aspects of the loading for wind where required (ASCE/SEI 7 Figure 6-9) and seismic loads (ASCE/SEI 7 Section 12.5) as prescribed by the applicable building code. Use the nominal geometry and nominal member properties in the first-order analysis.
2. Determine all gravity loads that are stabilized by the lateral load resisting system.
3. Determine the lateral loads corresponding to the wind, seismic and notional load requirements. In the FOM, the notional loads are calculated and applied as follows:
   (a) Select a target design drift limit, Δ/L, (not the same as the serviceability drift ratio, Δ_s/L, unless the lateral load factors are the same for service drift and the strength checks) to be enforced at all levels of the structure for each load combination at the LRFD strength design load levels if LRFD is used or the ASD strength design load levels if ASD is used.
   (b) Calculate $N_i = 2.1(\Delta/L)Y_i \geq 0.0042(Y_i/\alpha)$; the notional load at level i, where Y_{i1} = gravity load at level i from the LRFD or ASD load combination being considered.
   (c) Apply N_i as a minimum or an additional lateral load with all the load combinations. The notional loads should be applied only in the direction that adds to the effect of the lateral loads. One need not apply notional loads in a direction opposite from the lateral loads to minimize the reduction in internal forces in certain components due to the lateral load. The notional lateral loads, N_i, should be applied independently about each of two orthogonal building axes. These axes should be selected as approximate principal lateral stiffness directions for the overall building structure. Note: "independently" means that the notional loads are applied only in one direction at a time. For structures where N_i must be applied additively, and when considering combination load effects such as required by ASCE/SEI 7 for wind and seismic loading, the notional loads should be included as part of each independent lateral loading.

For instance, as part of the diagonal wind loading requirement in Figure 6-9 of ASCE/SEI 7, 75% of the respective notional loads should be applied simultaneously along each principal direction in addition to the wind load. No torsional notional load is required to mimic the code eccentric torsion wind and seismic loadings.

(d) For general structural analysis, the notional loads may be applied at each location where gravity load is transferred to the structural columns. The load Y_i is the gravity load transferred to the columns at each of these locations.

4. Perform a first-order analysis for gravity and lateral loads either separately or in combination with the gravity loads at the convenience of the designer. Note that since only a first-order analysis is performed with this method, superposition of load cases is applicable, so the gravity and lateral load analyses can be linearly combined. The lateral load analysis does not have to include the effects of gravity loads since this effect is embedded conservatively within the calculation of the required notional load.

5. Confirm for each load combination, without the notional loads applied, that the average story drifts, Δ/L, at each level of the frame are less than or equal to the target drift used in step 3(a) in setting N_i. Stiffen the story members as required to satisfy this story drift limit.

6. Confirm that $\alpha P_r/P_y \leq 0.5P_y$ for all columns whose flexural stiffness contributes to the lateral stability of the structure and adjust column sizes as required to conform to this limit.

7. Using AISC *Specification* Equations C2-3 and C2-6b [2010 AISC *Specification* Equations A-8-6 and A-8-7] confirm that B_2 for each level under all load combinations (without the notional loads applied) is less than or equal to 1.5. Stiffen the frame as required.

8. Apply the AISC nonsway amplifier, B_1, to the total member moments.

9. Design the various members and connections for the forces obtained from the above procedure using the applicable provisions of the AISC *Specification*.

10. For all beam-columns, apply the interaction AISC *Specification* Equations H1-1a and H1-1b (or where applicable, Equation H1-2 at the designer's option) using an effective length factor, $K = 1.0$.

11. Check the seismic drift limits according to ASCE/SEI 7 Section 12.12 and the maximum P-Δ effects as prescribed by ASCE/SEI 7 Section 12.8.7. Note that P-Δ effects are checked for a load factor no greater than 1.0 on all vertical design loads, P_x, in ASCE/SEI 7 Equation 12.8-16.

12. Check the wind drifts for service level wind loads (mean recurrence interval for wind at designer's option). Note that this check is a serviceability check, not a code requirement. Also note that for moment frames, drift under wind and/or seismic load levels will typically control the design. Therefore, this check should be made first in initial proportioning of member sizes for these frame types. It is recommended in this guide that serviceability wind drift checks be made considering second-order effects when the wind drift checks are intended to be used to accurately control actual damage to cladding and partitions.

The FOM is used to solve two example problems at the end of this chapter.

4.3 ADVANTAGES, DISADVANTAGES AND RESTRICTIONS ON USAGE

The advantages of the FOM are:

1. The FOM is simpler to use than either the DM or the ELM since it requires only a first-order analysis and superposition of the basic load cases is applicable. However, a second-order analysis check of B_2 is required to ensure that the ratio of second-order to first-order lateral deflections is less than or equal to 1.5 at each story for each load combination.
2. Superposition of load combinations is permissible.
3. $K = 1.0$ is used in the design of all columns.

The disadvantages of the FOM are:

1. The method is applicable only when columns in the lateral load resisting system are lightly loaded, such that $\alpha P_r \leq 0.5P_y$.
2. The method is restricted to frames in which the second-order amplification of the sidesway displacements, $\Delta_{2nd}/\Delta_{1st}$ or B_2, is less than or equal to 1.5.
3. The method, because it is based on an implicitly assumed $B_2 = 1.5$ in the derivation for the notional lateral load, N_i, can result in substantial conservatism for the design of some types of frames.

The restrictions on use of the FOM are those specified in disadvantages 1 and 2:

1. Columns in the lateral load resisting system are lightly loaded, such that $\alpha P_r \leq 0.5P_y$.
2. $\Delta_{2nd}/\Delta_{1st}$ or B_2 is less than or equal to 1.5.

4.4 SUMMARY OF DESIGN RECOMMENDATIONS

Following is a summary of design recommendations for application of the FOM.

1. Consider using the FOM for regular rectangular frames where a first-order analysis is desired and conservative results are acceptable. For this method to apply, B_2 must be less than or equal to 1.5 and αP_r must be less than $0.5P_y$ for all the members whose flexural resistances contribute to the lateral stability.
2. This method may also be used for fast, conservative preliminary designs where either the ELM or the DM is intended to be used for final design.

4.5 DESIGN EXAMPLES

Refer to Section 1.7 for a background discussion of the solution to the example problems. The design examples are implemented using the 2005 AISC *Specification* and 13th Edition AISC *Manual.*

Example 4.1—Two-Story Warehouse, Typical Braced Frame Building

Given:

Size the braced frame columns, beams and rod bracing for a typical bay of the braced frame building shown in Figure 4-1. Consider dead, live and wind load combinations using ASCE/SEI 7 load combinations and design by ASD. Consider wind load in the plane of the braced frame only. Solve using the first-order analysis method (FOM). All columns are braced out-of-plane at the floor and the roof. The lateral load resistance is provided by tension rod bracing only. All beam-to-column connections are simple "pinned" connections. Maintain interstory drift limit, $\Delta/L \leq 1/100$, under nominal wind load using a first-order analysis. Assume nominal out-of-plumbness of $\Delta_o/L = 0.002$. This is the AISC *Code of Standard Practice* maximum permitted out-of-plumbness. All steel is ASTM A992, except that the tension rods are ASTM A36 steel.

The loading is as follows:

Roof

Dead load, w_{RD}	= 1.0 kip/ft
Live load, w_{RL}	= 1.2 kip/ft
Estimated roof beam weight	= 0.076 kip/ft
Estimated roof interior column weight	= 0.065 kip/ft
Estimated roof end column weight	= 0.048 kip/ft
Wind load, W_R	= 10 kips

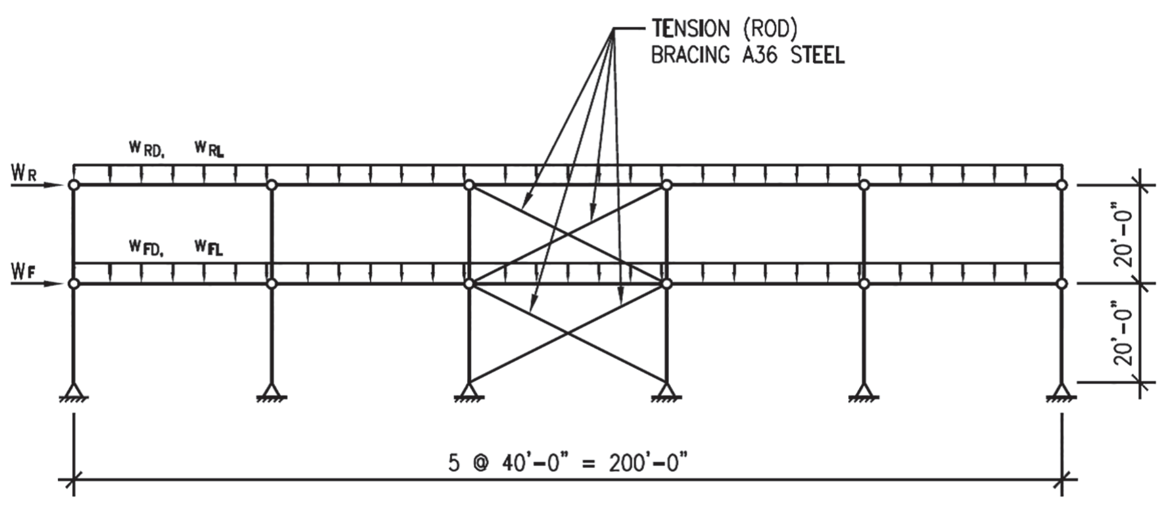


Fig. 4-1. Example 4.1 braced frame elevation.

Floor

Dead load, w_{FD} = 2.4 kip/ft
Live load, w_{FL} = 4.0 kip/ft
Estimated floor beam weight = 0.149 kip/ft
Estimated floor interior column weight = 0.065 kip/ft
Estimated floor end column weight = 0.048 kip/ft
Wind load, W_F = 20 kips

(Note: Wind load on roof surfaces as specified in ASCE/SEI 7 is not considered in this problem.)

Solution:

From AISC *Manual* Table 2-3, the material properties are as follows:

Beams and columns
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Bracing
ASTM A36
F_y = 36 ksi
F_u = 58 ksi

The analysis was performed using a general first-order elastic analysis computer program. See Figure 4-2 for the member labels used. In the first-order analysis method, the required strengths are permitted to be determined using a first-order analysis (AISC *Specification* Section C2.2b). Refer to Section 4.2 for a step-by-step description of the FOM.

Description of Framing

All lateral load resistance is provided by the tension only rod bracing. Tension rods are assumed as pinned connections using a standard clevis and pin (see the AISC *Manual* Tables 15-3 and 15-7). Beams within the braced frame are bolted into the column flanges using double angles (see AISC *Manual* Figure 13-2(a)). A single gusset plate connecting the tension rod is shop welded to the beam flange and field bolted to the column flange (see AISC *Manual* Figure 13-2(a)). All other columns outside the braced frames are leaning columns with simple beam-to-column connections.

Design Approach

Design is an iterative process. Preliminary sizes should be chosen based on experience or a preliminary analysis. Braced frame structures are often controlled by strength as opposed to wind or seismic drift. Thus, for this problem, sizes are estimated from a preliminary strength check and those sizes used in the first-order computer analysis. The member sizes used in the analyses are those shown in Figure 4-4.

Analysis Load Combinations

The member design forces are obtained by analyzing the structure for ASD load combinations. The load combinations in Table 4-1, from ASCE/SEI 7 Section 2.4.1, are used for the first-order analysis.

Additional Lateral Loads

The additional lateral loads that need to be applied are determined as follows:

$$N_i = 2.1(\Delta/L)Y_i \geq 0.0042(Y_i/\alpha) \quad \text{(from } Spec. \text{ Eq. C2-8)}$$

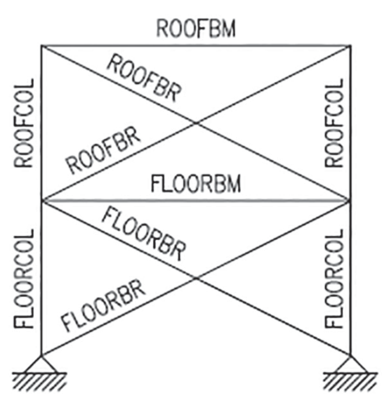


Fig. 4-2. Braced frame member labels.

Table 4-1. ASD Load Combinations
Comb1 = $D + N$
Comb2 = $D + L + N$
Comb3 = $D + L_r + N$
Comb4 = $D + 0.75L + 0.75L_r + N$
Comb5 = $D + W + N$
Comb6 = $D + 0.75W + 0.75L + 0.75L_r + N$
Comb7 = $0.6D + W + N$
D = dead load L = live load L_r = roof live load W = wind load N = additional lateral load for use with all load combinations = $0.002Y_i$ Y_i = gravity load from 1.6 times ASD load combinations applied at level i

Δ/L is the first-order drift ratio under the design loads without N_i applied and can be conservatively taken as the maximum ratio from all design load combinations. See Section 4.1 for discussion about AISC *Specification* Equation C2-8.

In order to estimate the interstory drift ratio, Δ/L, under design loads required for application of Equation C2-8, preliminary member sizes are assumed as shown in Figure 4-3.

From the first-order analysis, the drift at the floor level is 0.408 in. and the drift at the roof is 0.718 in.; therefore, the first-order drift ratios are:

Floor

$$\begin{aligned}\Delta/L &= 0.408\text{ in.}/240\text{ in.}\\ &= 0.00170\end{aligned}$$

Roof

$$\begin{aligned}\Delta/L &= (0.718\text{ in.} - 0.408\text{ in.})/240\text{ in.}\\ &= 0.00129\end{aligned}$$

The interstory drift ratio at the floor level controls; therefore, from the left side of AISC *Specification* Equation C2-8:

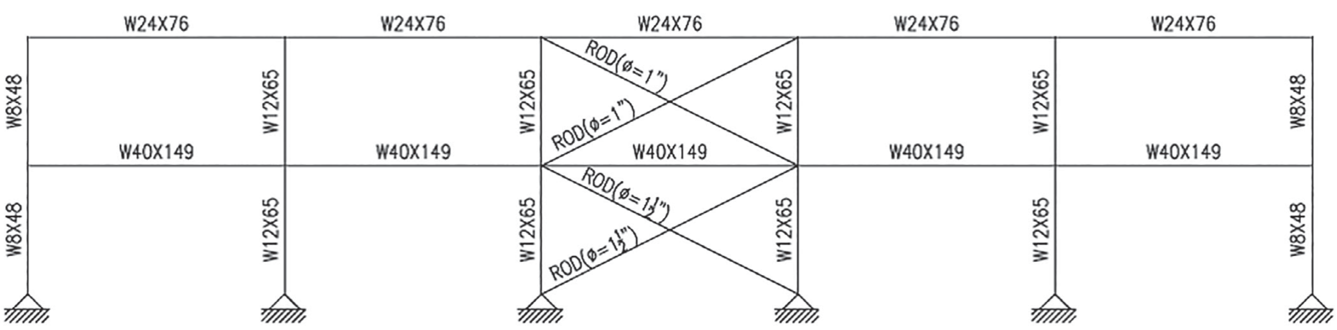


Fig. 4-3. Preliminary member sizes.

Table 4-2. Additional Lateral Loads, N_i, for Each Load Combination, kips							
	Comb1	**Comb2**	**Comb3**	**Comb4**	**Comb5**	**Comb6**	**Comb7**
Roof	1.27	1.27	2.64	2.30	1.27	2.30	0.761
Floor	2.95	7.52	2.95	6.38	2.95	6.38	1.77

$$
\begin{aligned}
N_i &= 2.1(\Delta/L)Y_i \\
&= 2.1(0.00170)Y_i \\
&= 0.00357Y_i
\end{aligned}
$$

From the right side of AISC *Specification* Equation C2-8, as modified in this Design Guide:

$$
\begin{aligned}
0.0042(Y_i/\alpha) &= 0.0042(Y_i/1.6) \\
&= 0.00263Y_i
\end{aligned}
$$

$N_i = 0.00357Y_i$ **controls**

Therefore, use $N_i = 0.00357Y_i$ at each level where Y_i = gravity loads from 1.6 times the ASD load combinations.

These loads, N_i, are to be applied as additional lateral loads with all load combinations. The gravity loads are:

Roof

$$
\begin{aligned}
\Sigma D &= (1.0\text{ kip/ft})(200\text{ ft}) + (0.076\text{ kip/ft})(200\text{ ft}) + (0.065\text{ kip/ft})(20\text{ ft})(4) + (0.048\text{ kip/ft})(20\text{ ft})(2) \\
&= 222\text{ kips} \\
\Sigma L_r &= (1.2\text{ kip/ft})(200\text{ ft}) \\
&= 240\text{ kips}
\end{aligned}
$$

Floor

$$
\begin{aligned}
\Sigma D &= (2.4\text{ kip/ft})(200\text{ ft}) + (0.149\text{ kip/ft})(200\text{ ft}) + (0.065\text{ kip/ft})(20\text{ ft})(4) + (0.048\text{ kip/ft})(20\text{ ft})(2) \\
&= 517\text{ kips} \\
\Sigma L &= (4.0\text{ kip/ft})(200\text{ ft}) \\
&= 800\text{ kips}
\end{aligned}
$$

The additional lateral load, N_i, must be evaluated for each load combination. For example, at the roof level for Comb 3 from Table 4-1:

$$
\begin{aligned}
N_i &= 0.00357Y_i \\
&= 0.00357(1.6)(D + L_r) \\
&= 0.00357(1.6)(222\text{ kips} + 240\text{ kips}) \\
&= 2.64\text{ kips}
\end{aligned}
$$

Additional lateral loads for Comb1 through Comb7 are listed in Table 4-2.

Design of Beams

Member required flexural strengths from a first-order analysis, simple beam moments, are tabulated in Table 4-3 for each load combination. The required flexural strength from the controlling load combination is presented in bold.

The following beam design is for gravity load alone without considering any axial load from the lateral load cases. The amount of axial load in the beams depends on the assumed load path of the forces applied to the lateral load resisting frames (through collector beams and/or floor and roof diaphragm connections to the beams). Even if a conservative estimate of the axial force in these members is included, the influence on the beam-column interaction value is small. The beams are assumed fully braced by the floor and roof diaphragms.

Table 4-3. Required Beam Flexural Strength (ASD)							
	M_r, kip-in.						
Member	**Comb1**	**Comb2**	**Comb3**	**Comb4**	**Comb5**	**Comb6**	**Comb7**
ROOFBM	2580	2580	**5460**	4740	2580	4740	1550
FLOORBM	6120	**15700**	6120	13300	6120	13300	3670

Table 4-4. Required First-Order Column Axial Strength							
	P_r, kips						
Member	**Comb1**	**Comb2**	**Comb3**	**Comb4**	**Comb5**	**Comb6**	**Comb7**
ROOFCOL	43.7	43.7	**91.7**	79.7	43.7	79.7	26.2
FLOORCOL	146	**306**	193	301	141	298	82.8

Roof Beams

The required strength of the roof beams is:

$$\begin{aligned} M_r &= 5{,}460 \text{ kip-in.}/12 \text{ in./ft} \\ &= 455 \text{ kip-ft } (\text{Comb3}) \end{aligned}$$

Try a W24×76. From AISC *Manual* Table 3-2, for a fully braced beam, the allowable flexural strength is:

$$\frac{M_n}{\Omega} = 499 \text{ kip-ft} > M_r \quad \textbf{o.k.}$$

Floor Beams

The required strength of the floor beams is:

$$\begin{aligned} M_r &= 15{,}700 \text{ kip-in.}/12 \text{ in./ft} \\ &= 1{,}310 \text{ kip-ft } (\text{Comb2}) \end{aligned}$$

Try a W40×149. From AISC *Manual* Table 3-2, for a fully braced beam, the allowable flexural strength is:

$$\begin{aligned} \frac{M_n}{\Omega_b} &= \frac{M_{px}}{\Omega_b} \\ &= 1{,}490 \text{ kip-ft} > M_r \quad \textbf{o.k.} \end{aligned}$$

Design of Columns in the Braced Frame

Member required axial compressive strengths from a first-order analysis program for each load combination are tabulated in Table 4-4 with the controlling value in bold.

Roof Columns

The required strength of the roof columns is:

P_r = 91.7 kips (Comb3)
K = 1.0 for braced frame columns

Table 4-5. First-Order Required Brace Axial Strength							
Member	P_r, kips						
	Comb1	Comb2	Comb3	Comb4	Comb5	Comb6	Comb7
ROOFBR	1.42	1.42	4.02	3.37	**12.6**	11.8	12.0
FLOORBR	4.72	13.4	8.40	14.0	38.3	**39.2**	36.4

Try a W8×31.

From AISC *Manual* Table 4-1 with $KL = 20$ ft, the allowable axial compressive strength is:

$$\frac{P_n}{\Omega_c} = 97.1 \text{ kips} > P_r \quad \textbf{o.k.}$$

Floor Columns

The required strength of the floor columns is:

$P_r = 306$ kips (Comb2)
$K = 1.0$ for braced frame columns

Try a W12×65. From AISC *Manual* Table 4-1 with $KL = 20$ ft, the allowable axial compressive strength is:

$$\frac{P_n}{\Omega} = 360 \text{ kips} > P_r \quad \textbf{o.k.}$$

Design of Braces

Member required tensile strengths for each load combination are given in Table 4-5 with the controlling value in bold.

Roof Braces

The required tensile strength of the roof braces is:

$P_r = 12.6$ kips (Comb5)

Try a 1-in.-diameter rod (ASTM A36 steel).

The allowable tensile yielding strength is determined from AISC *Specification* Section D2(a) as follows:

$$\begin{aligned}\frac{P_n}{\Omega_t} &= \frac{F_y A_g}{\Omega_t} \\ &= \frac{(36 \text{ ksi})(\pi/4)(1.00 \text{ in.})^2}{1.67} \\ &= 16.9 \text{ kips} > P_r \quad \textbf{o.k.}\end{aligned}$$

The allowable tensile rupture strength is determined from AISC *Specification* Section D2(b), assuming that the effective net area of the rod is $0.75A_g$, as follows:

$$\begin{aligned}\frac{P_n}{\Omega_t} &= \frac{F_u A_e}{\Omega_t} \\ &= \frac{(58 \text{ ksi})(0.75)(\pi/4)(1.00 \text{ in.})^2}{2.00} \\ &= 17.1 \text{ kips} > P_r \quad \textbf{o.k.}\end{aligned}$$

Floor Braces

The required tensile strength of the floor braces is:

$$P_r = 39.2 \text{ kips (Comb6)}$$

Try a 1⅝-in.-diameter rod (ASTM A36 steel). The allowable tensile yielding strength is determined from AISC *Specification* Section D2(a) as follows:

$$\begin{aligned}\frac{P_n}{\Omega_t} &= \frac{F_y A_g}{\Omega_t} \\ &= \frac{(36 \text{ ksi})(\pi/4)(1\tfrac{5}{8} \text{ in.})^2}{1.67} \\ &= 44.7 \text{ kips} > P_r \quad \textbf{o.k.}\end{aligned}$$

The allowable tensile rupture strength is determined from AISC *Specification* Section D2(b), assuming that the effective net area of the rod is $0.75A_g$, as follows:

$$\begin{aligned}\frac{P_n}{\Omega_t} &= \frac{F_u A_e}{\Omega_t} \\ &= \frac{(58 \text{ ksi})(0.75)(\pi/4)(1\tfrac{5}{8} \text{ in.})^2}{2.00} \\ &= 45.1 \text{ kips} > P_r \quad \textbf{o.k.}\end{aligned}$$

Check Drift under Nominal Wind Load (Serviceability Wind Drift Check)

Drift is checked under a nominal wind load using a first-order analysis. The nominal wind load is specified in this problem to be used in the serviceability load combination and the interstory drift is limited based on $\Delta/L \le 1/100$. The appropriate serviceability load combination is a matter of engineering judgment. The resulting drift at the roof level is 0.718 in. and at the floor level is 0.408 in. The interstory drift is checked as follows:

Floor Level

$$\begin{aligned}\Delta/L &= 0.408 \text{ in.}/240 \text{ in.} \\ &= 0.00170 < 0.01 \; (= 1/100) \quad \textbf{o.k.}\end{aligned}$$

Roof Level

$$\begin{aligned}\Delta/L &= (0.718 \text{ in.} - 0.408 \text{ in.})/240 \text{ in.} \\ &= 0.00129 < 0.01 \; (= 1/100) \quad \textbf{o.k.}\end{aligned}$$

Check Magnitude of Second-Order Effects after Finalizing the Design

Second-order effects are checked in the following to verify that the ratio of second-order to first-order drift is less than or equal to 1.5 as required for the FOM. Since the first-order analysis method is used, it is assumed that the second-order analysis results are not available using a computer program. Second-order effects will be determined using B_2 from AISC *Specification* Section C2.1b.

B_2 = amplification factor

$$= \frac{1}{1-\dfrac{\alpha\Sigma P_{nt}}{\Sigma P_{e2}}} \geq 1 \qquad (Spec. \text{ Eq. C2-3})$$

B_2 is calculated for the load combination, $D + 0.75W + 0.75L + 0.75L_r$, in the following; however, all load combinations must be checked. B_2 will be the maximum for the maximum gravity load that results from this load combination. The check at the floor and roof levels is as follows:

Floor Level

$\alpha\Sigma P_{nt}$ = 1.6 [222 kips + 517 kips + (0.75)(240 kips) + (0.75)(800 kips)]
= 2,430 kips

R_M = 1.0 for braced frames as specified in AISC *Specification* Section C2.1b

$$\Sigma P_{e2} = R_M \frac{\Sigma HL}{\Delta_H} \qquad (Spec. \text{ Eq. C2-6b})$$

$$= 1.0\left[\frac{(10 \text{ kips} + 20 \text{ kips})(20 \text{ ft})(12 \text{ in./ft})}{0.408 \text{ in.}}\right]$$

$$= 17{,}600 \text{ kips}$$

Therefore:

$$B_2 = \frac{1}{1-\dfrac{2{,}430 \text{ kips}}{17{,}600 \text{ kips}}}$$

$$= 1.16 < 1.5 \textbf{ o.k.}$$

Roof Level

$\alpha\Sigma P_{nt}$ = 1.6 [222 kips + (0.75)(240 kips)]
= 643 kips

R_M = 1.0 for braced frames as specified in AISC *Specification* Section C2.1b

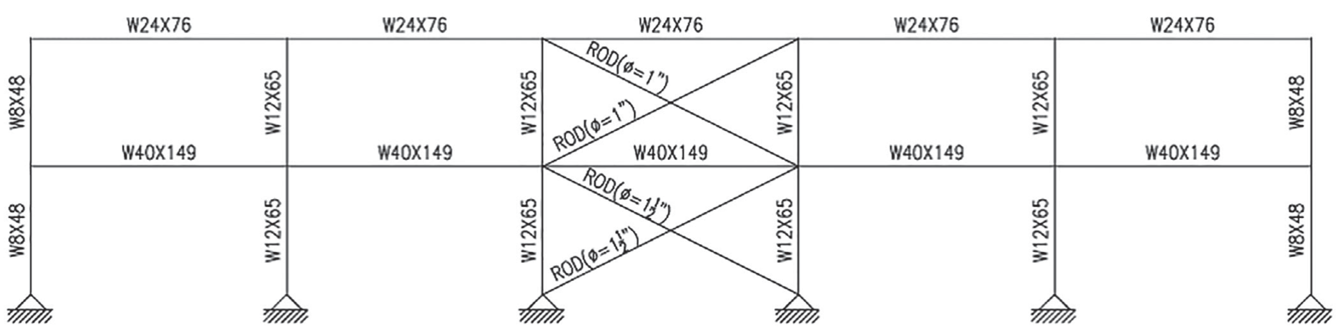


Fig. 4-4. Final member sizes using the first-order analysis method.

$$\Sigma P_{e2} = R_M \frac{\Sigma HL}{\Delta_H} \qquad (Spec.\text{ Eq C2-6b})$$

$$= 1.0\left[\frac{(10\text{ kips})(20\text{ ft})(12\text{ in./ft})}{0.718\text{ in.} - 0.408\text{ in.}}\right]$$

$$= 7{,}740\text{ kips}$$

Therefore:

$$B_2 = \frac{1}{1 - \dfrac{643\text{ kips}}{7{,}740\text{ kips}}}$$

$$= 1.09 < 1.5 \quad \textbf{o.k.}$$

Note: It is common practice to use two-story columns in tiered buildings to minimize splice connection costs. Thus, in this problem, the first floor columns are extended to the roof to omit the cost of the column splice.

Observations

1. Second-order effects are small but not insignificant for this problem. This is typical for many low-rise braced frame structures.
2. Drift does not control the design for this frame. This is typical of many braced frame structures in low-rise buildings.
3. The first-order analysis method produces a slightly larger size for the rod bracing in this problem. See Example 2.1 for the solution to this problem using the ELM and Example 3.1 for the solution by the DM.
4. The design of the tension bracing is governed by the strength requirements associated with one of the lateral load combinations. This is expected to be the case in braced frame structures.
5. The destabilizing effect of leaning columns should always be included in the design of the lateral load resisting system, regardless of the type of framing. In this example, the effect of the leaning columns is accounted for through the calculation of B_2 and through the additional lateral loads determined using the modified form of AISC *Specification* Equation C2-8.

Example 4.2— Large One-Story Warehouse Building

Given:

Design the braced frames and moment frames in the warehouse building shown in Figure 4-5 for gravity, snow and wind loads as specified by ASCE/SEI 7 load combinations using ASD. Use the first-order analysis method (FOM). Use the most economical size columns and beams (no limitations on W-series). Assume out-of-plumbness, $\Delta_o/L = 0.002$. Design for interstory drift control, $\Delta/L = 1/100$, for nominal wind load. Use ASTM A992 steel for wide-flange shapes and ASTM A36 steel for tension X-bracing. Assume the roof deck provides a rigid diaphragm and that the outside walls are light metal panels that span vertically between the roof and the ground floor. (Note: This defines the wind load path to the lateral load resisting system.)

The loading is as follows:
Dead Load, D = 25 psf (not including steel self-weight of roof beams, columns, braced and moment frames.)
Snow Load, S = 30 psf
Wind Load, W = 20 psf

(Note: Wind load on roof surfaces as required by ASCE/SEI 7 is not addressed in this problem.)

Solution:

From AISC Manual Table 2-3, the material properties are as follows:

Beams and columns
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Bracing
ASTM A36
$F_y = 36$ ksi
$F_u = 58$ ksi

Refer to Section 1.7 for a more detailed discussion of the analysis computer models utilized in this Design Guide. See Section 4.2 for a step-by-step description of the FOM.

Description of Framing

All lateral load resistance in the east-west direction is provided by tension only X-bracing in the north and south end bays as specified in Section A-A of Figure 4-5. All lateral load resistance in the north-south direction is provided by moment frames on each north-south column line. A moment connection is provided between the exterior column and beam at the end bay of each north-south frame as shown in Section B-B of Figure 4-5. Assume that this moment connection is field welded with a complete-joint-penetration weld at the beam flange-to-column flange connection. A beam-to-column web connection is provided using a bolted single-plate connection (see AISC *Manual* Figure 12-4(a) for the moment connection detail). The tension rod-to-gusset connections are assumed to be pinned connections using a standard clevis and pin (see the AISC *Manual* Tables 15-3 and 15-7). Beams within the braced frame are bolted into the column flanges using double angles (see AISC *Manual* Figures 13-2(a)). A single gusset plate connecting the tension rod is shop welded to the beam flange and field bolted to the column flange (see AISC *Manual* Figures 13-2(a)). All other columns outside the braced frames and moment frame bays are leaning columns with simple beam-to-column connections.

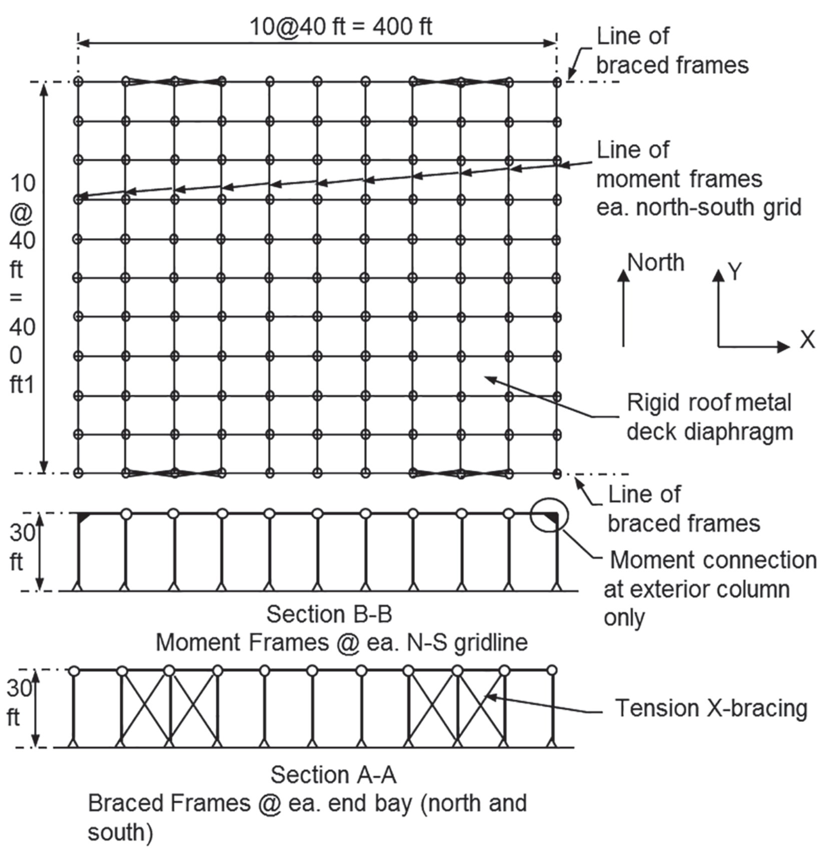


Fig. 4-5. Example 4.2 plan and sections.

Design Approach

Design is an iterative process. The first step is to estimate member sizes that are used in an analysis and then check for conformance to the strength requirements of the AISC *Specification* and serviceability drift limits that are established by the designer. In this type of building, braced frames are often controlled by strength limit states while moment frames are controlled by drift. The selection of preliminary sizes is not shown but is obtained based on consideration of service load drift for the moment frames and strength for the braced frames. The preliminary sizes for the members identified in Figures 4-6 and 4-7 are given in Table 4-6.

Table 4-6. Preliminary Member Sizes

Member	Size
Tension Brace (T1, T2)	1¼-in.-diameter rod
Moment Frame Beam (B1)	W24×131
Moment Frame Column (C1)	W24×117

Estimate of Framing Weight

Assume that roof purlins at 6 ft 8 in. center-to-center spacing in the east-west direction span to girders in the north-south direction located at column lines. Assume 7 psf framing self-weight for all steel framing, including purlins, columns, beams and braces. (Note: Use this weight for calculating the minimum lateral loads.)

The estimated dead load is:
Dead load = 25 psf + 7 psf = 32 psf

(Note: The weights specified include an allowance for all steel framing and exterior cladding.)

Analysis

Perform a first-order elastic analysis of the braced frames and moment frames. The analysis is performed using ASD load combinations.

Before applying the notional lateral loads as required for the FOM, verify the following:

1. Structure meets the interstory drift limit, $\Delta/L = 1/100$, under nominal wind load.
2. Determine whether $\Delta_{2nd}/\Delta_{1st} \le 1.5$, as required for use of this method (AISC *Specification* Section C2).

These checks will be included in the following design calculations.

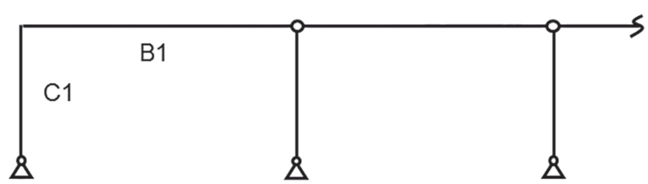


Fig. 4-6. North-south moment frame.

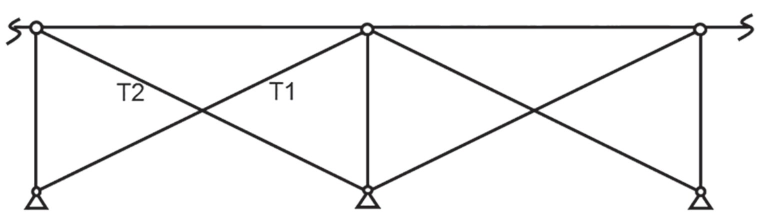


Fig. 4-7. East-west braced frame.

Serviceability Drift Limit

Wind drift for the moment frames in the north-south direction is evaluated first since drift often controls the sizes in moment frames. Wind drift is evaluated using the following serviceability load combination, where W is the specified nominal wind load:

$$1.0D + 0.5S + 1.0W$$

The roof deflection from a first-order computer analysis using the preliminary sizes given in Table 4-6 for wind in the north-south direction is:

$$\begin{aligned}\Delta &= 1.85 \text{ in.}\\ &= \frac{L}{195}\end{aligned}$$

Note that, for the purpose of calculating wind drift, the nominal wind load, W, is used as required in the problem statement. Wind drift limits and the appropriate serviceability load combination are not specified in the building code and are a matter of engineering judgment. Since the FOM assumes that second-order analysis is not performed using computer software, the amplification of the first-order deflection can be made by manual calculations or a spreadsheet as shown here:

$$B_2 = \frac{1}{1 - \dfrac{\alpha \Sigma P_{nt}}{\Sigma P_{e2}}} \qquad \text{(\textit{Spec.} Eq. C2-3)}$$

Because this is a serviceability check, $\alpha = 1.0$.

$$\begin{aligned}\Sigma P_{nt} &= (1.0)(32 \text{ psf})(400 \text{ ft})(400 \text{ ft}) + (0.5)(30 \text{ psf})(400 \text{ ft})(400 \text{ ft})\\ &= 7{,}520 \text{ kips}\\ \Sigma H &= (20 \text{ psf})(400 \text{ ft})(15 \text{ ft})\\ &= 120 \text{ kips}\\ R_M &= 0.85 \text{ for moment-frame systems}\end{aligned}$$

$$\begin{aligned}\Sigma P_{e2} &= R_M \frac{\Sigma HL}{\Delta_H}\\ &= 0.85\left[\frac{(120 \text{ kips})(30 \text{ ft})(12 \text{ in./ft})}{1.85 \text{ in.}}\right]\\ &= 19{,}800 \text{ kips}\end{aligned}$$

Therefore:

$$\begin{aligned}B_2 &= \frac{1}{1 - \dfrac{1.0(7{,}520 \text{ kips})}{19{,}800 \text{ kips}}}\\ &= 1.61\end{aligned}$$

The second-order roof deflection determination and check of the specified drift limit follows:

$$\begin{aligned}\Delta_{2nd} &= (1.85 \text{ in.})(1.61)\\ &= 2.96 \text{ in.}\\ &< (360 \text{ in.})/100 = 3.60 \text{ in.} \quad \textbf{o.k.}\end{aligned}$$

Note that, for the purpose of calculating wind drift, the nominal wind load, W, is used as required in the problem statement. Also, the analysis is based on using the nominal properties of the members. Wind drift limits and the appropriate serviceability load

Table 4-7. ASD Load Combinations
Comb1 = $D + N_x$
Comb2 = $D - N_x$
Comb3 = $D + N_y$
Comb4 = $D - N_y$
Comb5 = $D + S + N_x$
Comb6 = $D + S - N_x$
Comb7 = $D + S + N_y$
Comb8 = $D + S - N_y$
Comb9 = $D + W_x + N_x$
Comb10 = $D - W_x - N_x$
Comb11 = $D + W_y + N_y$
Comb12 = $D - W_y - N_y$
Comb13 = $D + 0.75W_x + 0.75S + N_x$
Comb14 = $D - 0.75W_x + 0.75S - N_x$
Comb15 = $D + 0.75W_y + 0.75S + N_y$
Comb16 = $D - 0.75W_y + 0.75S - N_y$
Comb17 = $0.6D + W_x + N_x$
Comb18 = $0.6D - W_x - N_x$
Comb19 = $0.6D + W_y + N_y$
Comb20 = $0.6D - W_y - N_y$
x, y = direction of forces in plan x = east-west braced frame direction y = north-south moment frame direction D = nominal dead load S = nominal snow load W_x, W_y = nominal wind load in x-, y-direction N_x, N_y = Additional lateral load in x-, y-direction Note: Because this structure is not sensitive to quartering winds, those load combinations are not included in this summary for brevity.

combinations are not specified in the building code and are a matter of engineering judgment. Note that there is a 60% increase in wind drift as a result of the gravity load acting on the frame and the second-order effects. This is an early indication that the frame does not have enough lateral stiffness to satisfy the requirement of $\Delta_{2nd}/\Delta_{1st} \leq 1.5$ under the maximum gravity strength load combinations.

It is assumed that wind drift for the braced frames in the east-west direction will be small and the sizes controlled by strength.

ASD Load Combinations

A complete list of all the ASD load combinations considered from ASCE/SEI 7 Section 2.4 for both orthogonal (x, y) directions is given in Table 4-7. Note that the FOM requires additional lateral load, N_i, to be included for all load combinations. FOM is only allowed when $\Delta_{2nd}/\Delta_{1st}$ is less than or equal to 1.5. It will be demonstrated later that $\Delta_{2nd}/\Delta_{1st}$ is less than or equal to 1.5 in both directions, after the members sizes are increased. The structure is analyzed for the load combinations in Table 4-7. The load combinations shown include positive and negative values for the lateral loads to allow automated design and eliminate the need for symmetry of loading and member sizes. For design that is performed manually, the number of load combinations can be reduced by half as long as symmetry of member size selection is maintained in the analysis and design.

Additional Lateral Loads, N_i

$$N_i = 2\ 1(\Delta/L)Y_i \geq 0.0042Y_i/\alpha \qquad \text{(from } \textit{Spec.} \text{ Eq. C2-8)}$$

Gravity loads, Y_i, used for determination of minimum lateral loads are calculated based on the estimated dead load previously discussed and the given snow load:

Dead load = 32 psf
Snow load = 30 psf

Note that the actual self-weight of the framing could be lighter than that assumed and could potentially change the minimum lateral load. This can be checked after the initial design is complete and the minimum lateral loads recalculated. Gravity and minimum lateral loads are not changed from these initial conservative values.

Gravity Load

$$\begin{aligned}\sum D &= (32 \text{ psf})(400 \text{ ft})(400 \text{ ft})/1{,}000 \text{ lb/kip} \\ &= 5{,}120 \text{ kips} \\ \sum S &= (30 \text{ psf})(400 \text{ ft})(400 \text{ ft}/1{,}000 \text{ lb/kip} \\ &= 4{,}800 \text{ kips}\end{aligned}$$

Y_i is 1.6 times the gravity load from ASD load combinations. N_i and Δ/L can be evaluated for each load combination. Alternatively, the maximum drift value, Δ/L, from any story can be used for all combinations. For this example, by inspection full wind load in the *x*- and *y*-directions will produce the maximum drift in each respective direction. The following interstory drift values were determined from a computer analysis:

For wind loading, W, in the *x*-direction (braced frame), the maximum ratio, Δ/L, is:

$$\Delta/L = 0.00110$$

Using the modified AISC *Specification* Equation C2-8, the additional lateral load is:

$$\begin{aligned}N_x &= 2.1(\Delta/L)\ Y_i \geq 0.0042Y_i/\alpha \qquad \text{(from } \textit{Spec.} \text{ Eq. C2-8)} \\ &= 2.1(0.00110)Y_i \\ &= 0.00231Y_i\end{aligned}$$

Because $0.00231Y_i < 0.0042Y_i/1.6 = 0.00263Y_i$, use $N_x = 0.00263Y_i$.

For wind loading, W, in the *y*-direction (moment frame), the maximum ratio, Δ/L, is:

$$\Delta/L = 0.00506$$

Using the modified AISC *Specification* Equation C2-8, the additional lateral load is:

$$\begin{aligned}N_y &= 2.1(\Delta/L)Y_i \geq 0.0042Y_i/\alpha \qquad \text{(from } \textit{Spec.} \text{ Eq. C2-8)} \\ &= 2.1(0.00506)Y_i \\ &= 0.0106Y_i\end{aligned}$$

Because $0.0106Y_i > 0.0042Y_i/1.6 = 0.00263Y_i$, use $N_y = 0.0106Y_i$.

The additional lateral loads are added to each load combination. For example, at the roof level in the *x*-direction for Comb1:

$$D + N_x$$

$$\begin{aligned}N_x &= 0.00263Y_i \\ &= (0.00263)(1.6)(5{,}120 \text{ kips}) \\ &= 21.5 \text{ kips}\end{aligned}$$

Table 4-8. Additional Lateral Loads, N_i, kips	
Combination	N_{roof}
Comb1	21.5
Comb2	21.5
Comb3	86.8
Comb4	86.8
Comb5	41.7
Comb6	41.7
Comb7	168
Comb8	168
Comb9	21.5
Comb10	21.5
Comb11	86.8
Comb12	86.8
Comb13	36.7
Comb14	36.7
Comb15	148
Comb16	148
Comb17	12.9
Comb18	12.9
Comb19	52.1
Comb20	52.1

Similarly, at the roof level in the y-direction for Comb15:

$$D + 0.75W_y + 0.75S + N_y$$

$$\begin{aligned} N_y &= 0.0106Y_i \\ &= 0.0106\{1.6\ [5{,}120 \text{ kips} + (0.75)(4{,}800 \text{ kips})]\} \\ &= 148 \text{ kips} \end{aligned}$$

Additional lateral loads, N_i, for Comb1 through Comb20 are listed in Table 4-8.

Analysis

A first-order analysis is performed using ASD load combinations. The analysis is performed using the unreduced member properties as described previously. The analysis reveals that the moment frame must be stiffened in order to meet the $\Delta_{2nd}/\Delta_{1st} \leq 1.5$ requirement. Several iterations are made between analysis and design to arrive at the final sizes that meet this requirement. Ultimately in this case, a drift limit of $L/467$ is required to maintain B_2 equal to or less than 1.5.

Check for $\Delta_{2nd}/\Delta_{1st}$ Drift in East-West Braced Frame Direction

Note that the $\Delta_{2nd}/\Delta_{1st}$ quantity must be less than or equal to 1.5 in order to use the FOM.

Therefore, from Table 4-9 the braced frame satisfies the $\Delta_{2nd}/\Delta_{1st}$ limit of 1.5 in the east-west direction.

Table 4-9. Second-Order Effect Checks in East-West Direction Using Initial Member Sizes					
Load Combination	Δ_H, **in.**	ΣP_{nt}, **kips**	ΣH, **kips**	$\dfrac{\alpha \Sigma P_{nt}}{R_M \dfrac{\Sigma HL}{\Delta_H}}$	$\Delta_{2nd}/\Delta_{1st}$ (B_2)
Comb1	0.0711	5120	21.5	0.0753	1.08
Comb2	0.0711	5120	21.5	0.0753	1.08
Comb5	0.138	9920	41.7	0.146	1.17
Comb6	0.138	9920	41.7	0.146	1.17
Comb9	0.468	5120	142	0.075	1.08
Comb10	0.468	5120	142	0.075	1.08
Comb13	0.419	8720	127	0.128	1.15
Comb14	0.419	8720	127	0.128	1.15
Comb17	0.439	3070	133	0.045	1.05
Comb18	0.439	3070	133	0.045	1.05

Table 4-10. Second-Order Effect Checks in North-South Direction Using Initial Member Sizes					
Load Combination	Δ_H, **in.**	ΣP_{nt}, **kips**	ΣH, **kips**	$\dfrac{\alpha \Sigma P_{nt}}{R_M \dfrac{\Sigma HL}{\Delta_H}}$	$\Delta_{2nd}/\Delta_{1st}$ (B_2)
Comb3	1.32	5120	86.8	0.406	1.68
Comb4	1.32	5120	86.8	0.406	1.68
Comb7	2.56	9920	168	0.788	4.72
Comb8	2.56	9920	168	0.788	4.72
Comb11	3.15	5120	207	0.407	1.69
Comb12	3.15	5120	207	0.407	1.69
Comb15	3.61	8720	238	0.692	3.25
Comb16	3.61	8720	238	0.692	3.25
Comb19	2.60	3070	172	0.243	1.32
Comb20	2.60	3070	172	0.243	1.32

Check for $\Delta_{2nd}/\Delta_{1st}$ Drift in North-South Direction

Note that $\Delta_{2nd}/\Delta_{1st}$ must be less than 1.5 in order to use the FOM method. It was found that the initial member sizes given in Table 4-6 (W24×117 column and a W24×131 beam) that met the serviceability drift check do not meet the $\Delta_{2nd}/\Delta_{1st}$ requirement as shown in Table 4-10. Therefore, the moment frame member sizes must be increased.

From a computer analysis, after the frame members have been adjusted, the drift from W in the moment frame (y) direction, Δ/L = 0.00198 and

$$\begin{aligned} N_y &= 2.1(0.00198)Y_i \\ &= 0.00416Y_i > 0.0042Y_i/1.6 \end{aligned}$$

Therefore:

$$N_y = 0.00416\ Y_i$$

Table 4-11. Additional Lateral Loads, N_i, in y-Direction After Adjustment of Member Sizes, kips	
Combination	**N_i, roof**
Comb3	34.1
Comb4	34.1
Comb7	66.0
Comb8	66.0
Comb11	34.1
Comb12	34.1
Comb15	58.1
Comb16	58.1
Comb19	20.5
Comb20	20.5

Table 4-12. Second-Order Effect Checks in North-South Direction Using Final Member Sizes					
Load Combination	**Δ_H, in.**	**ΣP_{nt}, kips**	**ΣH, kips**	$\dfrac{\alpha \Sigma P_{nt}}{R_M \dfrac{\Sigma HL}{\Delta_H}}$	**$\Delta_{2nd}/\Delta_{1st}$ (B_2)**
Comb3	0.203	5120	34.1	0.159	1.19
Comb4	0.203	5120	34.1	0.159	1.19
Comb7	0.393	9920	66.0	0.309	1.45
Comb8	0.393	9920	66.0	0.309	1.45
Comb11	0.916	5120	154	0.159	1.19
Comb12	0.916	5120	154	0.159	1.19
Comb15	0.880	8720	148	0.271	1.37
Comb16	0.880	8720	148	0.271	1.37
Comb19	0.835	3070	140	0.0957	1.11
Comb20	0.835	3070	140	0.0957	1.11

For example, at the roof level in the y-direction for Comb15:

$$D + 0.75W_y + 0.75S + N_y$$

$$\begin{aligned} N_y &= 0.00416Y_i \\ &= 0.00416\{1.6[5{,}120 \text{ kips} + (0.75)(4{,}800 \text{ kips})]\} \\ &= 58.1 \text{ kips} \end{aligned}$$

Additional lateral loads, N_i, for combinations in the (y) direction are listed in Table 4-11.

The B_2 stiffness limits with the final member sizes in the moment frame that meet the $\Delta_{2nd}/\Delta_{1st}$ limit of 1.5 and the strength requirements are shown in Table 4-12.

Final Member Forces

Tables 4-13 and 4-14 summarize the FOM member forces for the final design. The controlling load combination values are shown in bold print.

Table 4-13. First-Order Analysis Moment Frame Member Forces					
Load Combination		C1		B1	
		P_r		P_r	
		$M_{r\text{-}i}$	$M_{r\text{-}j}$	$M_{r\text{-}i}$	$M_{r\text{-}m}$
Comb3	**P_r, kips**	-28.1		0	
	M_r, kip-in.	0	1220	-1220	2460
Comb4	**P_r, kips**	-30.5		0	
	M_r, kip-in.	0	2330	-2330	1910
Comb7	**P_r, kips**	-54.5		0	
	M_r, kip-in.	0	2360	-2360	4770
Comb8	**P_r, kips**	-59		0	
	M_r, kip-in.	0	4520	-4520	3690
Comb11	**P_r, kips**	-24		0	
	M_r, kip-in.	0	-748	748	3440
Comb12	**P_r, kips**	-34.6		0	
	M_r, kip-in.	0	4300	-4300	924
Comb15	**P_r, kips**	-44.8		0	
	M_r, kip-in.	0	599	-602	**4930**
Comb16	**P_r, kips**	**-54.9**		0	
	M_r, kip-in.	0	**5450**	-5450	2510
Comb19	**P_r, kips**	-12.8		0	
	M_r, kip-in.	0	-1230	1230	2460
Comb20	**P_r, kips**	-22.4		0	
	M_r, kip-in.	0	3360	-3360	162

Table 4-14. First-Order Analysis Braced Frame Member Forces			
Combination	Action	T1, kips	
Comb1	Axial Load	3.36	0
Comb2	Axial Load	0	3.36
Comb5	Axial Load	6.51	0
Comb6	Axial Load	0	6.51
Comb9	Axial Load	**22.1**	0
Comb10	Axial Load	0	**22.1**
Comb13	Axial Load	19.8	0
Comb14	Axial Load	0	19.8
Comb17	Axial Load	20.8	0
Comb18	Axial Load	0	20.8

Table 4-15. First Order Analysis Method Final Member Sizes (used in final B_2 and final strength checks)	
Member	**Size**
Tension Brace (T1, T2)	1¼-in. diameter rod
Moment Frame Beam (B1)	W40×149
Moment Frame Column (C1)	W40×149
Note: These member sizes are significantly lighter than those selected by the ELM because the depth limit used in the ELM solution has not been stated as a requirement in this case.	

Representative Member Strength Design Checks

Member checks are given in the following for the representative final member sizes given in Table 4-15.

North-South Moment Frame, Typical Interior Frame, Column C1

The governing load combination is Comb16 from Table 4-13 where $P_r = 54.9$ kips (compression) and $M_r = 5{,}450$ kip-in.

From AISC *Manual* Table 1-1, the properties of the W40×149 are as follows:

W40×149
$A = 43.8$ in.^{2}
$S_x = 513$ in.^{3}
$Z_x = 598$ in.^{3}
$r_y = 2.29$ in.
$r_{ts} = 2.89$ in.
$h/t_w = 54.3$
$b_f/2t_f = 7.11$
$h_o = 37.4$ in.
$J = 9.36$ in.^{4}

For the FOM, $K = 1$, and the effective length is:

$$\begin{aligned} KL &= (30\text{ ft})(12\text{ in./ft}) \\ &= 360\text{ in.} \end{aligned}$$

AISC *Manual* Table 1-1 indicates that the W40×149 is slender for compression when $F_y = 50$ ksi; therefore, the design compressive strength is determined as follows from AISC *Specification* Section E7. The y-axis slenderness ratio, KL/r_y, controls the strength ($KL/r_y > KL/r_x$):

$$\begin{aligned} \frac{KL}{r_y} &= \frac{360\text{ in.}}{2.29\text{ in.}} \\ &= 157 \end{aligned}$$

The elastic critical buckling stress is:

$$F_e = \frac{\pi^2 E}{\left(\dfrac{KL}{r}\right)^2} \qquad (\textit{Spec}.\text{ Eq. E3-4})$$

$$\begin{aligned} &= \frac{\pi^2 (29{,}000\text{ ksi})}{(157)^2} \\ &= 11.6\text{ ksi} \end{aligned}$$

Because $F_e < 0.44F_y = 22$ ksi, the flexural buckling stress is:

$$\begin{aligned} F_{cr} &= 0.877F_e \\ &= 0.877(11.6 \text{ ksi}) \\ &= 10.2 \text{ ksi} \end{aligned} \qquad (\textit{Spec}. \text{ Eq. E3-3})$$

From AISC *Specification* Section E7:

$Q = Q_s Q_a$ for members with slender elements

Determine Q_a from AISC *Specification* Section E7.2. Determine whether $\dfrac{h}{t_w} = 54.3 \geq 1.49\sqrt{\dfrac{E}{F_{cr}}}$:

$$\begin{aligned} 1.49\sqrt{\frac{E}{F_{cr}}} &= 1.49\sqrt{\frac{29{,}000 \text{ ksi}}{10.2 \text{ ksi}}} \\ &= 79.4 \end{aligned}$$

Because $\dfrac{h}{t_w} = 54.3 < 79.4$, then $Q_a = 1.0$.

Determine Q_s from AISC *Specification* Section E7.1. Determine whether $\dfrac{b_f}{2t_f} \leq 0.56\sqrt{\dfrac{E}{F_y}}$:

$$\begin{aligned} 0.56\sqrt{\frac{E}{F_y}} &= 0.56\sqrt{\frac{29{,}000 \text{ ksi}}{50 \text{ ksi}}} \\ &= 13.5 \end{aligned}$$

Because $\dfrac{b_f}{2t_f} = 7.11 \leq 13.5,$ then $Q_s = 1.0$ and $Q = Q_s Q_a = 1.0$.

The nominal compressive strength is:

$$\begin{aligned} P_n &= F_{cr} A_g \\ &= (10.2 \text{ ksi})(43.8 \text{ in.}^2) \\ &= 447 \text{ kips} \end{aligned} \qquad (\textit{Spec}. \text{ Eq. E3-1})$$

The allowable compressive strength as defined in AISC *Specification* Section E1 is:

$$\begin{aligned} \frac{P_n}{\Omega_c} &= \frac{447 \text{ kips}}{1.67} \\ &= 268 \text{ kips} > 54.9 \text{ kips} \quad \textbf{o.k.} \end{aligned}$$

Determine the allowable flexural strength of the column. From AISC *Manual* Table 3-2, for a W40×149:

$L_p = 8.09$ ft
$L_r = 23.5$ ft
$L_b = 30$ ft

$$\frac{M_{px}}{\Omega_b} = (1{,}490 \text{ kip-ft})(12 \text{ in./ft})$$
$$= 17{,}900 \text{ kip-in.}$$

The required flexural strength at end i is $M_{ri} = 0$, and the required flexural strength at end j is $M_{rj} = 5{,}450$ kip-in.

From AISC *Manual* Table 3-1 and AISC *Specification* Equation F1-1, $C_b = 1.67$ (note that there is a linear moment diagram between the support and the first brace point). Because $L_b = 30 \text{ ft} > L_r$, determine the nominal strength from AISC *Specification* Equation F2-3, and the critical stress from Equation F2-4. For a doubly symmetric I-shape, $c = 1.0$. Alternatively, AISC *Manual* Table 3-10 can be used.

$$F_{cr} = \frac{C_b \pi^2 E}{\left(\frac{L_b}{r_{ts}}\right)^2} \sqrt{1 + 0.078 \frac{Jc}{S_x h_o}\left(\frac{L_b}{r_{ts}}\right)^2} \qquad \text{(}\textit{Spec}\text{. Eq. F2-4)}$$

$$= \frac{(1.67)\pi^2(29{,}000 \text{ ksi})}{\left[\frac{(30 \text{ ft})(12 \text{ in./ft})}{2.89 \text{ in.}}\right]^2} \sqrt{1 + 0.078 \frac{9.36 \text{ in.}^4}{(513 \text{ in.}^3)(37.4 \text{ in.})}\left[\frac{(30 \text{ ft})(12 \text{ in./ft})}{2.89 \text{ in.}}\right]^2}$$
$$= 38.8 \text{ ksi}$$

The available flexural strength is:

$$\frac{M_n}{\Omega_b} = \frac{F_{cr} S_x}{\Omega_b} \le \frac{M_p}{\Omega_b} \qquad \text{(from }\textit{Spec}\text{. Eq. F2-3)}$$
$$= \frac{(38.8 \text{ ksi})(513 \text{ in.}^3)}{1.67} \le 17{,}900 \text{ kip-in.}$$
$$= 11{,}900 \text{ kip-in.} \le 17{,}900 \text{ kip-in.}$$

Check the interaction of compression and flexure using AISC *Specification* Section H1:

$$\frac{P_r}{P_c} = \frac{54.9 \text{ kips}}{268 \text{ kips}}$$
$$= 0.205 > 0.2$$

Therefore use AISC *Specification* Equation H1-1a:

$$\frac{P_r}{P_c} + \frac{8}{9}\left(\frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}}\right) \le 1.0 \qquad \text{(}\textit{Spec}\text{. Eq. H1-1a)}$$

$$0.205 + \frac{8}{9}\left(\frac{5{,}450 \text{ kip-in.}}{11{,}900 \text{ kip-in.}} + 0\right) = 0.612 \le 1.0 \quad \textbf{o.k.}$$

Note: By inspection $P_r/(AF_y) < 0.5$ (AISC *Specification* Section C2.2b(1))

North-South Moment Frame, Beam B1

The governing load combination for member B1 is Comb15 from Table 4-13 where $M_r = 4{,}930$ kip-in.

From AISC *Manual* Table 3-2, for a W40×149:

$L_p = 8.09$ ft
$L_r = 23.5$ ft
$L_b = 6.67$ ft (purlin spacing) $< L_p = 8.09$ ft

$$\frac{M_{px}}{\Omega_b} = 1{,}490 \text{ kip-ft}$$

Because $L_b < L_p$, $\dfrac{M_n}{\Omega_b} = \dfrac{M_{px}}{\Omega_b} = 1{,}490 \text{ kip-ft}$, according to AISC *Specification* Section F2.

$$\begin{aligned}\frac{M_n}{\Omega_b} &= (1{,}490 \text{ kip-ft})(12 \text{ in./ft}) \\ &= 17{,}900 \text{ kip-in.} > 4{,}930 \text{ kip-in.} \quad \textbf{o.k.}\end{aligned}$$

East-West Braced Frame, Tension Only Member, T1

The required tensile strength of the 1¼-in.-diameter rod is:

$P_r = 22.1$ kips for Comb9 from Table 4-14

For tension-only rod (F_y =36 ksi), the allowable tensile yielding strength is:

$$\begin{aligned}\frac{P_n}{\Omega_t} &= \frac{(36 \text{ ksi})(\pi)(1\tfrac{1}{4} \text{ in.})^2/4}{1.67} \\ &= 26.5 \text{ kips} > 22.1 \text{ kips} \quad \textbf{o.k.}\end{aligned}$$

The allowable tensile rupture strength is (assume the effective net tension area is $0.75A_g$; this must be confirmed once connections are completed):

$$\begin{aligned}\frac{P_n}{\Omega_t} &= \frac{(58 \text{ ksi})(0.75)(\pi)(1\tfrac{1}{4} \text{ in.})^2/4}{2.00} \\ &= 26.7 \text{ kips} > 22.1 \text{ kips} \quad \textbf{o.k.}\end{aligned}$$

Observations

1. The FOM provides a conservative design for this problem compared to the DM because (1) it was done using the more conservative 1.6 overall load factor used by ASD; (2) the $\Delta_{2nd}/\Delta_{1st} \le 1.5$ stiffness limitation applies; and (3) other conservative approximations are invoked in its development (see Chapter 4 and Appendix B).
2. There were no seismic requirements for this problem, but the lateral load system in the north-south moment frame direction may not be satisfactory in higher seismic zones because of the magnitude of the second-order effects. This would need to be checked according to ASCE/SEI 7 Section 12.8.7.
3. The moment frame design is strongly influenced by the large leaning column load and the fact that there are relatively few moment connections.
4. The braced frame design is controlled by strength while the moment frame design is controlled by stiffness (necessary to satisfy the $B_2 \le 1.5$ requirement). This is common in many buildings of this type.

Chapter 5
Related Topics

5.1 APPLICATION TO SEISMIC DESIGN

5.1.1 Determination of Seismic Load Effect, *E*

The current approach to seismic design in the U.S. is based on provisions defined in ASCE/SEI 7 Chapter 12: Seismic Design Requirements for Building Structures. A design response spectrum is created (ASCE/SEI 7 Figure 11.4-1) from which a seismic base shear, V, is determined (ASCE/SEI 7 Equation 12.8-1) based on the occupancy importance factor of the building, I, the response modification factor, R, dependent on the type of structural system used (e.g., braced frame, moment frame, and the level of detailing applied to it), and the fundamental building period, T. In the equivalent lateral force procedure (ELF) defined in Section 12.8, the base shear, V, is distributed up the building and applied as a seismic lateral force at each floor, F_x, according to the mass of the structure at each level and the building period based on ASCE/SEI 7 Equations 12.8-11 and 12.8-12. In the modal response spectrum analysis procedure (MRSA) defined in Section 12.9, an analysis to determine the natural modes of vibration of the structure is made and the internal member forces are determined from the modes of vibration along with a defined response spectrum divided by the quantity R/I. The member forces calculated for each mode are combined using either the square root of the sum of the squares (SRSS) method or the complete quadratic combination (CQC) method. The internal member forces are scaled using the base shear, V, from the ELF according to Section 12.9.4. Refer to the section in the following, "Member Properties Used in Structural Analysis Modeling," for a discussion of member properties to use for these different types of structural analysis.

The designer should understand that the seismic load effect, E, from either the ELF or MRSA involves a reduction in the elastic forces by the R factor to a "first significant yield" level of response for ease in design. It is fully expected that the building will perform inelastically during the design earthquake. This is in contrast to the ordinary design for wind loads, where the level of response is essentially elastic even under ultimate wind loads. This philosophy of design is depicted in Figure 5-1 which shows a monotonic push-over curve for a well proportioned structural steel moment frame building. Its use is based on ductile detailing in the steel frame as required to ensure good energy dissipation and life safety during the design level earthquake. This design philosophy is important for economy and is based on a long history of acceptable performance of steel buildings during earthquakes.

Typical code designed structures are expected to undergo large inelastic reversed cyclic deformations during strong earthquake motion. Systems with good detailing of members and connections as defined in the 2005 AISC *Seismic Provisions for Structural Steel Buildings* are expected to have ductile behavior and are rewarded with smaller seismic design forces. Good inelastic behavior is characterized by full and stable hysteretic force-deformation loops which provide the structure with the necessary energy dissipation capability to survive the severe shaking and ensure life safety of the occupants. The expectation is that all forms of member buckling and overall system buckling are reasonably controlled so that adequate ductility and inelastic deformation are achieved before buckling influences the overall performance of the structure.

Despite the fact that considerable research has been conducted on the problem of overall stability of steel structures under seismic loading (Ziemian, 2010, Chapter 20), very little has found its way into the building code. The code treatment of the problem is very simplistic and stems from the current practice of elastic analysis at code specified reduced force levels. Consideration of an overall P-Δ effect under the specified seismic load effect, E, is required but only under nominal (not ultimate) gravity load levels. While this requirement (discussed further below) is admirable, it certainly does not address in any meaningful way the real stability problem during major ground shaking, where excursions into the inelastic range may induce negative post-yield stiffness and possible collapse. The "true" seismic response can only be addressed by running second-order inelastic dynamic time-history analyses under a suite of probable ground motions, which is permitted but not required by the current code. This is not done for the vast majority of steel structures. The Federal Emergency Management Agency (FEMA) is sponsoring long term research, directed by the Applied Technology Council (ATC) and currently known as the ATC-58 project, to develop a performance-based design approach based on assessing frame deformations under various levels of earthquake shaking using an appropriate method of analysis. These new design guidelines, when completed, are expected to be the foundation for the next generation of seismic design standards in the U.S.

Given this brief background, one can ask how design for stability using the methods described in this design guide fits into the seismic design process as defined by the building code. The answer lies in the fact that the building code has sanctioned the use of the traditional effective length method (ELM) by virtue of its widespread use and acceptance in all building codes, including the IBC 2006 code. Therefore, by virtue of the fact that comparable design results are obtained using the new DM, this method can also be used along with the ASCE/SEI 7 load combinations, including *E*, the seismic load effect, defined in Section 1605 of the 2006 IBC Code and Chapter 2 of ASCE/SEI 7. The successful use of either of these stability design methods is predicated upon satisfaction of all the seismic detailing requirements of the code, and conformance with the prescribed drift limits defined in Section 12.12 and the P-Δ limitations specified in ASCE/SEI 7 Section 12.8.7, both of which are discussed in the following.

5.1.2 Member Properties to Use in Structural Analysis Modeling

It is recommended in this Design Guide that the seismic forces, whether from the ELF or MRSA, be determined from an analysis model based on the traditional member properties (nominal member properties for steel structures and cracked member properties for concrete structures) of the frame as traditionally done prior to the introduction of the DM. The DM was not developed with any intention to modify the determination of the seismic load effect, *E*, required by the building code or the ASCE/SEI 7 load standard. Any

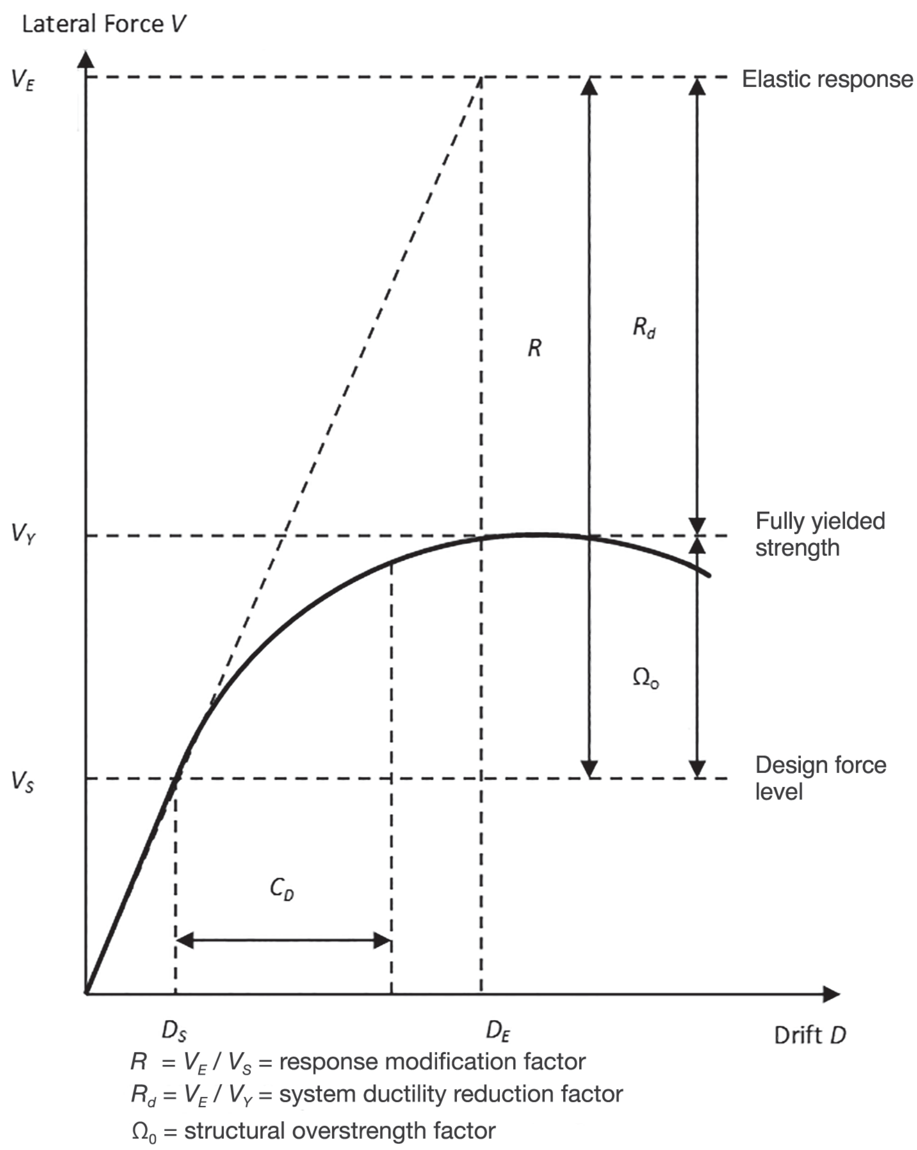


Fig. 5-1. Building seismic design philosophy.

subsequent static second-order analysis may be conducted by the DM using the reduced properties of the members and considering this load effect.

5.1.3 Drift Control Under Code Seismic Forces

Drift requirements for steel structures designed by the 2006 IBC are defined in ASCE/SEI 7 Section 12.8.6, Figure 12.8-2 (Story Drift Determination) and Section 12.12.1 (Story Drift Limit). In determining the story drift, an elastic analysis of the steel frame is conducted under the code prescribed seismic load effect, E, (the elastic load effect reduced by R) to determine elastic deflections, δ_e. These deflections are then magnified by the deflection amplification factor, C_d, which is determined from Table 12.2-1 of ASCE/SEI 7 based on the type of structural system. This yields a deflection estimate under the true inelastic excursion of the structure. The story drift, Δ, is then determined from the inelastic frame deflections, $\delta = \delta_e\,(C_d/I)$, and its value is compared against the code drift limits or allowable story drift, Δ_a, defined in Table 12.12-1 of ASCE/SEI 7. Note that the importance factor, I, is included in the deflection equation to cancel it out of the drift determination since its effect is already included in the seismic load effect, E. For most steel frames, $\Delta_a = 0.02\,h_{sx}$, where h_{sx} is the story height as defined in ASCE/SEI 7. For many moment frames, the member sizes are controlled by either wind or seismic drift limits rather than by the strength under the reduced code seismic load effect, E. These types of frames should be proportioned for drift first and then checked for strength. Conversely, most braced frame member sizes are controlled by strength rather than drift because of their inherent stiffness.

5.1.4 *P*-Δ Control Under Seismic Forces

In the current codes, the primary control for ensuring overall frame stability during earthquake ground shaking (and in the post-earthquake period from aftershocks) lies in drift control and in limiting the secondary effects by controlling the P-Δ moments. The control of P-Δ effects is covered in Section 12.8.7 of ASCE/SEI 7. The requirements of this section apply to all structures in a location assigned to seismic design categories B through F. The stability coefficient, θ, is determined from the following:

$$\theta = \frac{P_x \Delta}{V_x h_{sx} C_d}$$

(ASCE/SEI 7 Eq. 12.8-16)

where

P_x = total vertical design load at and above level x, computed with a load factor of not greater than 1.0 on all gravity loads, kips

Δ = design story drift under earthquake loads (including C_d and inelastic effects) occurring simultaneously with V_x, in.

V_x = seismic story shear below level x, kips

h_{sx} = story height below level x, in.

C_d = deflection amplification factor from Table 12.2-1

The maximum limit on θ is defined by ASCE/SEI 7 as:

$$\theta_{MAX} = \frac{0.5}{\beta C_d} \le 0.25$$

(ASCE/SEI 7 Eq. 12.8-17)

where β is the ratio of shear demand to shear capacity for the story below level x. The shear demand is simply the story shear obtained from the code seismic forces. The story shear capacity can be defined as the shear in the story that occurs simultaneously with the attainment of the first significant yield level of the overall structure, computed by amplifying the code applied seismic forces until first yield occurs in any one member of the frame. In addition, the story shear capacity can be determined conservatively by using the code seismic force level and determining the largest β value in the particular story under consideration. It is always conservative to assume $\beta = 1.0$. However, it can be important to consider the actual value of β (< 1.0) because many moment frames have extra reserve strength because their sizes are larger than required for strength in order to control the drift. This is reflected as a reduction factor on C_d, the code prescribed deflection amplification factor.

The technical justification for ASCE/SEI 7 Equations 12.8-16 and 12.8-17 is controversial. Further discussion of θ and β, along with additional references, can be found in the FEMA 450-2 Commentary (FEMA, 2003).

As discussed in ASCE/SEI 7 Section 12.8.7, the P-Δ effect (i.e., the P-Δ amplification) can be determined using computer software as part of the frame analysis or it can be calculated as follows:

$$P\text{-}\Delta \text{ effect} = \frac{1}{1-\theta} \tag{5-1}$$

ASCE/SEI 7 allows this effect to be ignored when $\theta \le 0.10$ (a P-Δ amplification from the above equation equal to 1.11). Note that Equation 5-1 is equivalent to the AISC B_2 factor (Equations C2-3 and C2-6a in Chapter C) [in a different form in the 2010 AISC *Specification* Appendix 8, Equations A-8-6 and A-8-7], except that it uses $R_M = 1.0$ rather than 0.85 for moment frames and combined systems and it uses a load factor on the gravity load no greater than 1.0.

Table 5-1 demonstrates the overall influence of the 2006 IBC Code on second-order effects using the common lateral load combination $1.2D + 0.5L + 1.6W$ (from IBC

Table 5-1. 2006 IBC and ASCE/SEI 7 Limitations on Second-Order Effects						
L/D	**0.75**		**1.50**		**3**	
P_u/P_s	**1.15**		**1.11**		**1.08**	
P-Δ effect	**MF**	**BF**	**MF**	**BF**	**MF**	**BF**
B_{2S}	1.42	1.33	1.42	1.33	1.42	1.33
B_2	1.51	1.40	1.48	1.38	1.47	1.37

Equation 16-4) or $1.2D + 0.5L + 1.0E$ (from IBC Equation 16-5). For these LRFD load combinations and the normal range of live, L, to dead, D, load ratios found in most multi-level steel buildings, the second-order P-Δ effect (B_2) is limited to about 1.40 for steel braced frames and 1.50 for steel moment frames because of these seismic requirements. Higher values of B_2 would exist for gravity-only load combinations in both one-story and multi-level buildings.

The values, B_{2S} and B_2, given in Table 5-1 are determined as follows:

$$B_{2S} = \frac{1}{1-\theta_{max}} \tag{5-2}$$

$$B_2 = \frac{1}{1-\dfrac{\theta_{max}(P_U/P_S)}{R_M}} \tag{5-3}$$

where

$\theta_{MAX} \le 0.25$ (from ASCE/SEI 7 Eq. 12.8-17)
$P_S = D + 0.5L$
$P_U = 1.2D + 0.5L$ (from 2006 IBC Eq. 16-4 and 16-5)
$R_M = 0.85$ for moment frames (MF)
$R_M = 1.00$ braced frames (BF)

Equation 5-2 is based on ASCE/SEI 7 Section 12.8.7. Equation 5-3 is the AISC *Specification* B_2 equation rewritten with ASCE/SEI 7 terminology. Several important observations can be made from these calculations:

1. The P-Δ effect is effectively calculated at the design earthquake lateral load level (see Figure 5-1) and not the inelastic or amplified lateral load level.

2. The P-Δ effect is based on gravity loads at the nominal (or lower) code load level and not the ultimate load level.

3. The code calculates the above P-Δ effects pertaining to the seismic load combinations using the reduced code level earthquake deflections, not the amplified or expected inelastic deflections.

4. The P-Δ amplifier expressed by Equation 5-1 does not account for any reduction in column stiffness in structural steel moment frames due to P-δ effects (these effects are accounted for by the use of $R_M = 0.85$ in the AISC *Specification*).

On the surface, all of these assumptions appear to be unconservative. The structural responses due to static P-Δ effects and dynamic P-Δ effects are of course very different. For many frames subjected to seismic shaking, the structure may indeed see the larger inelastic drifts. However, the inertial effects associated with the dynamic response force the frame back in the opposite direction from a potential sidesway failure before collapse can occur.

The justification for the current code calculation of the P-Δ effects is explained in the Commentary to FEMA 450-2 (FEMA, 2003), also called the NEHRP *Provisions Commentary*, which is the basis for the ASCE/SEI 7 code provisions. First, the procedure used is seemingly justified by the fact that there have not been many stability related failures observed during actual earthquakes. This can be attributed to the apparent overstrength beyond the code level design forces due to drift control. Also, the likelihood of a stability failure decreases with increased intensity of ground shaking because the stiffness of structures designed for extreme ground shaking is significantly greater than the stiffness of the same structure designed for a lower ground shaking or for wind alone. Low intensity earthquake damage is rare and there would likely be little observable damage. Secondly, consideration of the P-Δ moments based on the inelastic drift, even using nominal gravity loads, would result in a large increase in the design forces. This appears unwarranted based on observations under actual earthquakes. For instance, suppose that the P-Δ amplification from Equation 5-1 is 1.18 for an intermediate moment frame with a C_d = 4.0. Therefore, θ = 0.15. If one were to use $C_d\,\theta = 0.60$ in Equation 5-1 to approximate the influence of the inelasticity, the P-Δ effect would increase to 2.50. Clearly, this would have a large impact on building costs and does not seem justified based on past performance. In addition, Equation 5-1 is truly just a static elastic P-Δ amplification factor. Use of $C_d\,\theta$ in this equation amounts to the use of a secant stiffness tied to the inelastic drift, not a tangent stiffness associated with the inelastic response. Further discussion of steel frame response under dynamic P-Δ effects can be found in Gupta and Krawinkler (2000).

Summary of Design Recommendations:

1. The direct analysis method (DM) can be used along with the ASCE/SEI 7 load combinations, including the reduced seismic load effect, E, defined in Section 1605 of the 2006 IBC Code and Chapter 2 and Section 12.4 of ASCE/SEI 7.

2. The successful use of either the ELM or the DM is predicated upon satisfying all the seismic detailing requirements of the code and conforming to the prescribed drift limits defined in Section 12.12 plus the P-Δ effect limits of ASCE/SEI 7 Section 12.8.7.

3. It is recommended that for steel structures the seismic load effect, E, whether from the ELF or the MRSA, be determined using the nominal properties of the members. Any subsequent static second-order analysis may be conducted by the DM using the reduced properties of the members and considering this load effect.

5.2 COMMON PITFALLS AND ERRORS IN STABILITY ANALYSIS AND DESIGN

Since the advent of the effective length concept in 1961, many articles and textbooks have been published about problems in the application of stability methods used in steel design. Some of these problems have led to refinements in various later editions of the AISC *Specification* and Commentary to alert the designer to common pitfalls. Some of these common problems are discussed in the following as an aid to the designer.

1. *Improper Second-Order Analyses.* Many errors in stability design can be traced to improper second-order analysis techniques. Since any stability analysis basically requires the consideration of equilibrium on the deformed geometry of the structure, it is important that the deflections of the frame be captured with sufficient accuracy. This means that all significant deformations of the structure must be considered, including flexural deformations of beams and columns, shear deformations in beams and columns, axial deformations of columns and braces, panel zone deformation in beam-column joints, differential foundation settlements and rotational restraint, and out-of-plumbness effects to name a few of the more important ones. The software used must capture all of the significant deformations and all of the considerations discussed here must be at least evaluated for their importance. Depending on frame geometry such as bay spacing and member proportions, as well as the height and aspect ratio of the frame, the different effects take on different levels of importance.

   The computer analysis should accurately capture the effect of individual column P-δ effects (reduction in column stiffness due to axial load) on the overall lateral drift of each story. For frames with large column axial loads (αP_r larger than $0.05\pi^2 EI/L^2$ in some cases), this effect may become significant and require additional nodes in the column length for the analysis. If the software does not accurately account for the P-δ effects in the formulation of its frame elements, the engineer may need to apply the B_1 amplifier to approximate these effects or use multiple elements per member.

2. *Neglect of Leaning Column Effect.* It is important to properly include all gravity loads in determining the destabilizing effect on the lateral frame. The P-Δ effect of all gravity loads in the building must be accounted for in the analysis.

3. *Improper Use and Calculation of K_2.* The accurate calculation of K_2 can be a challenge for complex frames with a large number of leaning columns or irregular frame geometry. This problem can be avoided with use of the direct analysis method (DM) or the first-order analysis method (FOA) where $K = 1.0$.

4. *Torsional Loading Effects in Frames.* Simplification of a building analysis into two dimensions can be problematic for eccentric code wind and seismic loading requirements, irregular building shapes that are not orthogonal, and when there is eccentricity between the center of stiffness and mass or lateral wind loading. This can lead to significant errors in internal frame forces.

5. *Drift Control for Stability.* Structures with very light wind loads and little or no seismic requirements can be very flexible in sidesway while still satisfying tight service drift requirements. Thus, drift control by itself is not sufficient to control the magnitude of second-order effects. Significant sidesway flexibility and large column loads and/or leaning column effects can lead to large second-order effects. Note that the B_2 amplifier, and thus the second-order effects, becomes large when the total story gravity load is large when compared to the frame lateral buckling strength. Drift limits applicable to steel frame structures under seismic loading combinations are given in Section 12.12 and Table 12.12-1 of ASCE/SEI 7. However, even with these drift limits, second-order effects can be quite large for some gravity-only load combinations, even for frames that satisfy these requirements.

6. *Stiffness of Non-Steel Elements.* The stiffness of non-steel elements such as cladding and partitions (particularly masonry) can both help and hurt a steel frame. It can help reduce frame drift and second-order effects at small amplitudes of lateral loading but it can also hurt the distribution of story shears within the frame resulting in unintended behavior (particularly torsional effects) under high wind and seismic loading. In such cases, the masonry infill should be isolated from the lateral load resisting frame using properly detailed "soft joints."

7. *Soft Stories.* A designer should strive to have nearly uniform stiffness at each story as encouraged by seismic codes. This will reduce demands on the frame under severe lateral loading and reduce high concentrated second-order effects.

8. *Drift Control at Service Load Levels.* Serviceability checks for drift under wind load should include second-order effects whenever realistic drift limits that reflect actual potential damage are used. Torsional deformations in plan from eccentricity of loading, mass and stiffness should also be evaluated when checking story drift limits.

9. *Enveloping Frame Stiffness in Mixed Frame Systems.* When steel moment or braced framing is combined with concrete or masonry shear walls or composite columns or walls used in lateral frame resistance, consideration should be given to cracking of the concrete and masonry elements under different degrees of lateral loading. It is wise to assume various degrees of stiffness for these elements to check the sensitivity of the story shear participation of the steel elements and to design them for the worst effects from various degrees of stiffness. Guidance for stiffness of concrete elements can be found in the ACI 318-08 building code (ACI, 2008).

APPENDIX A
Basic Principles of Stability

A.1 WHAT IS STABILITY?

The SSRC Guide (Ziemian, 2010) defines stability and instability as follows:

Stability: The capacity of a compression member, element or frame to remain in position and support load, even if forced slightly out of line or position by an added lateral force.

Instability: A condition reached during buckling under increasing load in a compression member, element or frame at which the capacity for resistance to additional load is exhausted and continued deformation results in a decrease in load resisting capacity.

Both of these terms are also defined within the AISC *Specification* as follows:

Stability: Condition reached in the loading of a structural component, frame or structure in which a slight disturbance in the loads or geometry does not produce large displacements.

Instability*:* Limit state reached in the loading of a structural component, frame or structure in which a slight disturbance in the loads or geometry produces large displacements.

Perhaps an even simpler definition for stability is equilibrium in a deformed position under the applied load. While theoretical buckling of a perfectly straight member or frame is a bifurcation behavior, real members and frame systems have an initial out-of-straightness such that application of a compressive load immediately results in transverse deflections relative to the initial crooked position. At the onset of instability, the effective lateral stiffness of the structure approaches zero. It is very fortuitous that this behavior often occurs in real structures, as it provides a warning of failure.

A.2 FACTORS INFLUENCING FRAME STABILITY

There are many factors that can influence the stability of steel frame structures. The SSRC Guide (Ziemian, 2010) lists the primary factors in two tables of its Chapter 16: (1) Physical Attributes of the Structure and Loading and (2) Modeling Parameters and Behavioral Assumptions. The tables are repeated here for reference as Tables A-1 and A-2. All of these factors have been considered in the formulation of the AISC *Specification*. A discussion of the most significant of these factors is included in the AISC *Specification* Appendix 7 Commentary [2010 AISC *Specification* Chapter C Commentary]. A brief discussion of several of these factors follows to help provide insight into the AISC *Specification* requirements.

A.2.1 Second-Order Effects, Geometric Imperfections, and Fabrication and Erection Tolerances

The AISC *Specification* provisions are based on the premise that member internal forces are determined using a second-order elastic analysis, where equilibrium is satisfied on the deformed geometry of the structure. Two of the predominant second-order effects on frame members are the P-Δ and the P-δ effects. Figure A-1 illustrates the fundamental meaning of these two effects. In tiered building structures, the P-Δ effect is usually considered as the effect of the vertical loads acting through the lateral sway displacements of the columns and other vertical load supporting elements. However, in general, this effect is simply the couple generated by the axial force acting through the relative transverse displacements of the ends of a given segment. Conversely, the P-δ effect is due to transverse displacement of a member cross section relative to a straight chord caused by the bending deformation of the member. Several attributes are important to note about the P-Δ and the P-δ effects:

1. If there is no sidesway of a member, then there are no member P-Δ moments and no member P-Δ effect.
2. If there is no curvature of the deformed member, that is, if the member remains straight (e.g., an ideal truss element), there are no P-δ moments and there is no P-δ effect.
3. If bending deformations occur in a member due to sidesway, then the member is subjected to both P-Δ and the P-δ effects. For beam-columns under axial compression, the additional P-δ moments at a given cross section cause additional member bending deformations, and hence increase the member sidesway displacements, Δ. This increases the member P-Δ effect.
4. If a member is subdivided into multiple segments, e.g., if a member is modeled using multiple frame elements, the P-δ moments in each segment become smaller and smaller and the second-order effect in the plane of bending is captured by the P-Δ moments from the combination of all of the segments.

Consideration must be given to initial geometric imperfections in the structure due to fabrication and erection

Table A-1. Factors Affecting Steel Frame Stability—Physical Attributes of Structure and Loading
Frame geometry and configuration
Centerline framing dimensions
Member geometry and material
Connection details
Foundation and support conditions
Shear connections to slab
Infill walls or secondary structural elements
Finite member and joint size effects
Out-of-plane bracing elements
Material properties
Elastic moduli
Expected versus nominal strengths
Ductility and fracture toughness
Geometric imperfections
Erection out-of-plumbness
Member out-of-straightness
Incidental connection or loading eccentricities
Internal residual stresses
From manufacturing/fabrication processes
From erection fit-up
From construction sequencing
From incidental thermal loadings or support settlements
Loadings
Magnitude and distribution
Loading rate and duration

tolerances. In the traditional effective length method (ELM) and the first-order analysis method (FOM), the structure is assumed perfectly straight in the structural analysis model. The FOM is calibrated so that these effects are accounted for in the design of the frame; these effects are more implicit than explicit in the ELM. In the direct analysis method (DM), using the concept of notional loading, initial geometric imperfections are conservatively assumed to be equal to the maximum fabrication and erection tolerances specified by the AISC *Code of Standard Practice for Steel Buildings and Bridges* (AISC, 2005d). The user is free to modify these assumptions in the analysis based on evidence of stricter control. This is discussed in more detail later in this appendix. In ASTM A6/A6M (ASTM, 2012), for a W-shape with a flange width greater than 6 in. the member out-of-straightness works out to be approximately $L/1000$, where L is the member length in inches between bracing or framing points. This is explicitly stated in the AISC *Code of Standard Practice* as the frame out-of-plumbness tolerance of $L/500$, where L is the story height, subject to specified absolute limits.

A.2.2 Residual Stresses and Spread of Plasticity

Residual stresses inherent in all rolled and built-up shapes during the rolling and fabrication process cause early yielding as the strength limit state is approached. This softening of the structure or spread of plasticity through the member cross section and along the member length is directly accounted for in the DM by reducing the axial and flexural properties of the members that are part of the lateral load resisting frame. In the ELM and the FOM, this effect is calibrated into the design process to account for this effect using the nominal properties of the members. Residual stresses also contribute to the stiffness reduction factors defined as τ_a and τ_b in the AISC *Specification* and Commentary.

A.2.3 Member Limit States

Strength of members in the lateral load resisting frame may be controlled by cross-sectional yielding, local buckling, flexural buckling, and lateral-torsional or flexural-torsional

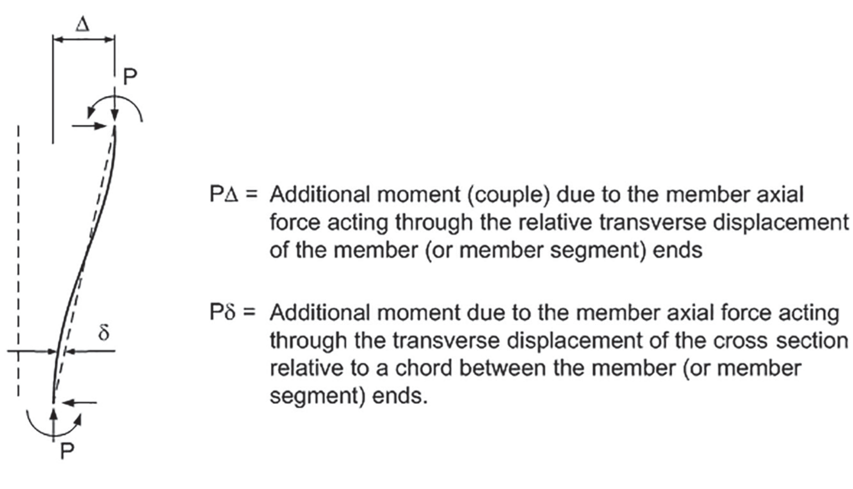


Fig. A-1. Second-order P-Δ *and* P-δ *effects.*

Table A-2. Factors Affecting Steel Frame Stability—Modeling Parameters and Behavioral Assumptions

Linear elastic response
Flexural, axial, and shear deformations of members
Deformations of connections and beam-column panel zones
Uniform torsion and/or nonuniform warping torsion deformations in members
Foundation and support movement
Dynamic and inertial effects
Geometric nonlinear (second-order) response
P-δ effects: Influence of axial force on stiffness and internal moments in beam-columns
P-Δ effects: Influence of relative joint displacements on forces and displacements
Local buckling and cross section distortion
Finite rotation effects (three-dimensional behavior)
Material nonlinear response
Member plastification under the action of axial force and biaxial bending (spread of plasticity versus plastic hinge idealizations)
Member plastification due to shear forces, uniform torsion, and nonuniform warping torsion (bi-moments)
Yielding in connection components and joint panels
Tension rupture of members and connections
Strain hardening behavior
Cyclic plasticity effects
Load path effects, shakedown, and incremental collapse

buckling. These limit states must be checked with separate member design equations from the various chapters of the AISC *Specification* and through the use of the beam-column interaction equations. A feature in the 2005 AISC *Specification* reducing some of the conservatism in the design allows compact-element section wide-flange members subjected to single-axis flexure and axial compression to be checked with interaction equations in AISC *Specification* Section H1.3.

A.3 SIMPLE STABILITY MODELS

Many of the key aspects of frame stability can be demonstrated with simple stability models. Three such models are shown in Figure A-2. The models are described in the following.

Model A: Model A depicts the simplest of all moment frames—a cantilever column with a second pin-connected column carrying only gravity load, often called a "leaning column." This model is used to explain the basic principles of second-order effects in frames including the P-δ and P-Δ effect, the reduction in column stiffness from axial load (represented by the C_L factor defined by LeMessurier (1977)) and the destabilizing effect of leaning columns.

Model B: Model B depicts a one-story braced frame, represented by a simple lateral spring support, along with a leaning column. This model is used to explain the same basic principles as Model A except in the context of a braced frame structure.

Model C: Model C depicts a subassemblage with one leaning column representing the gravity columns of a floor stabilized by a moment frame. The sub-assemblage may be thought of as a typical bay of a moment frame consisting of a beam above and below a moment frame column (see Figure A-3). Each of the single beams at the top and bottom of the column may be thought of as representing two beams, one framing in from each side of the column. The beams in the analysis model are assumed pinned at the mid-length of the physical beams and the properties (EI_g) are doubled to properly mimic an interior subassemblage. This model is used to demonstrate some of the same basic principles of stability as Model A, but in a more realistic setting.

These models were also used to demonstrate the stability design procedure for the ELM (Chapter 2) and the DM (Chapter 3) covered in Chapter C and Appendix 7 of the 2005 AISC *Specification* [Appendix 7 and Chapter C of the 2010 AISC *Specification*].

A.3.1 Model A

A simple cantilever column with and without leaning columns can be used to demonstrate many key principles of frame stability as demonstrated in Figure A-2 as Model A. This type of model was studied extensively by LeMessurier (1977) and a number of conclusions reached in that study are present in the building codes today.

Consider first the case without a leaning column. The first- and second-order moment diagrams are illustrated in Figure A-4. Analytical expressions for the second-order base moment and tip deflection in this problem are as follows (Timoshenko and Gere, 1961; Chajes, 1974; Chen and Lui, 1987):

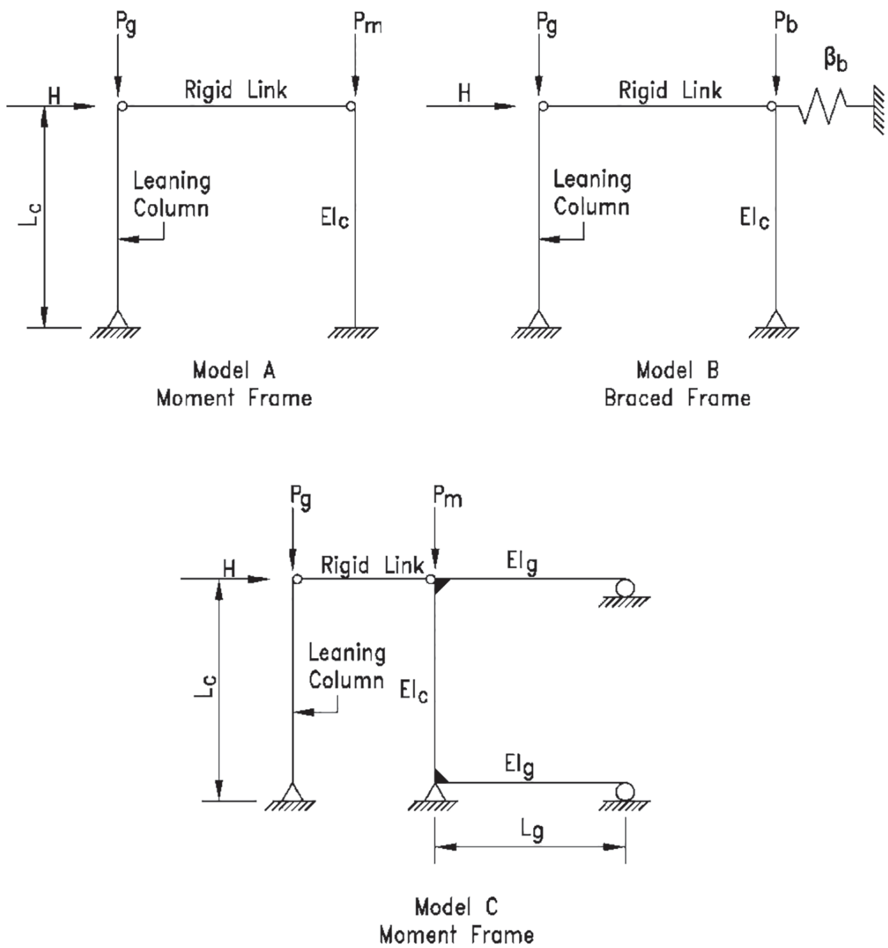


Fig. A-2. Simple stability models.

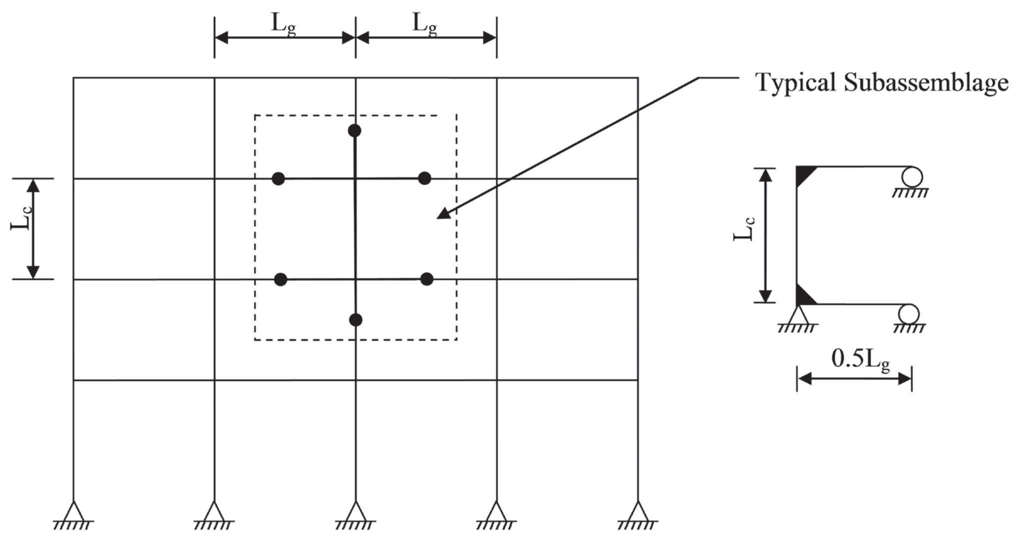


Fig. A-3. Typical moment frame subassemblage.

$$M_{max} = HL\frac{\tan u}{u} \tag{A-1}$$

$$\Delta_{2nd} = \Delta_H\left[\frac{3(\tan u - u)}{u^3}\right] \tag{A-2}$$

where

FAF = force amplification factor

$$= \frac{M_{max}}{HL}$$

$$= \frac{\tan u}{u} \tag{A-3}$$

DAF = deflection amplification factor

$$= \frac{\Delta_{2nd}}{\Delta_H}$$

$$= \left[\frac{3(\tan u - u)}{u^3}\right] \tag{A-4}$$

$$u = \pi\left(P/P_{eL}\right)^{0.5} \tag{A-5a}$$

$$P_{eL} = \pi^2 EI/L^2 \tag{A-5b}$$

(Euler load based on *actual* length); therefore, $u = \sqrt{\dfrac{PL^2}{EI}}$

$$\Delta_H = HL^3/3EI \tag{A-5c}$$

The FAF and the DAF are the analytical solutions for the ratios of the second-order to the first-order maximum moment and deflection values, respectively.

LeMessurier derived comparable expressions for the base moment, tip deflection, FAF and DAF. LeMessurier's FAF and DAF expressions are:

$$FAF = \frac{1}{\left(1 - \dfrac{P}{P_L - C_L P}\right)} \tag{A-6}$$

$$DAF = \frac{1}{\left(1 - \dfrac{P(1 + C_L)}{P_L}\right)} \tag{A-7}$$

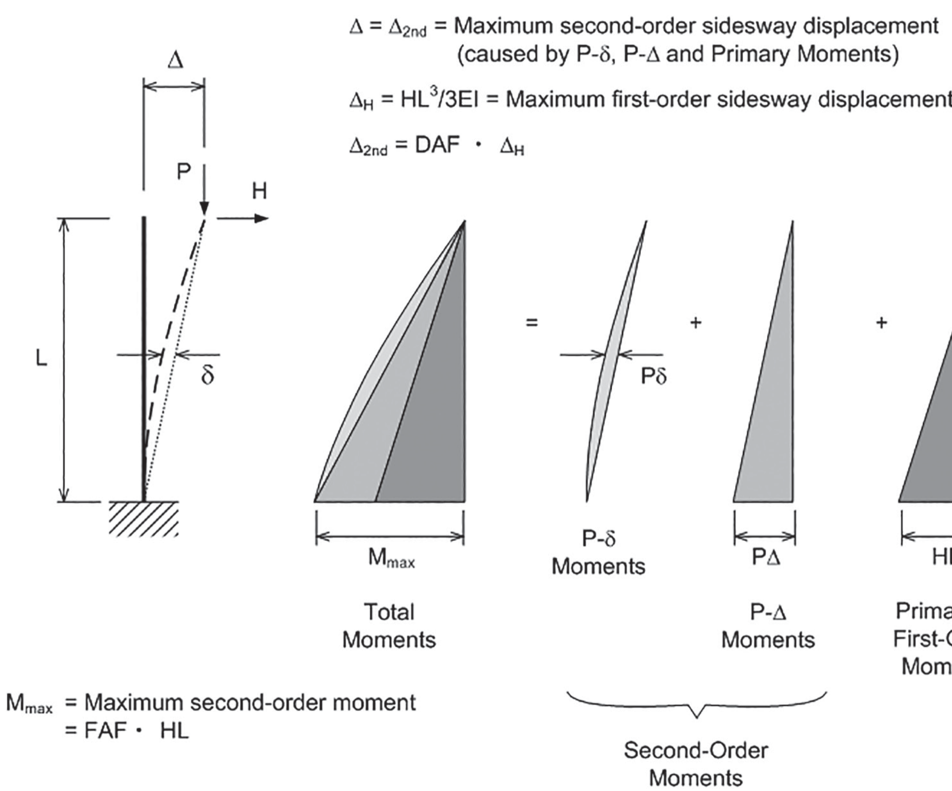


Fig. A-4. First- and second-order effects—cantilever column.

P_L is defined as a story lateral load required to produce a unit first-order story drift ratio, Δ_H/L, where Δ_H is the first-order story drift under a lateral load, H. In the case of the cantilever column:

$$\Delta_H = \frac{HL^3}{3EI} \quad \Rightarrow \quad \frac{\Delta_H}{L} = \frac{HL^2}{3EI}$$

For $\frac{\Delta_H}{L} = 1$ (such that $H = P_L$):

$$1 = \frac{P_L L^2}{3EI} \quad \Rightarrow \quad P_L = \frac{3EI}{L^2} \tag{A-8}$$

The factor C_L accounts, in an approximate way, for the reduction in the column sidesway stiffness due to the presence of the axial load on the column, i.e., the P-δ effect on the sidesway deflections. For most practical cases, C_L varies from 0 to 0.216. For a cantilever column with infinitesimal axial load it is equal to 0.2. As the buckling load on the cantilever column is approached, the value of C_L approaches 0.216. LeMessurier derived C_L by evaluating the effect of the additional moments caused by the column axial force acting through the deflection relative to its chord (δ). These deflections, δ, are caused by the bending curvature. As the buckling load is approached, the column deflected shape approaches a sine curve (see Figure A-4).

By comparing the analytical solutions for the *FAF* and the *DAF* with the results from LeMessurier's expressions, it is found that they are nearly identical. The *FAF* and *DAF* are plotted in Figure A-5 based on Equations A-6 and A-7. Also, the result from these expressions is compared to two different forms of the AISC B_2 Equation A-12 developed in the following, with and without the consideration of P-δ effects on the sidesway moments and sidesway deflections. The abscissa of the plot in Figure A-5 is the ratio P/P_{eL}, where P_{eL} is the column Euler buckling load based on $K = 1$, given by Equation A-5b. The amplification effect also is illustrated in Figure A-6 where the tip deflection of the cantilever column in Model A is shown magnified from a first-order deflection value to include the second-order effect of the axial load in the column. A W10×60 in major-axis bending with $L_c = 22$ ft ($L/r = 60$), and proportional loading with $H/P = 0.01$ is used for the cantilever column in developing this plot. The foregoing discussion reveals the following key points:

1. The *DAF* is larger than the *FAF*.
2. If C_L is taken equal to zero, the *DAF* and *FAF* are identical. In effect, if C_L is taken equal to zero, the solutions

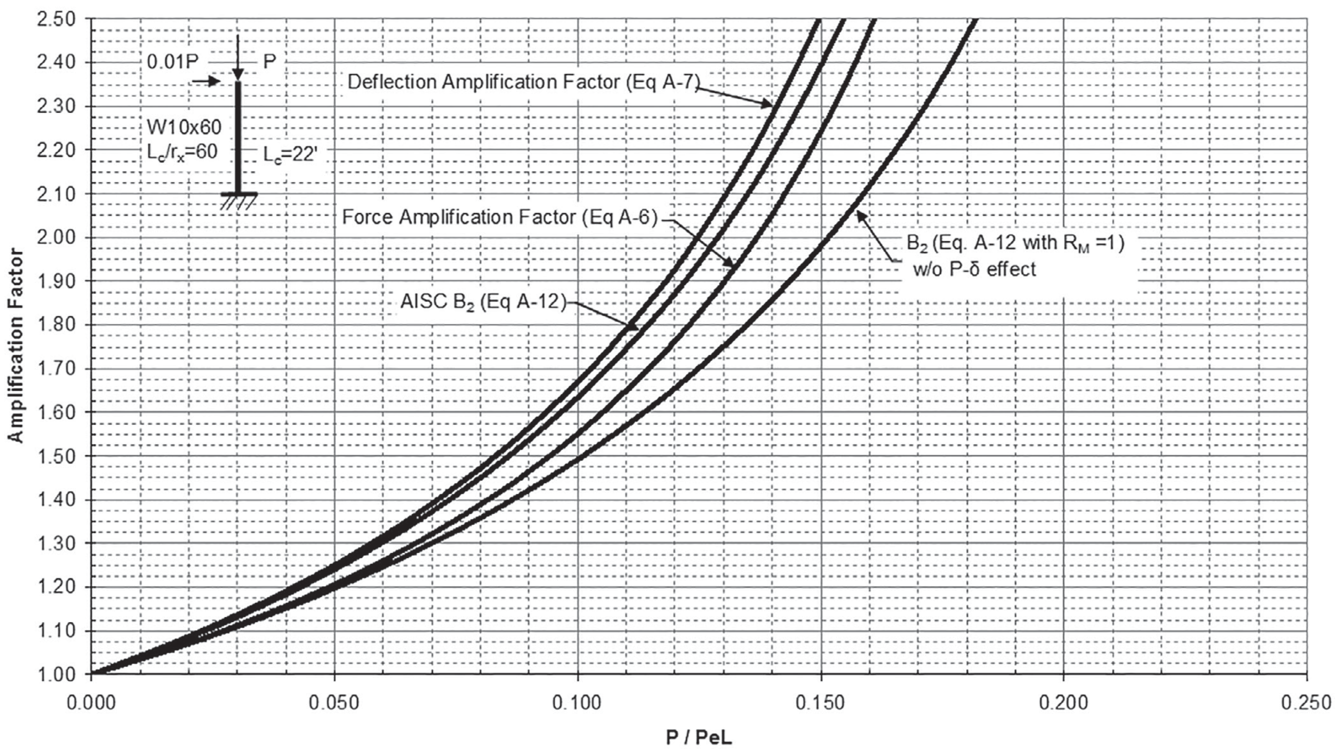


Fig. A-5. Amplification factors, cantilever column—Model A.

ignore the effect of stiffness reduction due to P-δ moments on the amplification of the base moment and tip deflection. This can lead to significant error in the true amplification factor at high axial loads. To demonstrate this point, B_2 is also plotted in Figure A-5 without the effect of the flexural stiffness reduction due to the P-δ moments ($R_M = 1$ or $C_L = 0$).

3. Within the practical range of $B_2 \le 2.5$, the AISC B_2 values are reasonably close to the analytical amplification values for the DAF and the FAF.
4. The presence of axial load amplifies the base moment and the tip deflection—the greater the axial load the greater the amplification.

It should be emphasized that the P-δ effects captured by the C_L term are associated with sidesway and are different than the P-δ effects related to "non-sidesway" deflections that many engineers are accustomed to considering. The "non-sidesway" P-δ effects are captured by the B_1 modifier in an amplified first-order elastic analysis. The B_1 modifier does not have any influence on the sidesway deflections and moments.

If a leaning column is added to the problem, the solutions for the FAF and DAF are similar, with P replaced with $\Sigma P = P_m + P_g$, the sum of the gravity load on the cantilever column (P_m) and the leaning column (P_g):

$$FAF = \frac{1}{\left(1 - \dfrac{\Sigma P}{P_L - \Sigma C_L P}\right)} \tag{A-9}$$

$$DAF = \frac{1}{\left(1 - \dfrac{\Sigma P + \Sigma C_L P)}{P_L}\right)} \tag{A-10}$$

C_L is always zero for leaning columns because they are theoretically straight in the deflected configuration (such that the P-δ moments are zero along their length). Also, stocky columns with flexible end restraints, where the drift is predominantly due to deformations in the restraining elements and the columns are nearly straight in the deflected structure, have C_L values that approach zero.

Figure A-7 shows the amplification factor for Model A with a leaning column load (P_g) equal to the same load on the cantilever column ($P_g = P_m$) and a load of four times the cantilever column load ($P_g = 4P_m$). Note the significant effect of leaning column loads on the amplification factors. Figure A-8 shows the axial load-moment (P-M) interaction curves (AISC *Specification* Equations H1-1a and H1-1b) for Model A (Figure A-2) based on the ELM for the case of a W10×60 in strong-axis bending with $L_c = 7.32$ ft and $L/r = 20$. The curves rising from the origin on this plot depict

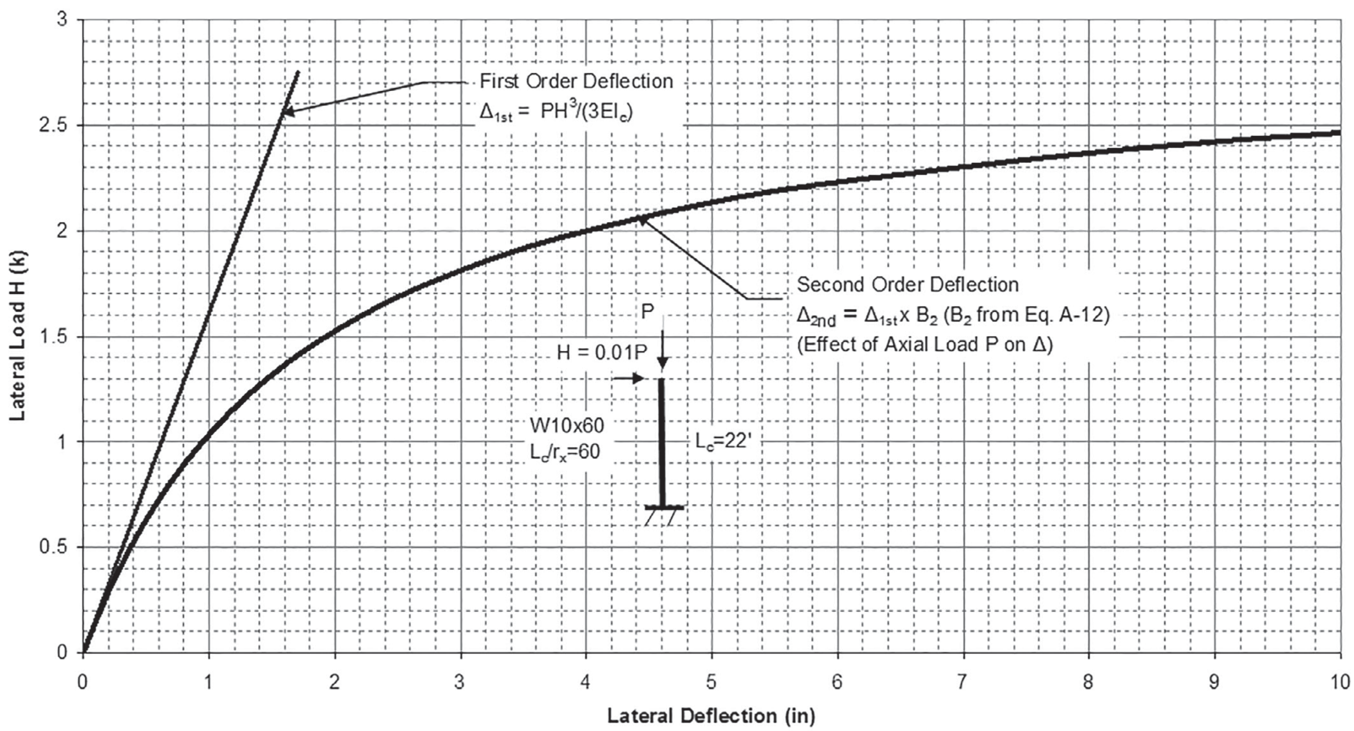


Fig. A-6. Lateral load versus cantilever column tip deflection—Model A.

the second-order elastic internal axial force and moment at the base of the cantilever due to proportionally increasing lateral and vertical load on the frame ($H/P_m = 0.01$). There are three column interaction curves shown, one for each case of vertical load, since the effective length factor, K, changes with the amount of leaning column load stabilized by the cantilever column. K is calculated from Equation C-C2-8 in the AISC *Specification* Commentary Chapter C [Equation C-A-7-8 in the 2010 AISC *Specification* Commentary Appendix 7]. Note the significant reduction in axial load available strength when the leaning column load is added to the frame. This emphasizes the extreme importance of including all leaning column loads in frame stability design.

Equations A-9 and A-10 hold true for any moment frame story assemblage of columns. Examination of these two expressions for the *FAF* and the *DAF* reveals the following:

1. Once again, the $DAF > FAF$ in all cases.
2. If $C_L = 0$, ignoring the P-δ effect on the sidesway deflections, the two expressions are identical.

The AISC B_2 equation (AISC *Specification* Equation C2-3 in combination with Equation C2-6b [2010 AISC *Specification* Equations A-8-6 and A-8-7, with different symbols]) can be derived from the DAF equation as follows. If ($\Sigma P + \Sigma C_L P$) in the denominator of Equation A-10 is conservatively taken as $\Sigma P + C_{Lavg}\Sigma P = \Sigma P\left(1 + C_{Lavg}\right)$ and if C_{Lavg} is taken as a relatively large value of 0.18 (it is difficult to obtain ideal full base fixity, and C_L is generally smaller when some flexibility is accounted for in the end restraint(s)), then:

$$\begin{aligned} R_M &= \frac{1}{\left(1 + C_{Lavg}\right)} \\ &= \frac{1}{1.18} \\ &= 0.85 \end{aligned} \tag{A-11}$$

$$\begin{aligned} B_2 &\cong \frac{1}{1 - \dfrac{1.18\Sigma P}{P_L}} \\ &= \frac{1}{1 - \dfrac{\Sigma P}{0.85 P_L}} \\ &= \frac{1}{1 - \dfrac{\Sigma P}{R_M P_L}} \end{aligned} \tag{A-12}$$

where LeMessurier's equation, $P_L = \Sigma HL/\Delta_H$ (LeMessurier,

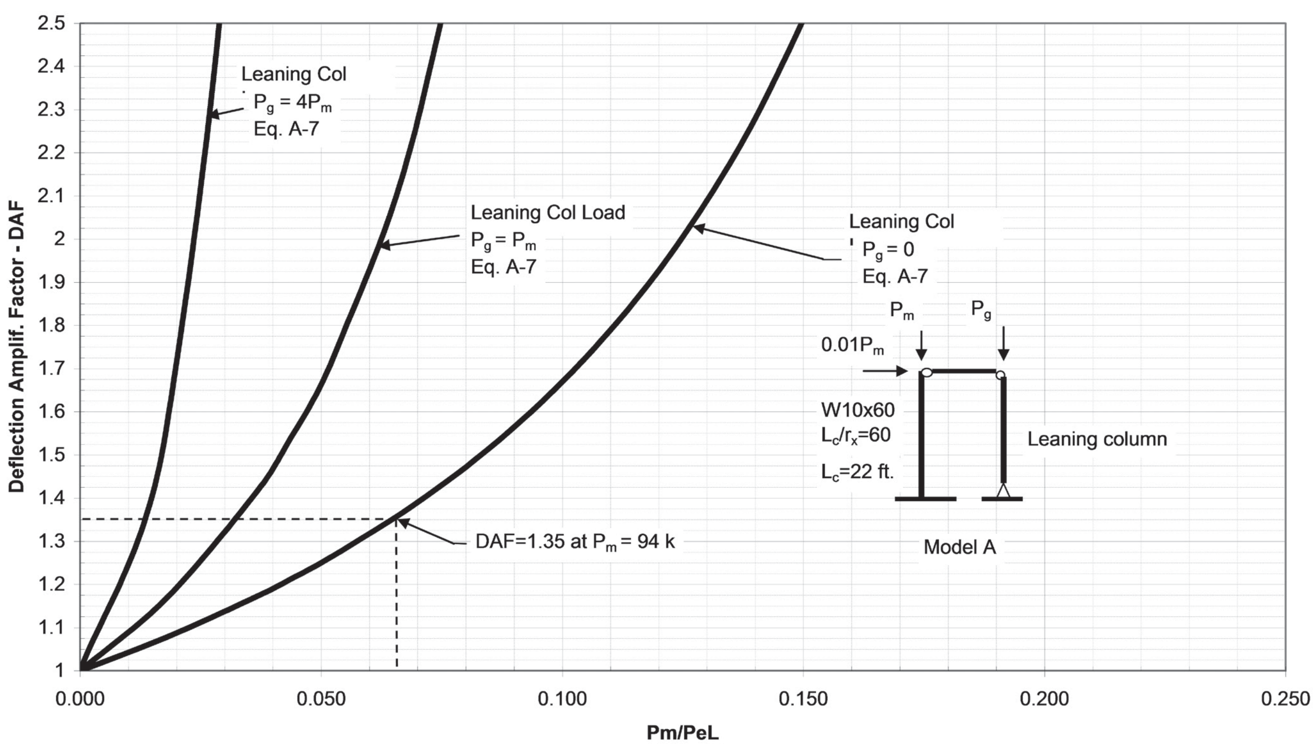


Fig. A-7. Leaning column effect on amplification factors—Model A.

1977). As illustrated in Figure A-5, the resulting B_2 equation in the AISC *Specification* is a reasonable approximation of both the *DAF* and *FAF* for the extreme case of the cantilever column. One can gain a more detailed understanding of this approximation by considering more detailed equations for C_L discussed at the end of this section.

The R_M factor only applies for moment frames. It is taken as unity for braced frames because the members are idealized as straight and there are theoretically no P-δ moments in members of braced frames (thus, there is no stiffness reduction related to the P-δ effect on the lateral drift in these types of structures).

Related equations for calculation of sidesway buckling loads, provided in the Commentary to the AISC *Specification* and discussed subsequently in this guide (specifically AISC *Specification* Commentary Equations C-C2-5 and C-C2-6 [2010 AISC *Specification* Equations C-A-7-5 and C-A-7-6]), effectively utilize the following equation for R_M, which depends on the relative proportion of the gravity load supported by moment frames and gravity and/or braced frames in the story:

$$R_M = 0.85 + 0.15R_L \qquad \text{(A-13)}$$

where

$$R_L = \frac{\Sigma P \text{ leaning columns and braced frames}}{\Sigma P \text{ all columns}} \qquad \text{(A-14)}$$

Equation A-13 may be used as an accurate expression for R_M, which is justified by the considerations discussed previously. For the Model A frames, $R_M = 0.925$ for $P_g = P_m$ and $R_M = 0.97$ for $P_g = 4P_m$. This gives a very close approximation to the results from Equation A-10.

Update Note: The 2010 AISC *Specification* includes the expression for R_M discussed here. Equation A-8-8 in the 2010 AISC *Specification* is the same as Equation A-13, with different symbols and in a different form.

Precise calculations of C_L could be conducted and Equations A-9 and A-10 could be employed for calculation of second-order moments and drifts. However, the additional precision is rarely justified. Equations A-11 and A-12 provide a reasonably accurate calculation of the *FAF* and *DAF* in practical building frames. The following section provides additional background on this topic and gives equations for calculation of more precise C_L values in the unusual cases where higher precision is merited.

The influence on the second-order amplification (B_2) by ignoring the flexural stiffness reduction (C_L) in moment frames can be very large when C_L is large and the axial load

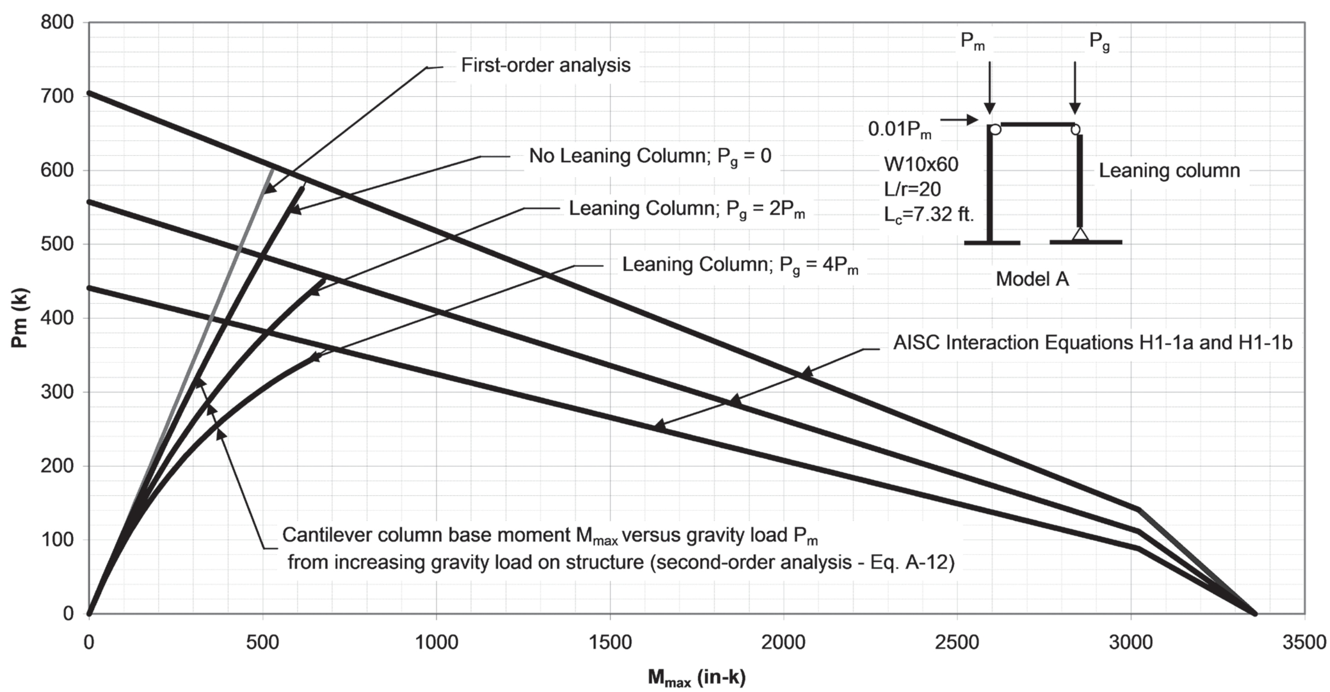


Fig. A-8. Force-point traces and beam-column strength interaction curves (effective length method)—effect of leaning columns on Model A.

in the column is high. The error in B_2 is illustrated in Figure A-9, where B_2 from Equation A-12 is plotted for Model A with gravity load placed on the cantilever column ($P_g = 0$) with and without the effect of R_M (equivalent to calculating the *DAF* in Equation A-10 with and without C_L). Note that for $B_2 = 1.5$, the error is about 7% but at $B_2 = 2.5$, the error is about 22%. This comparison also demonstrates why it is important for a computer program to accurately account for P-δ effects in the calculation of story drifts and sidesway moments.

A.3.1.1 Further Background on C_L and the P-δ Effect in Moment Frames

This section provides detailed equations for determining refined estimates of C_L in individual moment-frame columns. These individual C_L values may be substituted in Equations A-9 and A-10 to calculate refined estimates of the story force and sidesway amplification factors. As noted previously, this level of precision is not considered to be justified in the context of story amplification approaches for second-order analysis. However, the developments in this section are useful for gaining a clear understanding of the influence of column P-δ moments on the second-order sidesway behavior of building structures.

The term C_L can be defined numerically for any given column participating in the flexural resistance in a story of a moment frame by a simple equation that relates the elastic end restraints, G, on the column with the elastic effective length factor, K, from the sidesway buckling equation that defines the sidesway nomograph (Kavanagh, 1962). The equations are defined as follows:

$$C_L = \frac{\beta K^2}{\pi^2} - 1 \tag{A-15}$$

$$\beta = \frac{6\left(G_A + G_B\right) + 36}{2\left(G_A + G_B\right) + G_A G_B + 3} \tag{A-16}$$

$$0 = \frac{6\dfrac{\pi}{K}}{\tan\dfrac{\pi}{K}}\left(G_A + G_B\right) - \left(\frac{\pi}{K}\right)^2 G_A G_B + 36 \tag{A-17}$$

K can be solved for directly with the following approximate equation, which is accurate within 2%:

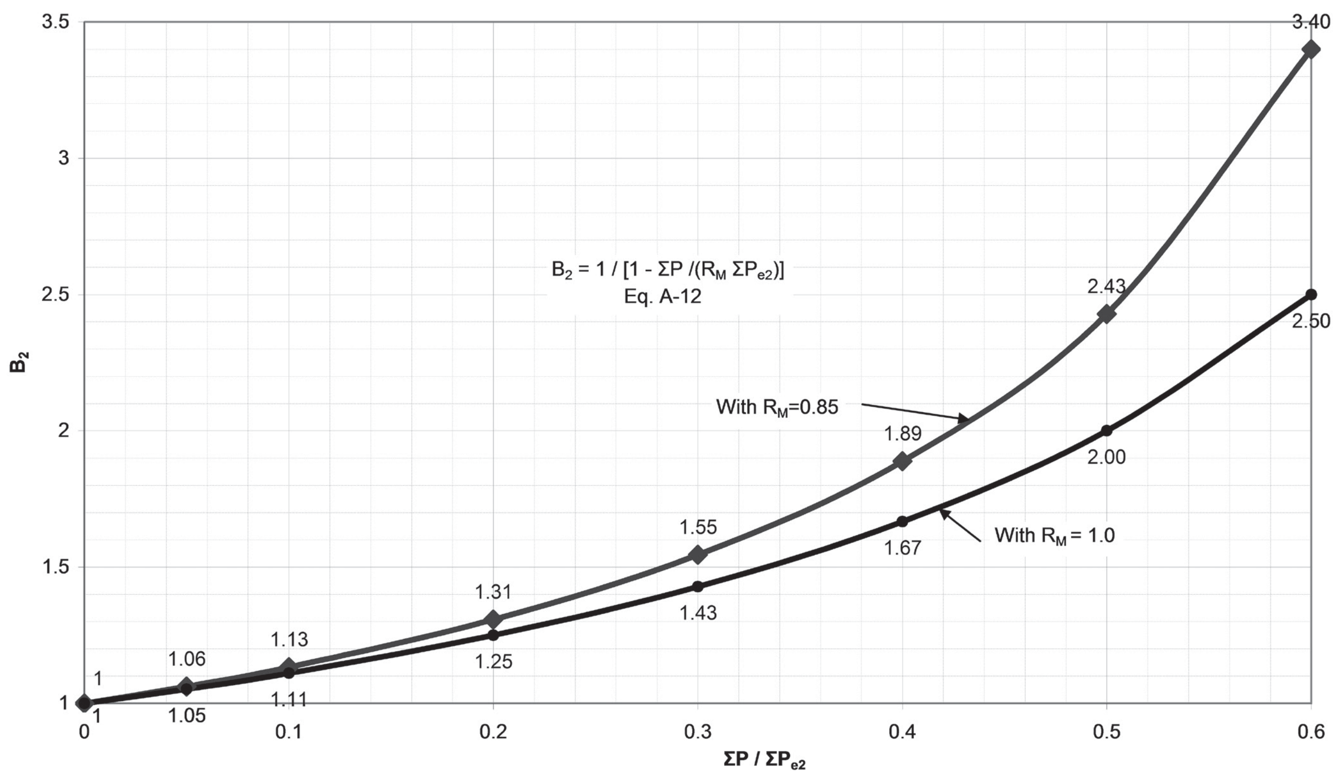


Fig. A-9. Amplification factor comparison, effect of R_M.

$$K = \sqrt{\frac{1.6G_A G_B + 4.0(G_A + G_B) + 7.5}{G_A + G_B + 7.5}} \quad \text{(A-18)}$$

where

$$G = \frac{\Sigma(EI_c/L_c)}{\Sigma(EI_g/L'_g)} \quad \text{(A-19)}$$

$$L'_g = L_g(2 - M_F/M_N) \quad \text{(A-20)}$$

Refer to the AISC *Specification* Commentary Chapter C [2010 AISC *Specification* Commentary Appendix 7, Section 7.2] for further discussion of L'_g. G is calculated for each end, A and B, of the column. C_L varies between zero and 0.216 based on the equations presented. It tends to be larger for columns having small G values at one or both ends. This is because, for a given sidesway deflection, the column curvature is largest (and therefore the displacement of the column relative to its deflected chord and the column P-δ moments are largest) when there is significant end restraint. When G is large at both column ends, indicating that the column end restraints are small and the sidesway deflections are associated with rigid body rotation of the column with the deformations occurring predominantly in the restraining beams, C_L approaches zero.

The foregoing equations require the use of the sidesway uninhibited alignment chart. LeMessurier (1993) provided the following equations that allow the determination of C_L solely in terms of the results from a first-order sidesway analysis:

$$C_L = 0.216\left(\frac{\beta_L}{3 + 9\alpha}\right)^2 \quad \text{(A-21)}$$

$$\beta = \frac{\beta_L EI}{L^3} \quad \text{(A-22)}$$

where

$$\beta_L = \frac{H}{\Delta_H}\left(\frac{L^3}{EI}\right) \quad \text{(A-23)}$$

= coefficient corresponding to the contribution of the moment-frame column to the first-order story sidesway stiffness ($\beta_L = 3$ for a cantilever column)

$$\alpha = \frac{M_S}{M_L} \quad \text{(A-24)}$$

= ratio of the smaller to the larger end moment in the moment-frame column, determined from a first-order sidesway analysis

H = column horizontal shear determined from the first-order sidesway analysis, kips

Δ_H = column drift associated with the shear force, H, determined from the first-order sidesway analysis, in.

L = column (and story) height, in.

EI = column flexural rigidity, kip-in.^{2}

For practical purposes, the results from a first-order drift analysis for wind load can be used to determine the ratios H/Δ_H and M_S/M_L. However, strictly speaking, notional lateral loads proportional to the gravity load at each story level, e.g., $N_i = 0.002Y_i$, are more closely related to the true sidesway stability effects. These lateral loads are recommended for unusual cases where the frame is not designed for any applied lateral loads (ASCE, 1997). LeMessurier (1993) shows that Equation A-7 gives results that are within a few percent of the analytical Equation A-4 for all cases involving an isolated sidesway column restrained by top and bottom elastic springs.

Important attributes of the behavior that can be discerned from Equation A-7 are as follows:

1. In the limit that β_L approaches zero, such that the column is approaching a leaning column or a simply connected column in a braced framing system, C_L approaches zero. These types of columns do not have any significant curvature under sidesway of the frame. Therefore, their P-δ effects associated with the sidesway are zero.
2. At the limit of rigid rotational restraint at one end and ideally free rotational conditions at the other, $\beta_L = 3$, $\alpha = 0$, and $C_L = 0.216$.
3. At the limit of rigid rotational restraint at both ends, $\beta_L = 12$, $\alpha = 1$, and $C_L = 0.216$.
4. Cases 2 and 3 both involve maximum deflections of the deformed sidesway moment-frame column relative to its rotated chord, with the exception of very unusual frames in which one or both column end rotations due to the frame sidesway are in the opposite direction to the rotation of the column chord. ASCE (1997) provides some discussion of these unusual cases.
5. With the exception of the unusual cases stated here, whenever there is some finite rotational restraint at the column ends, C_L has an intermediate value between 0.0 and 0.216. The most important factor influencing the value of C_L is the relative stiffness of the restraining beams.

A.3.2 Model B

Model B in Figure A-2 demonstrates the behavior of a braced frame structure with and without the influence of leaning columns. Note that this model uses an elastic lateral spring support with stiffness β_b. The linear spring support can be thought of as representing the stiffness of a bracing system, such as the diagonal brace and righthand column shown in Figure A-10. LeMessurier (1976) derived an equation for the sidesway story stiffness of this bracing system, shown as β_b in Figure A-10, and showed that the force amplification factor for a braced frame takes the same form as that for a moment frame, but with $C_L = 0$. This is because in a braced frame there is no P-δ effect on the lateral deflection of the structure since all members are loaded predominantly only by axial force. The amplification factor is:

$$DAF = FAF \qquad \text{(A-25)}$$
$$= \frac{1}{1-\dfrac{\Sigma P}{P_L}}$$
$$= \frac{1}{1-\dfrac{P_g + P_b}{P_L}}$$

For the specific example in Figure A-11, the stiffness parameter P_L may be expressed as:

$$P_L = \frac{HL_c}{\Delta_H} \qquad \text{(A-26a)}$$
$$= \frac{A_c E}{\tan^2\theta + \lambda\left(\tan^2\theta + 1\right)}$$

using virtual work (LeMessurier, 1976). Note that this P_L is proportional to the axial rigidity, A_cE, modified by the term in the denominator, which is a function of the brace angle, θ, and the ratio of column-to-brace area parameter, $\lambda = A_c/(A_b \sin\theta)$. Also, it is useful to note that in many cases, the area of the bracing diagonal will be significantly smaller than the area of the column. At this limit of $EA_c/L_c >> EA_b/L_d$, where L_d is the length of the diagonal brace, Equation A-26a reduces to:

$$P_L = \frac{HL_c}{\Delta_H} \qquad \text{(A-26b)}$$
$$= \frac{A_b E L_c}{L_d}\cos^2\theta$$

Equation A-25 has the same form as the AISC *Specification* B_2 equation shown in Equation A-12 with $R_M = 1$ as specified in AISC *Specification* Section C2.1b [2010 AISC *Specification* Appendix 8, Section 8.2.2] for braced frame systems.

A simple braced frame example using Model B is used here to demonstrate some important features of low rise braced frames. The same W10×60 column with $L_c = 22$ ft as used previously for the cantilever column in Model A and in generating Figure A-5 is used with a tension diagonal brace attached to it providing stability as shown in Figure A-11. The brace (A_b = 0.247 in.^{2}, λ = 148) is sized so that it has approximately the same lateral capacity as the cantilever column for a lateral load, H = 9.4 kips, and a column load, P_b = 94 kips (H/P_b = 0.1, with no leaning column load). The problem is solved first with a zero leaning column load (P_g = 0) and secondly as a case with a large leaning column load ($P_g = 4P_b$, representing a large number of gravity columns stabilized by the bracing system). The resulting amplification factor, B_2, is plotted against the normalized column

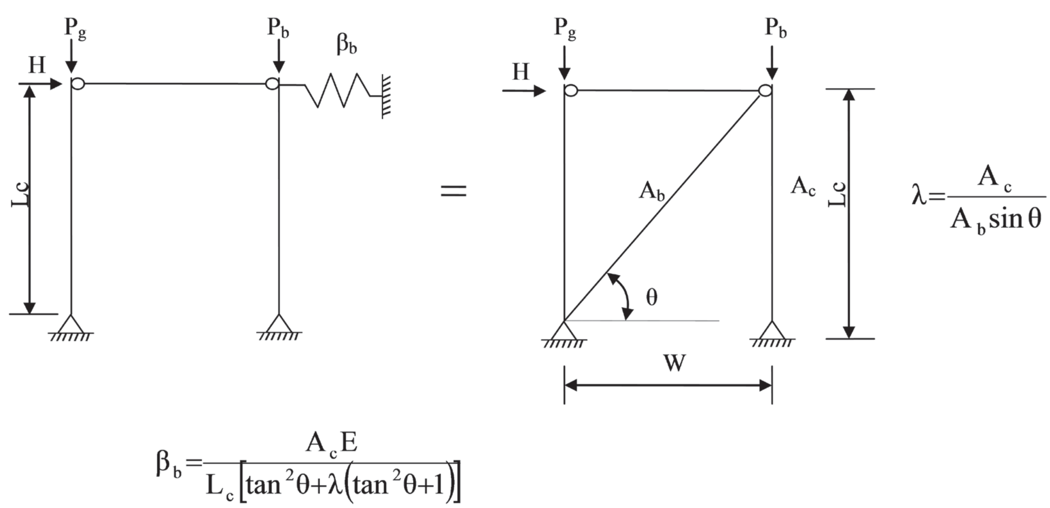


Fig. A-10. Calculation of braced frame stiffness—Model B.

axial load $\left(P_b/P_{eL}\right)$ in Figure A-12, where $P_{eL} = \pi^2 EI/L_c^2$.

The following conclusions can be reached by comparing the amplification factor, B_2, in Figure A-12 for the braced frame to that shown in Figure A-7 for the cantilever column moment frame:

1. Second-order effects are much more pronounced for the cantilever moment frame than the braced frame. The amplification factors increase at a much faster rate as the axial load is increased.
2. The leaning column effect is much more pronounced for the moment frame than the braced frame. For a given leaning column load, the increase in the amplification factor for a braced frame is much smaller than for a moment frame. This behavior is true in general because braced frames tend to be much stiffer than moment frames.

The story deflections for the two frame types are compared in Figure A-13. In this figure, the braced frame deflection and the cantilever column deflection are both plotted for

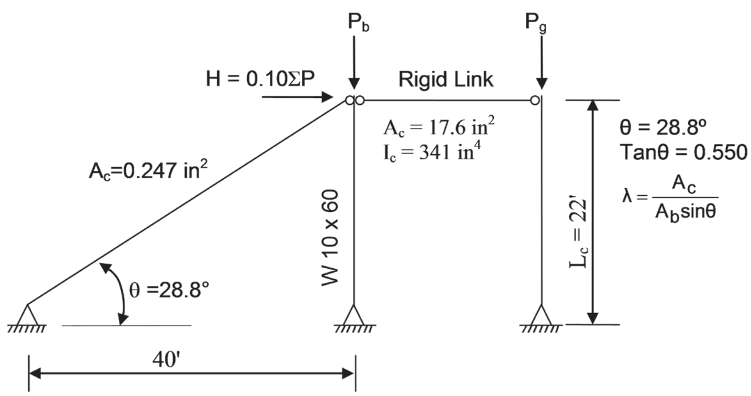


Fig. A-11. Braced frame example.

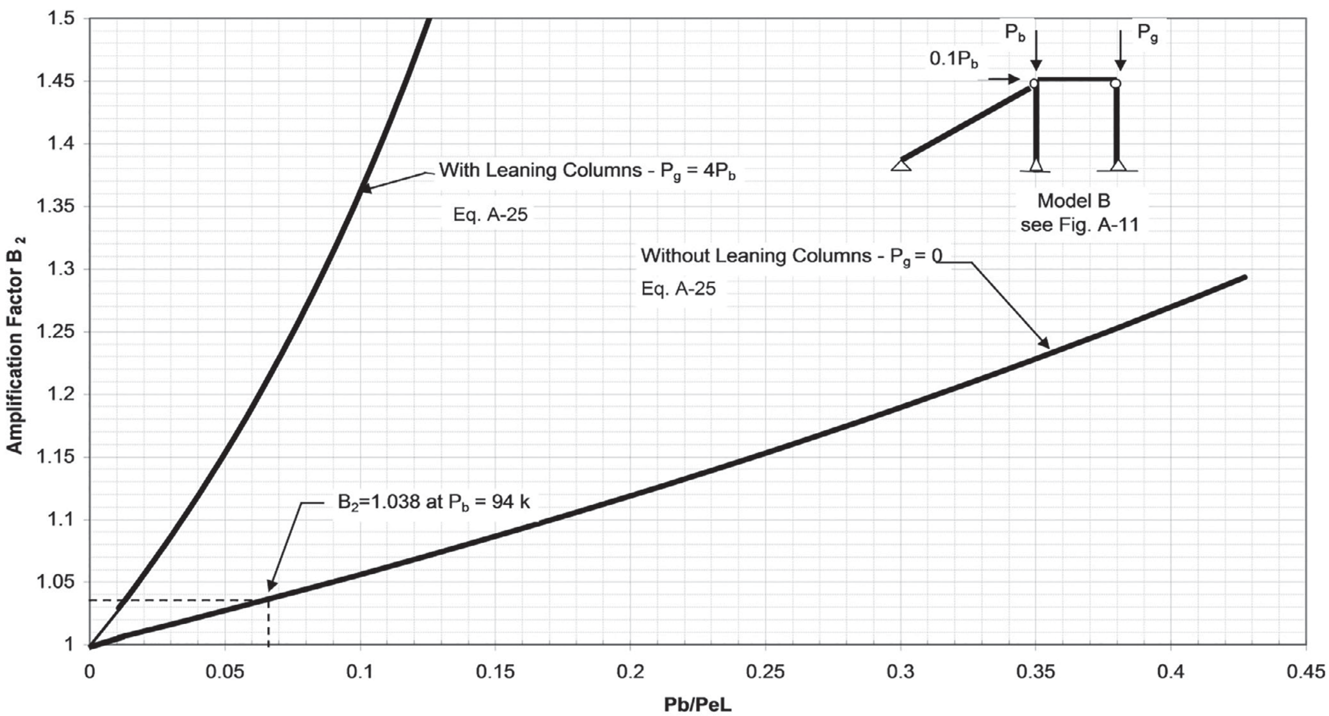


Fig. A-12. Amplification factors for representative braced frames—Model B.

the case of $H = 0.1P$, where P is the load on the column. Zero load is applied to the leaning column load in this structure. It can be seen that the braced frame is much stiffer overall than the moment frame. At a lateral load, H = 9.4 kips, the strength design load for the braced frame and cantilever column, the braced frame deflection is only 1 in. compared to the moment frame deflection of 8 in. Note that a typical drift limit for this story height would be between 0.67 in. ($L/400$) and 2.64 in. ($L/100$). It is evident that second-order deflections are much less for the braced frame than for the moment frame. Also, they are influenced to a much lesser degree as the lateral and axial load increase. Figure A-14 shows the braced frame deflection ratio (L_c/Δ) with $H = 0.1P_b$ for no leaning column load, $P_g = 0$, and with a leaning column load, $P_g = 4P_b$. These comparisons help to explain why drift usually controls moment frame design while strength usually controls braced frame design.

A.3.3 Model C

Model C in Figure A-2 has the ability to simulate many different conditions found in practice. If the same girder is used top and bottom, the model simulates the case of a typical subassemblage in a story of a multi-story moment frame. The story shear is applied as a horizontal joint load. The influence of the story shears and the column second-order effects in the stories above and below the model may be included by reducing the girder stiffnesses, EI_g, in the model to account for the missing primary and P-Δ moments at the beam-to-column joints from the columns above and below the story. If the stiffness of the bottom girder is taken equal to zero, the model represents the case of a one-story moment frame pinned at the base. If the bottom girder stiffness, EI, and the length, L, is selected so that G =10 at the bottom of the column, the model represents a moment frame column with the foundation stiffness recommended in AISC *Specification* Commentary Chapter C [2010 AISC *Specification* Commentary Appendix 7] for use in the sidesway-uninhibited column alignment charts.

Model C is demonstrated here with a large column (W14×455) that is part of a moment frame. The girder is a W24×62. The moment of inertia, I_g, used in the model is increased by a factor of two to represent the effect of girders on each side of the joint in the prototype by one girder framing into the joint in the analysis model. The influence of any primary and P-Δ moment from columns above and below the story is neglected. Also, no leaning column is included in the model. The story height is 9 ft and the bay width is 25 ft (L_g = 12.5 ft in the analysis model). A lateral wind load is applied at the top beam-to-column joint. The gravity and lateral load are applied simultaneously and increased

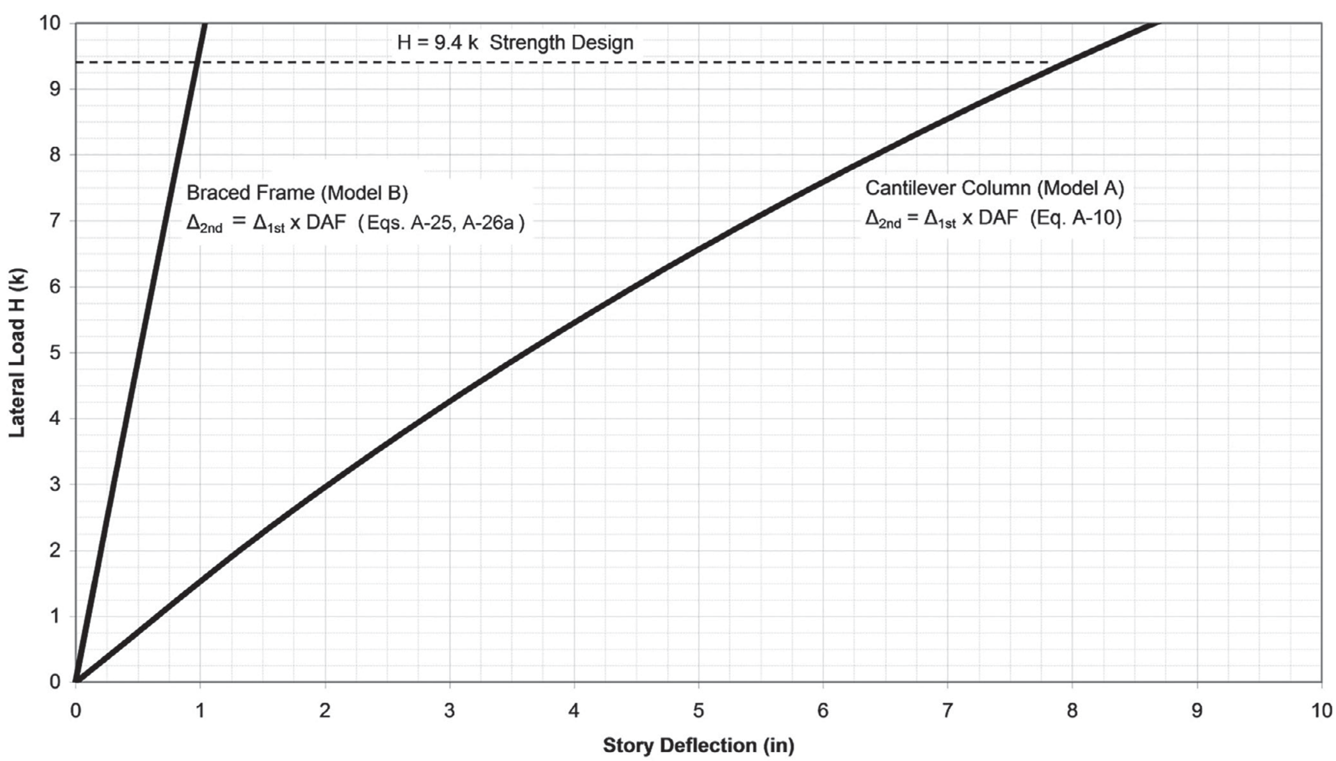


Fig. A-13. Story sidesway deflection—Model A versus Model B.

proportionally from zero up to their maximum values (P = 3,940 kips and H = 28.5 kips). The lateral load is plotted versus the first- and second-order drift in Figure A-15. In this problem, the second-order amplification is 1.37. The stability is highly dependent on the relatively small girder in proportion to the much larger column. Therefore, the problem is reanalyzed with a larger girder size (W30×99). The second-order amplification is significantly reduced along with a reduction in the story drift by more than a factor of three. Clearly, the choice of girder size in this case is very key to the story drift and the overall frame stability.

A.3.4 Summary of Design Recommendations

The following summarizes the major differences between moment frames and braced frames as discussed here:

1. In general, braced frames in low rise buildings are much stiffer than moment frames and are more economical in resisting lateral loads. This distinction between braced frame and moment frame buildings often loses significance in high rise and super-tall buildings.
2. Most practical braced frames in low rise buildings will have relatively small second-order effects and can be proportioned for strength and checked for drift. Second-order effects can be significant in braced frames with large leaning column loads, particularly when the magnitude of leaning column load is four or more times the load carried by the braced frame itself.
3. Moment frames are often controlled by drift. They should be proportioned for drift control and checked for strength.
4. Moment frame stability can be very sensitive to girder sizes when the columns are large and the girders are comparatively smaller.
5. The leaning column effect is more pronounced in moment frames and can produce relatively large second-order effects. The destabilizing effect of leaning columns must be accounted for in the design of all frames.
6. The reduction in column flexural stiffness due to large axial load (i.e., the reduction in column flexural stiffness due to the P-δ effect) can be significant and should be accounted for in the second-order sidesway analysis of moment frames where it is significant.

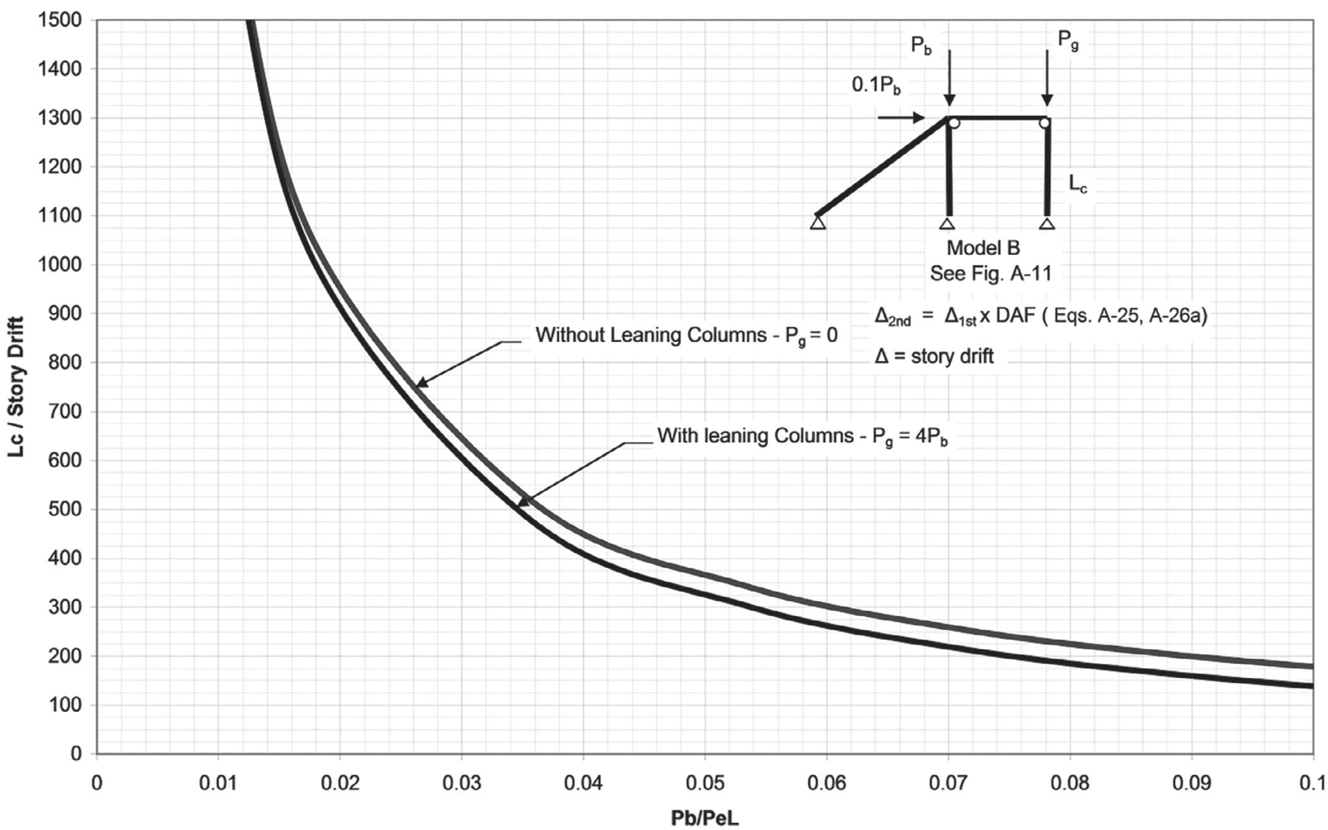


Fig. A-14. Drift ratios—braced frame Model B.

A.4 COLUMN CURVE FOR FLEXURAL BUCKLING OF MEMBERS WITHOUT SLENDER CROSS-SECTION ELEMENTS

The basic column strength equations in AISC *Specification* Section E3 are essentially the same as those used in the previous editions of the LRFD *Specification* (AISC, 1999) with the exceptions that the equations are presented directly in terms of the more traditional slenderness ratio, KL/r, or the elastic column buckling stress, F_e, and the resistance factor, ϕ_c, is changed from 0.85 to 0.90. The limit between elastic and inelastic buckling is defined to be at $KL/r = 4.71\sqrt{E/F_y}$ or $F_e = 0.44F_y$, where F_e is the elastic buckling stress. The nominal compressive strength is given in the AISC *Specification* as:

$$P_n = F_{cr} A_g \qquad (Spec.\text{ Eq. E3-1})$$

When $KL/r \le 4.71\sqrt{\dfrac{E}{F_y}}$ $\left(\text{or } F_e \ge 0.44F_y\right)$

$$F_{cr} = \left(0.658^{\frac{F_y}{F_e}}\right)F_y \qquad (Spec.\text{ Eq. E3-2})$$

When $KL/r > 4.71\sqrt{\dfrac{E}{F_y}}$ $\left(\text{or } F_e < 0.44F_y\right)$

$$F_{cr} = 0.877F_e \qquad (Spec.\text{ Eq. E3-3})$$

where

$$F_e = \frac{\pi^2 E}{\left(\dfrac{KL}{r}\right)^2} \qquad (Spec.\text{ Eq. E3-4})$$

The AISC column curve is presented in Figure A-16 by plotting F_{cr} versus KL/r for the commonly used ASTM A992 Grade 50 steel. The designer should note that the use of AISC *Specification* Equation E3-4, as well as the expression $KL/r = 4.71\sqrt{E/F_y}$ as the limit between elastic and inelastic buckling, is generally relevant only for flexural bucking limit states. The use of F_e/F_y in the preceding equations is more general and accommodates other more general column limit states. In addition, column resistances for sidesway buckling may be determined using F_e obtained using the AISC story-stiffness based equations without ever calculating a column effective length factor, K.

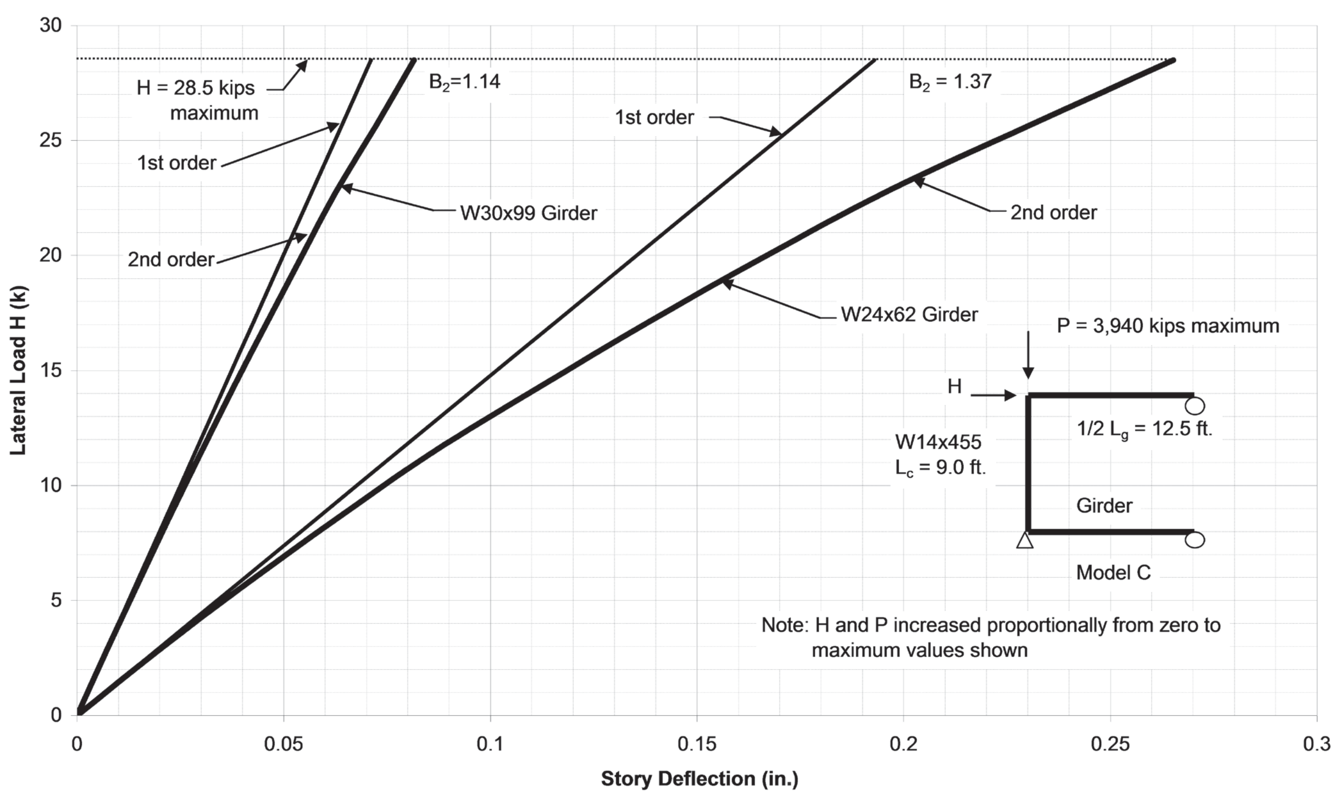


Fig. A-15. Story deflection—Model C.

The AISC column curve includes the effect of residual stresses and out-of-straightness, both of which have a major effect on column and frame stability. The AISC column curve fits most closely with SSRC Column Curve 2P, which is based on a mean out-of-straightness of 1/1,470 of the equivalent simply supported length. This curve gives an accurate to conservative estimate of the residual stress and geometric imperfection effects in all types of steel columns with $F_y \geq 50$ ksi, with the exception of minor-axis buckling of welded built-up H-shapes composed of universal mill plate (Ziemian, 2010). The main factors leading to the increase in ϕ_c from 0.85 to 0.90 in the AISC *Specification* (2005a) are that $F_y \geq 50$ ksi is now commonly used and welded built-up shapes are no longer manufactured from universal mill plate. Other factors have been considered in the development of the column strength equations as well (see the previous section of this appendix—Factors Influencing Frame Stability). Ziemian (2010) provides a detailed summary of the development of various steel column strength curves.

$K = 1.0$ for the direct analysis method (DM) and for certain conditions in the effective length method (ELM) and first-order analysis method (FOM). See Chapters 2, 3 and 4.

Update Note: The following discussion, involving the stiffness reduction factor, τ_a, is not directly applicable to equations in the 2010 AISC *Specification*, which use the single reduction factor, τ_b. A detailed discussion of τ_a and τ_b is presented in Section A.5. Also see the Update Note in Section A.6 on the effective length factor, K.

It is convenient for some design purposes to express AISC *Specification* Equations E3-2 and E3-3 as one equation (Baker, 1987; Yura, 1995; ASCE, 1997) involving the stiffness reduction factor, τ_a, (discussed in AISC *Specification* Commentary Chapter C). This may be accomplished by writing the nominal column strength generally as

$$P_n = 0.877\tau_a P_e \tag{A-27}$$

for both elastic and inelastic buckling. For the elastic buckling case, $\tau_a = 1.0$. For the inelastic buckling case, τ_a is less than 1.0 and may be determined by rewriting AISC *Specification* Equation E3-2 as:

$$P_n = \left(0.658^{P_y/P_e}\right)P_y \tag{A-28}$$

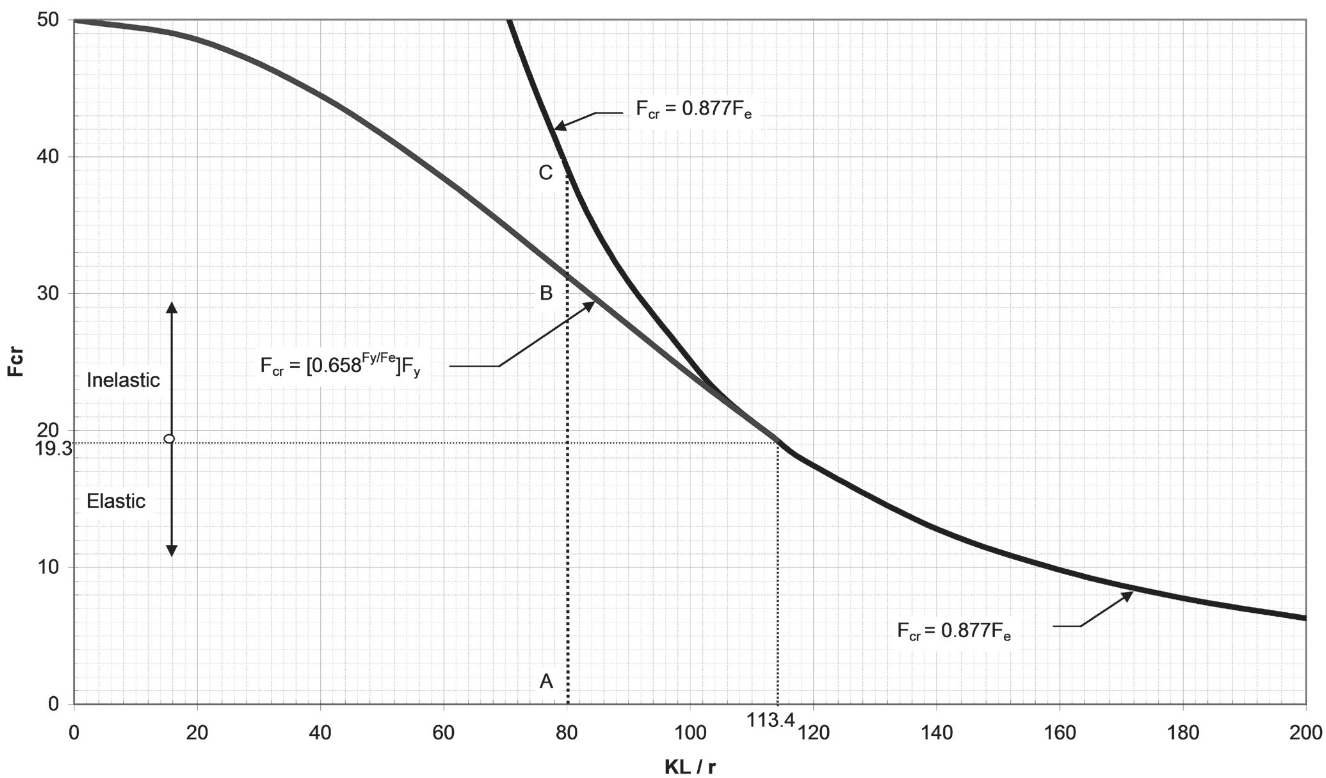


Fig. A-16. AISC column strength curve for ASTM A992 Grade 50 steel columns.

Substituting $P_e = P_n/0.877\tau_a$ from Equation A-27, and then solving for τ_a, the result is:

$$\tau_a = -2.724\left(P_n/P_y\right)\ln\left(P_n/P_y\right)$$

(*Spec.* Comm Eq. C-C2-12)

The use of Equation A-27 and AISC *Specification* Commentary Equation C-C2-12 requires iteration, because Equation C-C2-12 depends on P_n. For members subjected solely to concentric axial compression, it is acceptable to substitute the following for P_n into AISC *Specification* Commentary Equation C-C2-12 to determine τ_a noniteratively, where ϕ_c = 0.90 and $\Omega_c = 1.67$.

$$P_n = P_u/\phi_c \quad \text{for LRFD} \tag{A-29a}$$

$$P_n = P_a\Omega_c \quad \text{for ASD} \tag{A-29b}$$

The column inelastic stiffness reduction factor may be calculated in this way for two purposes:

1. Determining inelastic effective length factors, K, and
2. For members subjected solely to concentric axial compression, calculating an estimate of the available design resistance, $\phi_c P_n$ or P_n/Ω_c, for checking against the required strength, P_u or P_a.

For $P_u < \phi_c P_n$ or $P_a < P_n/\Omega_c$, this noniterative approach gives a larger value of τ_a than that obtained by iteration using Equation A-27 and AISC *Specification* Commentary Equation C-C2-12. This usually translates to a conservative estimate of the influence of column inelasticity on K (because the elastic end restraint is relatively larger for the inelastic column); hence, inelastic K factors calculated using this approach are usually larger than the corresponding values obtained from the "exact" iterative approach. However, the noniterative approach results in an overestimate of the nominal column resistance, P_n. This overestimate is not a problem for members subjected solely to concentric axial compression. For these types of members, if $P_u < \phi_c P_n$ (LRFD) or $P_a < \Omega_c P_n$ (ASD) based on the noniterative estimate, these inequalities will always be satisfied if the "exact" iterative approach is used. However, when calculating strength ratios, $P_u/\phi_c P_n$ or $P_a\Omega_c/P_n$, for use in the AISC beam-column strength interaction equations, the noniterative estimate would result in an unconservative design assessment. Therefore, the use of the noniterative approach for the calculation of $\phi_c P_n$ does not comply with the AISC *Specification* requirements and should never be employed for beam-columns.

In summary, for general design of columns and beam-columns, AISC *Specification* Commentary Equation C-C2-12 and Equation A-29a or Equation A-29b may be used to obtain a useful noniterative calculation of inelastic column effective length factors. Table 4-21 in the 2005 AISC *Manual* gives the value of τ_a from these noniterative equations for ASD and for LRFD. After one has calculated the inelastic K factor for a given column, AISC *Specification* Equations E3-1 to E3-4 are used to determine the column resistance for the corresponding flexural buckling limit state. One should never calculate τ_a from AISC *Specification* Commentary Equation C-C2-12 and Equation A-29a or Equation A29b and then substitute this value back into Equation A-27 for beam-column members. This can produce a substantial overestimate of P_n and a corresponding substantial underestimate of $P_u/\phi_c P_n$ or $P_a\Omega_c/P_n$ in the beam-column interaction equations. However, Equation A-27 and AISC *Specification* Commentary Equation C-C2-12 may be used very effectively for calculation of the "exact" compressive flexural buckling resistances, $\phi_c P_n$ or P_n/Ω_c, based on iterative nonlinear inelastic eigenvalue buckling solutions readily implemented in computer software. This application is discussed further in Appendix E, Section E.5.

A.4.1 Summary of Design Recommendations

Following is a summary of the design procedures discussed in this section:

1. When determining the effective length factor, K, with modification for column inelasticity as discussed in the AISC *Specification* Commentary Chapter C and Appendix A.6 in this Design Guide, use AISC *Specification* Commentary Equation C-C2-12 and Equation A-29a or Equation 29b, or equivalently, use Table 4-21, which provides τ_a given the applied stress, P_a/A (ASD) or P_u/A (LRFD), from the ASD or LRFD load combinations, respectively.
2. Given a calculated K factor ($K = 1$ when using the DM or the FOM), determine the available column strength using the design aids in Part 4 of the AISC *Manual* or any equivalent procedure.
3. Alternately, use Equation A-27, AISC *Specification* Commentary Equation C-C2-12, and Equation A-29a or Equation A-29b for a quick and direct solution to the AISC column strength equations for design or checking of columns subjected solely to axial load. The result from Equation C-C2-12 and Equation A-29a or Equation A-29b is provided in Table 4-21 of the AISC *Manual*. Note that this approach may not be used for beam-columns.
4. Use Table 6-1 in the AISC *Manual* for a quick and easy method to design wide-flange beam-columns.

A.5 COLUMN INELASTICITY

As discussed in the previous section, the influence of column inelasticity is addressed in part in the AISC column curve. The column curve includes the impact of residual stresses induced in the rolling process and also the out-of-straightness that exists in all steel columns permitted in the ASTM A6 Specification for rolled shapes. A typical nominal residual stress pattern often assumed in rolled column and beam-column research studies is shown in Figure A-17. The residual stresses induce early yielding at the flange tips, which in turn reduces the effective cross-section moment of inertia, I. A simple but convenient way to account for this effect is to modify the moment of inertia from I to τI where τ is a stiffness reduction factor. The reduced stiffness, τI, can be thought of as the ratio of the rigidity of the "effective elastic core" at incipient buckling to the elastic rigidity of the full cross section. It also may be visualized as a reduction in the modulus of elasticity from E to τE, where $\tau E = E_T$, the tangent modulus.

The AISC *Specification* and Commentary refer to two different values for the stiffness reduction factor, τ, used in design, τ_a and τ_b. The factor τ_a is defined in the Commentary to Chapter C as Equation C-C2-12. It is derived inherently from the AISC column strength curve as described in the previous section. Therefore, it implicitly includes the effect of residual stresses and member out-of-straightness. The equations for τ_a are summarized here for ease of reference:

For $P_n/P_y \le 0.39$ $\qquad \tau_a = 1.0$

For $P_n/P_y > 0.39$ $\qquad \tau_a = -2.724\left(P_n/P_y\right)\ln\left(P_n/P_y\right)$

(*Spec*. Comm. Eq. C-C2-12)

These equations for τ_a can be understood by reference to the AISC column curve in Figure A-16. In the inelastic buckling range of this figure, τ_a is simply the ratio AB/AC defined as:

$$\tau_a = E_T/E$$

$$\approx F_{cr}(\text{inelastic})/F_{cr}(\text{elastic})$$

$$= \left(0.658^{F_y/F_e}\right)F_y \Big/ \left(0.877F_e\right) \qquad \text{(A-30)}$$

In this form, τ_a can be thought of as a modifier applied to the elastic buckling equation, $0.877F_e$, (the upper curve in Figure A-16) to reduce the value of F_{cr} in the inelastic range down to the inelastic value (the lower curve in Figure A-16). At the point where the elastic and inelastic curves intersect ($F_{cr} = 19.3$ ksi and $KL/r = 113$ for ASTM A992 steel), $\tau_a = 1$. In addition to the use of Equation C-C2-12, τ_a can be found from Table 4-21 in the AISC *Manual* (where τ_a is plotted as a function of column stress, P_a/A or P_u/A, for ASD and LRFD respectively). Table 4-21 is based on Equation C-C2-12, with P_n replaced by $\phi_c P_n$ for LRFD and P_n/Ω_c for ASD, where $\phi_c = 0.90$ and $\Omega_c = 1.67$.

The factor τ_b is defined in Appendix 7 of the AISC *Specification* [Chapter C of the 2010 AISC Specification] as part of the direct analysis method (DM). It is used to modify the properties EI to $EI^* = \tau_b EI$. An expression for τ_b is defined in AISC *Specification* Appendix 7, Section 7.3 [2010 AISC *Specification* Section C2.3]. It is repeated here for ease of reference:

For $\alpha P_r/P_y \le 0.5$ $\qquad \tau_b = 1.0$ $\qquad$ (A-31a)

For $\alpha P_r/P_y > 0.5$ $\qquad \tau_b = 4\dfrac{\alpha P_r}{P_y}\left(1-\dfrac{\alpha P_r}{P_y}\right)$ $\qquad$ (A-31b)

where α = 1.0 (LRFD) and 1.6 (ASD). This equation is the original CRC parabolic equation for the column tangent modulus (see Ziemian (2010), Equations 3.5 to 3.7). Conceptually, it may be considered to include the effect of residual stresses but to not include the effects of column geometric imperfections, while τ_a is considered to include both of these effects. Therefore, τ_a is smaller than τ_b as can be seen in Figure A-18.

It is appropriate to use τ_a in all applications where an inelastic buckling solution is required, such as in using the Alignment Charts to calculate an inelastic K factor as discussed in the AISC *Specification* Commentary to Chapter C or in any other type of inelastic buckling analysis. Its use in these applications is consistent with the AISC column curve in Chapter E of the AISC *Specification*.

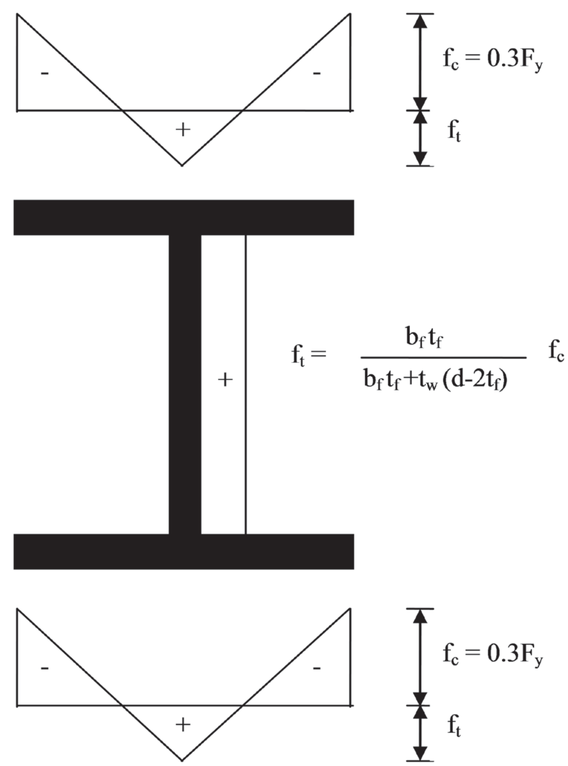


Fig. A-17. Typical assumed residual stress pattern for rolled wide-flange shapes (Galambos and Ketter, 1959).

It is appropriate to use τ_b when modifying member properties (e.g., $\tau_b EI$ for flexural members participating in the lateral load resistance of a moment frame) for use in the direct analysis of a frame at ultimate load conditions, or in any similar analysis where behavior of the frame is desired at ultimate load and frame out-of-straightness or out-of-plumbness is considered explicitly (either using notional loads or including the imperfections directly in the model geometry).

A.5.1 Summary of Design Recommendations

Following is a summary of design procedures discussed in this section:

1. Use AISC *Specification* Commentary Equation C-C2-12 for τ_a when using the AISC column strength in the form given by Equation A-27 and when determining inelastic K factors. τ_a can be conservatively taken equal to 1.0 when determining K factors.
2. Use τ_a when modifying member properties (e.g., $\tau_a EI$ for members participating in the buckling resistance of the frame) in an inelastic buckling analysis of a frame.
3. Use Equation A-31a or Equation A-31b for τ_b when the direct analysis method (DM) is employed to reduce member flexural rigidities and in other analysis situations, where member properties are modified to predict frame behavior at ultimate load (under LRFD load combinations or 1.6 times ASD load combinations) and frame geometric imperfections are accounted for explicitly in the model by application of notional loads or by directly modeling the geometric imperfections.

A.6 EFFECTIVE LENGTH FACTOR, *K*

Perhaps no other single topic within the structural engineering community has stirred more discussion, debate and confusion within the last fifty years than the effective length factor, K. Effective length factors for stability design were first introduced into the 1963 AISC *Specification* (AISC, 1963), although the concept appeared without the variable K in the 1961 AISC *Specification* (AISC, 1961). Since then, more and more extensive guidance has been provided in the commentaries to the AISC *Specifications* on proper accounting for second-order effects in frames including the amplification term on bending as well as proper determination of the effective length factor, K, within the first term of the interaction equation—this parameter being the mainstay of the method and the subject of intensive coverage in the literature. Some of the areas where extensive guidance is necessary include accounting for the effects of leaning columns and adjustments required in using tools such as the

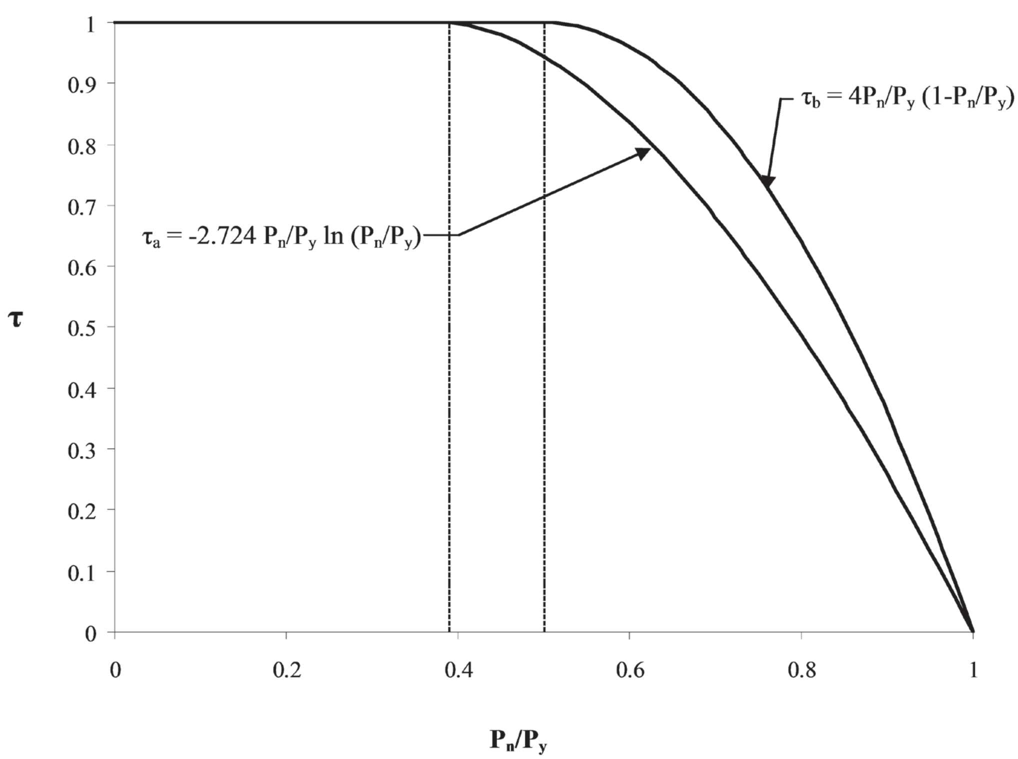


Fig. A-18. Stiffness reduction factors, τ_a and τ_b.

K-factor Alignment Charts to avoid substantial conservative and unconservative errors related to the differences between actual structures and the underlying highly simplified models on which these tools are based.

Update Note: Both the 2005 and 2010 editions of the *Specification* use an effective length factor, K, in calculating available strengths of members and a factor K_1 for determining P-δ effects (which amplify the required flexural strengths of certain members). The 2005 AISC *Specification* also uses a factor K_2 in one of the methods for calculating story buckling strength (which, in turn, is used to determine P-Δ effects through the B_2 factor in the approximate B_1-B_2 second-order analysis procedure). That method of determining story buckling strength has been eliminated in the 2010 edition, along with the K_2 factor. Much of the following discussion would not, therefore, apply to use of the 2010 AISC *Specification*. Updates for 2010 in section and equation references are not indicated in the remainder of this discussion of effective lengths.

The following discussion begins with the questions: what is the effective length factor, K; why is it important in the design of steel structures; and how is it determined and used in the AISC *Specification*? In some cases, K is noted as K_2 to emphasize the adjustments that often must be made when calculating the effective length. K_2 is illustrated in the Commentary to the AISC *Specification*.

The AISC *Specification* defines the effective length factor, K, as the ratio of the effective length—itself defined as "the length of an otherwise identical column with the same strength when analyzed with pinned end conditions"—and the actual unbraced length of the member (refer to Table C-C2.2 in the Commentary to AISC *Specification* Chapter C). The critical buckling load of a column with ideally pinned end conditions is defined by the well-known Euler buckling equation as:

$$P_e = \frac{\pi^2 EI}{L_c^{\,2}} \tag{A-32}$$

Columns in most practical steel moment frames have some measure of end restraint provided by the girders framing into the column joints along with possible destabilizing end conditions from sidesway caused by leaning columns. Thus, the Euler buckling load for such a column is modified to:

$$P_{e2} = \left(P_e\right)\left(factor\right) \tag{A-33}$$

If the "factor" in Equation A-33 is defined as $1/K_2^2$, then:

$$P_{e2} = \frac{\pi^2 EI}{(K_2 L_c)^2} \tag{A-34}$$

In this form, the effective length factor, K_2, can be seen as a modifier to the column length, L_c, reflecting the actual end restraints or end conditions on the column. The symbol K_2 is used here and in the AISC *Specification* to distinguish it from the effective length factor, K_1, determined from a sidesway-uninhibited (nonsway) buckling analysis. Thus, the effective length factor is simply seen as a mathematical adjustment to the perfect pin-ended column to define the buckling capacity of the column in the actual moment frame. All equations or charts attempting to define K_2 are nothing more than attempts to define the actual column buckling capacity as a function of the perfectly pin-ended Euler column. The critical buckling load for a column can also be expressed in terms of buckling stress by dividing the load by the cross-sectional area, A. In this form, the critical stress, F_{cr}, is represented by AISC *Specification* Equation E3-4 as follows:

$$F_{cr} = \frac{\pi^2 E}{\left(\dfrac{K_2 L_c}{r}\right)^2} \tag{A-35}$$

The term r is the radius of gyration of the column about the axis under consideration ($r^2 = I/A$). The most common method for determining K (K_1 or K_2) is with the use of the Alignment Charts (also commonly referred to as the nomographs) shown in AISC *Specification* Commentary Chapter C as Figure C-C2.3—Sidesway Inhibited and Figure C-C2.4—Sidesway Uninhibited, respectively. The appropriate subassemblies upon which the charts are based, the equations associated with the two cases, and the assumptions used in the derivation of each are given in the AISC *Specification* Commentary C2.2b. Theoretical K values obtained from the Alignment Charts for various extreme end conditions (rotation fixed or free and translation fixed or free) are shown in AISC *Specification* Commentary Table C-C2.2 along with practical recommended design values for K for use in actual design. The Alignment Charts are based on certain assumptions of idealized conditions that seldom exist in real structures. Therefore, adjustments are sometimes required when these assumptions are violated. These adjustments are provided in AISC *Specification* Commentary Section C2.2b.

K_2, from the sidesway uninhibited Alignment Chart, can be determined directly with the following approximate equation (Dumonteil, 1992), accurate within 2%:

$$K_2 = \sqrt{\frac{1.6 G_A G_B + 4.0\left(G_A + G_B\right) + 7.5}{G_A + G_B + 7.5}} \tag{A-36}$$

where

$$G = \frac{\Sigma\left(EI_c/L_c\right)}{\Sigma\left(EI_g/L_g\right)} \tag{A-37}$$

G is calculated for end A and B of the column.

Other analytical equations for K_2 for sidesway uninhibited (unbraced) frames are given in the Commentary and discussed in the following. The Alignment Chart for K_2 is nothing more than a "sidesway buckling analysis" solution as defined in the AISC *Specification* Commentary Section C2 in the definition of K_2. Note that the Commentary provides recommendations for various modifications to column boundary conditions from those assumed in the Alignment Chart derivation.

Some modern computer programs have the ability to determine the elastic critical buckling load for a steel frame directly. (Note: It is important to verify that such programs accurately account for the P-Δ and P-δ effects prior to using them as discussed here.) Also, for frames designed using moment-resisting base details, the flexibility at the base detail typically should be considered rather than assuming that the base condition offers rigid rotational restraint. This approach provides a general purpose buckling solution and is a practical way (albeit limited as discussed in the following) to determine K_2 factors for all the columns in the frame since it incorporates all the correct boundary conditions—something that the Alignment Chart often fails to do accurately. The computer buckling analysis can also incorporate the effect of leaning columns—one area where the sidesway uninhibited Alignment Chart requires adjustments. If a designer uses a computer program to determine the critical buckling load for a moment frame, he/she can define the effective length factors for each participating column, i, in the frame by taking the critical buckling load for each column, $P_{e2\text{-}i}$, and solving for $K_{2\text{-}i}$ using Equation A-33 as follows:

$$K_{2\text{-}i} = \sqrt{\frac{\pi^2 EI}{P_{e2\text{-}i} L_{c\text{-}i}^2}} \tag{A-38}$$

While practical and easy to apply when the software is available, this approach also has its challenges and limitations. In some cases, K_2 values can sometimes be very large, indicating that the column may not be participating in the buckling resistance of the frame significantly or, may be serving more as a restraining member for buckling of other columns. The member may be undergoing rigid-body motion in the governing buckling mode, or it may have a light axial load and is predominantly serving to restrain the buckling of other members. Some of the cases requiring the greatest exercise of judgment to avoid excessively large K_2 factors include (1) columns in the upper part of tall buildings, (2) columns with flexible or weak connections, and (3) beams or rafters in portal frames, which may have significant axial compression due to the horizontal thrust from the base of the frame (White et al., 2006). In such cases, engineering judgment is required in determining a proper K_2 for use in the design. Some guidance can be found in this decision by re-running the buckling analysis assuming pinned-end members for these questionable columns and noting if the buckling load for the frame is impacted. If it is not, then it may be appropriate to set K_2 =1 for these columns in the design. Of course, the member force and its destabilizing effect must still be accounted for in the stability analysis model.

If the buckling analysis is conducted using the modified stiffness properties, $\tau_a I$, for all members participating in the moment resisting frame, then a reasonable estimate of the inelastic buckling load of the moment frame can also be determined (Yura, 1995).

Software programs that accurately assess second-order effects will also estimate the sidesway buckling load for the steel frame in a lateral load analysis if the loads on the frame (H and P) are increased proportionally until the sidesway becomes extremely large. This response is shown in Figure A-19 for a typical moment frame where both the elastic and inelastic (plastic hinge) response is shown, as well as the overall frame buckling load determined from an eigenvalue buckling analysis. An elastic buckling analysis solution would be based on the nominal member properties (I, A) while the inelastic buckling analysis solution would be approximated by using $\tau_a I$ and A.

Why is the critical buckling load for the frame and its corresponding columns important? The answer lies in the fact that this critical load, P_{e2}, expressed in terms of stress, F_e, is an inherent part of the column capacity equations as used in the AISC *Specification* in Chapter E, e.g., Equations E3-2 and E3-3.

The calibration of the ELM in the AISC *Specification* has been done in a way that relies heavily on checking story stability in the P_r/P_c term in the beam-column interaction Equations H1-1a and H1-1b. The method requires an appropriate calculation of the effective column buckling length, KL, to determine P_c. The appropriate calculation can be difficult to determine for complex frames. The Commentary to Chapter C contains extended discussion about the effective length factor, K, and the designer is encouraged to review it carefully when using this method. A brief overview of K and how it is used in the AISC *Specification* is presented here.

There are two uses for the effective length factor, K, within the AISC *Specification*:

1. *Amplified first-order analysis.* K is used in the calculation of the elastic buckling load, P_{e1}, for a member. Also, it may be used, but is not required, for calculation of ΣP_{e2} for a building story. The values P_{e1} and ΣP_{e2} are

then used for determining the amplification factors, B_1 and B_2, in the amplified first-order elastic analysis procedure of AISC *Specification* Section C2.1b to account for second-order effects. Appendix B in this Design Guide discusses the B_1-B_2 amplification factor procedure and other methods in some detail.

2. *Column flexural buckling strength, P_n.* K may be used in the determination of the column flexural buckling strength, P_n, from Chapter E. This K calculation may be based either on an elastic or inelastic buckling analysis. This usage is the focus in this section.

Sidesway instability of a moment frame is a story phenomenon involving the sum of the sway resistances of each column in the story and the sum of the gravity loads in the columns in that story. No individual column in a story can buckle in a sidesway mode without all the columns in that story also buckling. Many common framing systems are used that distribute the story P-Δ effects to the columns in that story in proportion to their individual stiffnesses. This distribution is accomplished using floor diaphragms or horizontal trusses. In a moment frame containing columns that contribute little or nothing to the sway stiffness of the story, such columns are referred to as leaning columns and can be designed using K =1.0. The other columns in the story must be designed to support the destabilizing P-Δ moments developed from the axial loads on these leaning columns. Similarly, the more highly loaded columns in a story will distribute some of their P-Δ moments to the more lightly loaded columns. This phenomenon must be considered in the determination of K for all the columns in the story for the design of moment frames. The proper K-factor for calculation of P_n in moment frames, accounting for these effects, is denoted in the AISC *Specification* Commentary Section C2.2b by the symbol K_2. Note that for both of the applications for the effective length factor discussed here, the symbol K_2 is used. The designer is cautioned about the proper application for both uses (refer to AISC *Specification* Commentary Section C2.2b).

Two common methods for evaluating story frame stability, as measured by ΣP_{e2} for a story, are recognized: the story stiffness method (LeMessurier, 1976; LeMessurier, 1977) and the story buckling method (Yura, 1971). These are reflected in AISC *Specification* Chapter C with

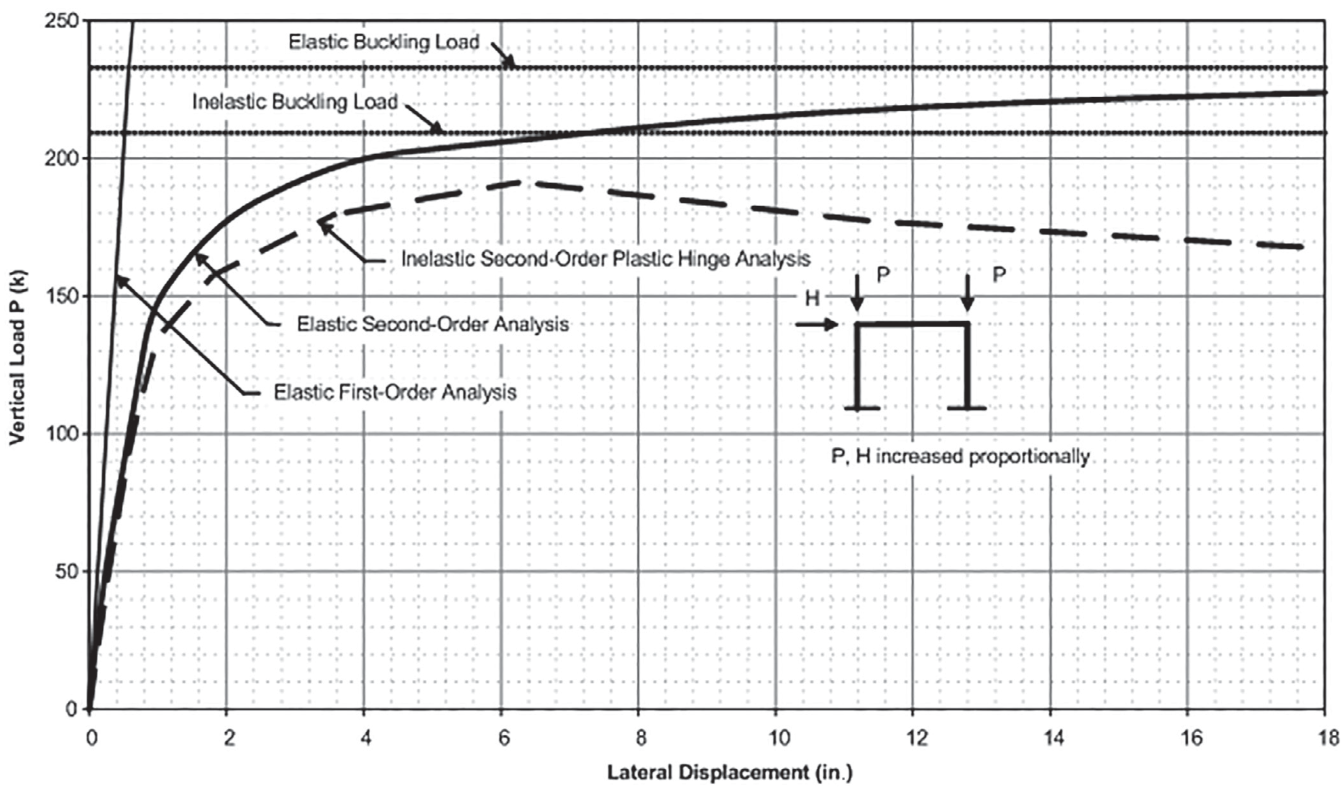


Fig. A-19. Determination of sidesway buckling load by buckling or load-deflection analysis.

Equations C2-6b and C2-6a, respectively. For the story stiffness method, K_2 is defined by:

$$K_2 = \sqrt{\frac{\Sigma P_r}{(0.85+0.15R_L)P_r}\left(\frac{\pi^2 EI}{L^2}\right)\left(\frac{\Delta_H}{\Sigma HL}\right)} \geq \sqrt{\frac{\pi^2 EI}{L^2}\left(\frac{\Delta_H}{1.7HL}\right)}$$

(*Spec*. Comm. Eq. C-C2-5)

This value of K_2 may be used in AISC *Specification* Equation C-C2-3 or directly in the equations of Chapter E. It is possible that certain columns, having only a small contribution to the lateral load resistance in the overall frame, will have a K_2 value less than 1.0 based on the term on the left of the inequality. The limit on the right-hand side is a minimum value for K_2 that accounts for the interaction between sidesway and nonsidesway buckling (ASCE, 1997; White and Hajjar, 1997). The term H is the shear in the column under consideration, produced by the lateral forces used to compute Δ_H.

$$R_L = \frac{\Sigma P_r \textit{ leaning columns}}{\Sigma P_r \textit{ all columns}} \qquad \text{(\textit{Spec}. Comm. Eq. C-C2-7)}$$

ΣP_r in AISC *Specification* Commentary Equations C-C2-5 and C-C2-6 includes all columns (and other components supporting gravity loads) in the story, including any leaning columns, tilt-up walls, etc. The term P_r is the required second-order axial strength for the column under consideration. R_L is the ratio of the vertical column load for all leaning columns in the story to the total vertical load supported by the story. This factor approaches 1.0 for systems with a large percentage of leaning columns. The purpose of R_L is to account for the debilitating influence of the P-δ effect on the sidesway stiffness of the columns of a story. The term $(0.85 + 0.15R_L)$ is a refined calculation of the R_M term in Equation C2-6b of the AISC *Specification* as discussed previously in this appendix (see Equation A-13).

AISC *Specification* Commentary Equation C-C2-5 for K_2 can be reformulated to obtain the column buckling load, P_{e2}, (AISC *Specification* Commentary Equation C-C2-6) and elastic column buckling stress, F_e, (AISC *Specification* Commentary Equation C-C2-4) for direct incorporation into the AISC column equations (AISC *Specification* Equations E3-2 and E3-3). Refer to the AISC *Specification* Commentary for further discussion.

In the story stiffness method, K_2 is expressed in terms of a building's story drift ratio, Δ_H/L, from a first-order lateral load analysis at a given applied lateral load level. In preliminary design, this may be taken in terms of a target maximum value for this drift ratio. The designer's focus is on the most fundamental stability requirement in building frames, providing adequate overall story stiffness in relation to the total vertical load, $\alpha\Sigma P_r$, supported by the story. The elastic story stiffness, $\Sigma H/\Delta_H$, is expressed in terms of the drift ratio, Δ_H/L, and the total horizontal load acting on the story, ΣH (see $(\Delta_H/\Sigma HL)$ in the equation for K_2).

For the story buckling method, K_2 is defined by:

$$K_2 = \sqrt{\frac{\frac{\pi^2 EI}{L^2}}{P_r}\left(\frac{\Sigma P_r}{\Sigma \frac{\pi^2 EI}{(K_{n2}L)^2}}\right)} \geq \sqrt{\frac{5}{8}}K_{n2}$$

(*Spec*. Comm. Eq. C-C2-8)

where K_{n2} is defined as the K value determined directly from the Alignment Chart in Figure C-C2.4 of the AISC *Specification* Commentary. Again, the value for K_2 calculated from AISC *Specification* Commentary Equation C-C2-8 may be less than 1.0. The limit on the right-hand side of this equation is a minimum value for K_2 that accounts for the interaction between sidesway and sidesway inhibited buckling (ASCE, 1997). Alignment Chart solutions for the effective length factor are based on frames whose story drifts are primarily due to flexural deformations in the moment frame. Errors in story drift can result if shear deformations in beams and columns (as may be significant and dominant for high-rise "tube" type buildings with closely spaced perimeter columns and deep spandrel beams) or other deformations (like panel zone deformation) are not considered where they are a significant portion of the story drift. In such cases, the story stiffness based equation should be used where the story drift results from all components that contribute to story deflection.

ΣP_r in AISC *Specification* Commentary Equation C-C2-8 includes all components resisting vertical load in the story, including any leaning columns, diagonal bracing transferring vertical loads, tilt-up walls, etc., and P_r is for the column under consideration. K_{n2} in Equation C-C2-8 is determined from the Alignment Chart in Figure C-C2.4 of the AISC *Specification* Commentary Chapter C. Note also that the value of P_n, calculated using K_2 by either method, cannot be taken greater than P_n based on sidesway inhibited buckling.

Equation C-C2-8 for K_2 can be reformulated to obtain the column buckling load, P_{e2} (AISC *Specification* Commentary Equation C-C2-9), and elastic column buckling stress, F_e, (AISC *Specification* Commentary Equation C-C2-4) for direct incorporation into the column equations (AISC *Specification* Equations E3-2 and E3-3). Refer to the AISC Commentary for further discussion.

Another simple formula for K_2 (LeMessurier, 1995), based only on the column end moments, is:

$$K_2 = \left[1+\left(1-M_1/M_2\right)^4\right]\sqrt{1+\frac{5}{6}\frac{\Sigma P_r \textit{ leaning columns}}{\Sigma P_r \textit{ nonleaning columns}}}$$

(*Spec*. Comm. Eq. C-C2-10)

M_1 is the smaller and M_2 the larger end moment in the column. These moments are determined from a first-order analysis of the frame under lateral load. Column inelasticity is considered in the derivation of this equation. The unconservative error in P_n using AISC *Specification* Commentary Equation C-C2-10 is less than 3%, as long as the following inequality is satisfied:

$$\left(\frac{\Sigma P_y \textit{ nonleaning columns}}{\Sigma HL/\Delta_H}\right)\left(\frac{\Sigma P_r \textit{ all columns}}{\Sigma P_r \textit{ nonleaning columns}}\right) \le 0.45$$

(*Spec.* Comm. Eq. C-C2-11)

Similar to Equation C-C2-8, Equation C-C2-11 does not account for the influence of shear deformations on the building sidesway stiffness and buckling resistance. Equation C-C2-5, however, is capable of incorporating the influence of shear deformations on the sidesway stability, via the inclusion of shear deformations in calculating Δ_H, the first-order interstory drift due to lateral forces.

If the designer elects to use other K equations or methods, it is necessary to recognize what assumptions are made in the development. For instance, the nomograph K is based on a set of assumptions and column boundary conditions often violated in real frames. Therefore, various adjustments are commonly required. Refer to the AISC *Specification* Commentary to Chapter C for a complete discussion of these assumptions and the necessary adjustments. Careful attention needs to be given to the leaning column effect in determining K for use in the interaction P_c term (via the use of AISC *Specification* Commentary Equation C-C2-8).

Adjustments in the effective length factor, K_2, for use in the calculation of the column strengths, P_c, can be made based on an inelastic buckling analysis of the frame and the inelasticity inherent in the column under the governing load combination (Yura, 1971; ASCE, 1997). Columns loaded into the inelastic range of behavior can be viewed as having a tangent modulus, E_T, that is smaller than E. For such columns, $E_c = E_T$ in the equation for G, which usually gives smaller G values, and therefore, smaller K-factors than those based on elastic behavior. Note that it is usually conservative to base the calculation of P_n on elastic K-factors. For more accurate solutions, inelastic K-factors can be determined from the alignment chart method by using τ_a times E_c for E_c in the equation for G, where $\tau_a = E_T/E$ is the column inelastic stiffness reduction factor.

Column inelasticity also can be considered in determining K_2 (AISC *Specification* Commentary Equations C-C2-5 and C-C2-8) for the story stiffness method and the story buckling method. In the story stiffness method, $\tau_a I_c$ is substituted for I_c for all columns in the frame analysis used to determine Δ_H. In addition, $\tau_a I_c$ is used in place of I in Equation C-C2-5. In the story buckling method, τ_a is used in the determination of K_{n2} from the Alignment Chart in Equations C-C2-8 and C-C2-9 and also in those same equations by replacing I_c with $\tau_a I_c$. The previous section provided additional discussion of column inelasticity and gave equations for τ_a.

Where determining K_2 values for a column and story by any of the methods discussed here, it is not so much the K_2 values that are important. Rather, the important consideration is that sufficient story lateral stiffness is maintained to support the imposed vertical load on the story. K_2 in this context is simply a means used to measure this fundamental requirement. In the story stiffness approach, the column load at sidesway buckling (P_{e2}) may be determined directly without the need to calculate K_2. This is given in the Commentary to AISC *Specification* Chapter C as Equation C-C2-6. This load may be divided by the column gross area to obtain $F_e = P_{e2}/A_g$ for use in the column strength equations (i.e., see AISC *Specification* Equations E3-3 and E3-4 in Chapter E).

Additional discussion about methods to determine the effective length factor, K, can be found in ASCE (1997).

A.6.1 Summary of Design Recommendations

Following is a summary of design procedures discussed in this section:

1. The effective length factor, K, has two distinct but different uses in steel building design:
   (a) K can be used in amplified first-order analysis to calculate P_{e1} and ΣP_{e2} to determine amplification factors, B_1 and B_2. Refer to Appendix B for details on the B_1-B_2 method.
   (b) K is used in calculating the column flexural buckling strength, P_n, in Chapter E of the 2005 AISC *Specification*. K can be calculated using either an elastic buckling analysis or an inelastic buckling analysis. It is conservative to base K on an elastic buckling analysis (i.e., $\tau_a = 1.0$).

2. K_2 (for sidesway uninhibited frames) can be calculated by any of the following methods:
   (a) Use the sidesway uninhibited Alignment Chart (AISC *Specification* Commentary Chapter C) and conform to all boundary conditions and other requirements as stated in the footnotes.
   (b) Use the closed-form solution of the sidesway-uninhibited Alignment Chart from Equation A-36, accurate to within 2% when compared to a similar Alignment Chart solution.
   (c) Use AISC *Specification* Commentary Equation C-C2-5, which is based on the story stiffness approach.
   (d) Use AISC *Specification* Commentary Equation C-C2-8, which is based on the story buckling approach.

(e) Use AISC *Specification* Commentary Equation C-C2-10 (LeMessurier, 1995), where the inequality of AISC *Specification* Commentary Equation C-C2-11 is satisfied.

(f) Use a second-order elastic analysis program or one set up to perform a buckling analysis.

(g) Use any other method that gives the correct solution as discussed in ASCE (1997).

3. Consider using column inelasticity in any of the methods in Item 2 by substituting $\tau_a I_c$ for I_c in all columns in the story.
4. Be sure to include the destabilizing effect of all leaning columns in the ΣP_r term of all equations.

A.7 BEAM-COLUMN INTERACTION EQUATIONS

In the AISC *Specification*, beam-columns are proportioned according to interaction equations defined in Chapter H. The interaction of flexure and compression in doubly symmetric members, such as wide-flange columns bent about both the strong and weak axis are designed according to Equations H1-1a and H1-1b as follows:

For $\dfrac{P_r}{P_c} \geq 0.2$

$$\frac{P_r}{P_c} + \frac{8}{9}\left(\frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}}\right) \leq 1.0 \quad (\textit{Spec}.\ \text{Eq. H1-1a})$$

For $\dfrac{P_r}{P_c} < 0.2$

$$\frac{P_r}{2P_c} + \left(\frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}}\right) \leq 1.0 \quad (\textit{Spec}.\ \text{Eq. H1-1b})$$

where

P_r = required axial compressive strength, kips
P_c = available axial compressive strength, kips
M_r = required flexural strength, kip-in.
M_c = available flexural strength, kip-in.
x = subscript relating to strong-axis bending
y = subscript relating to weak-axis bending

For design by LRFD:

P_r = required axial compressive strength using LRFD load combinations, kips
P_c = $\phi_c P_n$
= design axial compressive strength determined in accordance with Chapter E, kips
M_r = required flexural strength using LRFD load combinations, kip-in.
M_c = $\phi_b M_n$
= design flexural strength determined in accordance with Chapter F, kip-in.
ϕ_c = resistance factor for compression = 0.90
ϕ_b = resistance factor for flexure = 0.90

For design by ASD:

P_r = required axial compressive strength using ASD load combinations, kips
P_c = P_n/ϕ_c
= design axial compressive strength determined in accordance with Chapter E, kips
M_r = required flexural strength using ASD load combinations, kip-in.
M_c = M_n/Ω_b
= design flexural strength determined in accordance with Chapter F, kip-in.
Ω_c = safety factor for compression = 1.67
Ω_b = safety factor for flexure = 1.67

The terms in the denominator establish the end points of the interaction curve shown in Figure A-20 for single axis bending, where the AISC *Specification* interaction Equations H1-1a and H1-1b are plotted for reference. The nominal flexural strength, M_n, encompasses the limit states of yielding, lateral-torsional buckling, flange local buckling, and web local buckling. The term P_n can accommodate compact or slender columns and encompasses the limit states of major and minor axis buckling, torsional, and flexural-torsional buckling. Furthermore, these equations are calibrated to account for stability effects in frames. Thus, the interaction equations are very versatile and include the effect of multiple limit states in the column including biaxial bending.

The AISC *Specification* includes a third interaction equation, Equation H1-2, applicable to doubly symmetric members subjected to uniaxial flexure and compression with moments primarily in one plane of bending (the general application to all types of doubly-symmetric members implied by the *Specification* is not intended). Equation H1-2 permits the consideration of two independent limit states for compact doubly symmetric members—in-plane instability and out-of-plane buckling, or flexural torsional buckling separately, in lieu of the combined approach in Equations H1-1a and H1-1b. Equation H1-2 is also plotted in Figure A-20.

Update Note: The 2010 AISC *Specification* limits the application of Equation H1-2 to rolled compact sections. The equation has been modified to clarify its application and to improve its accuracy, as indicated in the next Update Note.

Equation H1-2 can be very useful for checking the out-of-plane resistance of typical column-type wide-flange section members (where $d/b_f \cong 1$). The true out-of-plane resistance for these types of members is typically relatively large. Therefore, when these types of members are subjected predominantly to in-plane bending and axial compression, they can often be designed based on their controlling in-plane resistance. Equation H1-2 then provides a check of the out-of-plane resistance. Because of its quadratic term, this equation is not as convenient for the selection of an appropriate member section, but it can be used easily for checking the out-of-plane resistance.

For the design of columns subjected to compression and flexure primarily about one axis:

1. Use AISC *Specification* Equations H1-1a and H1-1b to check the limit state of in-plane instability, with P_c, M_r and M_c determined in the plane of bending.
2. Use the following equation to check the limit state of out-of-plane buckling for compact doubly symmetric members:

$$\frac{P_r}{P_{co}} + \left(\frac{M_r}{M_{cx}}\right)^2 \leq 1.0 \qquad (\textit{Spec}.\ \text{Eq. H1-2})$$

where

P_{co} = available compressive strength out of the plane of bending, kips

M_{cx} = available lateral-torsional buckling strength for strong axis flexure determined from Chapter F, kip-in.

If bending occurs only about the weak axis, the out-of-plane design check of Equation H1-2 reverts to a simple check of out-of-plane flexural buckling using only the axial load term. For members with significant biaxial bending ($M_r/M_c \geq 0.05$ in both directions), Equations H1-1a and H1-1b must be used. The AISC *Specification* allows the engineer to neglect the out-of-plane bending moment when $M_{ry}/M_{cy} < 0.05$. Equations H1-1a and H1-1b may be used as a conservative check of the out-of-plane buckling limit state for compact doubly symmetric members. They are required to check the out-of-plane resistance for doubly symmetric sections with noncompact or slender cross section elements or for singly symmetric sections.

The term M_{cx} in Equation H1-2 is interpreted conservatively in Figures C-H1.4 through C-H1.6 of the AISC *Specification* Commentary. In these figures, M_{cx} is shown as the final calculated LRFD resistance, $\phi_b M_n$, from Chapter F of the AISC *Specification*. This interpretation is implied also by

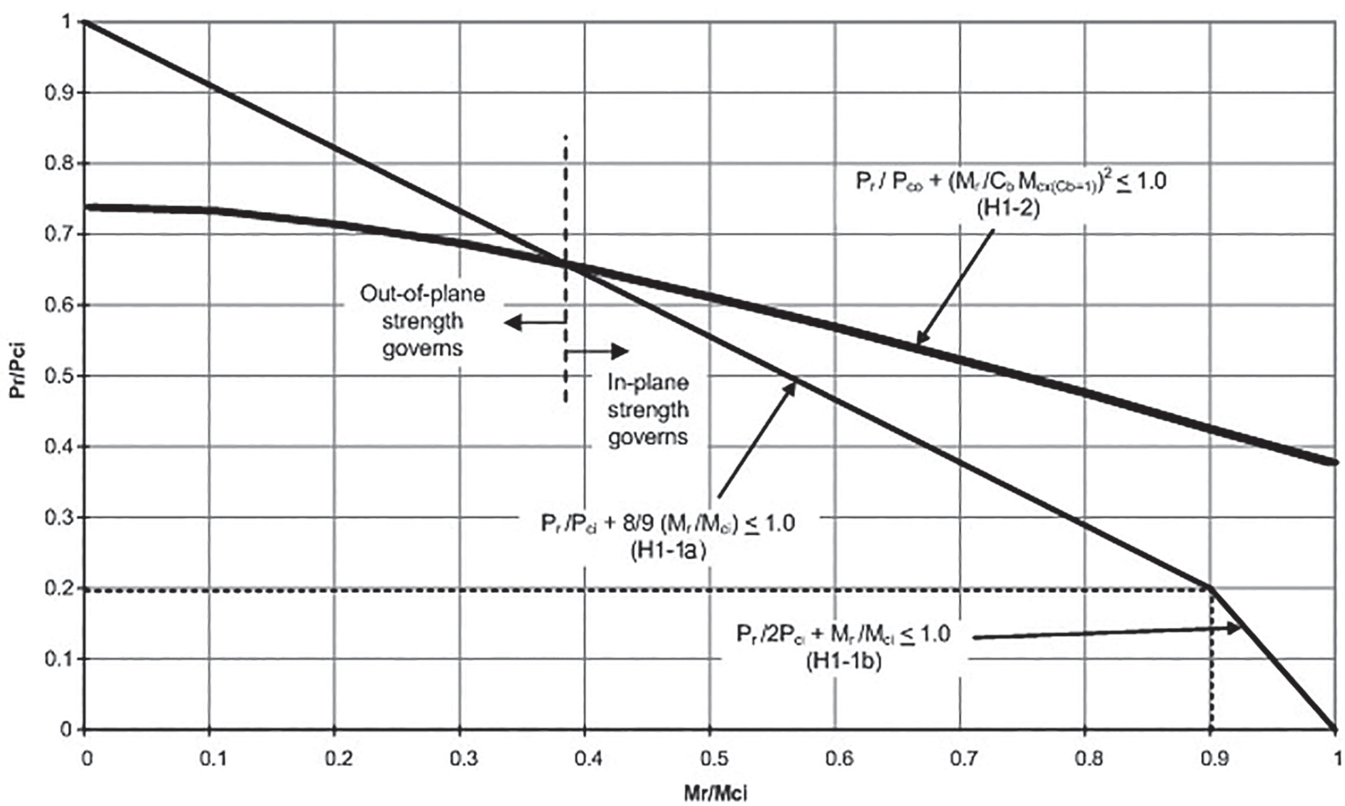


Fig. A-20. AISC beam-column strength interaction Equations H1-1a and H1-1b.

the definition of M_{cx} from the AISC *Specification* and stated herein. However, the intended calculation of M_{cx} in Equation H1-2 is:

$$M_{cx} = C_b M_{cx(C_b=1)}$$

where C_b is the moment gradient modifier for lateral-torsional buckling, and $M_{cx(C_b=1)}$ is the lateral-torsional buckling resistance for the uniform bending case (C_b = 1). M_{cx} calculated in this way is permitted to be greater than ϕM_{px} in LRFD or M_{px}/Ω in ASD. The out-of-plane resistance from Equation H1-2, calculated in this way, is "capped" by the in-plane resistance from Equations H1-1, similar to the way that $\phi_b M_{px}$ or M_{px}/Ω_b caps the lateral-torsional buckling resistance for the case of flexure alone (zero axial force). The strength is governed by the smaller of the in-plane and out-of-plane resistances, with the in-plane resistance calculated using AISC *Specification* Equation H1-1a or H1-1b, and the out-of-plane resistance calculated using Equation H1-2.

Figure A-20 shows a correct interpretation of Equation H1-2 for a typical case where the column out-of-plane axial resistance is smaller than the corresponding in-plane resistance ($P_{co} < P_{ci}$), and where, due to moment gradient, the lateral-torsional buckling strength, M_{cx}, is larger than the in-plane flexural resistance, M_{ci}. For typical cases like this one, Equation H1-2 does not govern unless the applied axial load is nearly equal to the out-of-plane resistance, P_{co}.

Update Note: Equation H1-2 in the 2005 AISC *Specification* is based on the simplifying assumption that the column elastic torsional buckling resistance, P_{ez}, is much larger than P_{co}. This is not always the case. The 2010 AISC *Specification* provides the following updated form of Equation H1-2, aimed at making this equation more general and also at clarifying the definition of M_{cx}:

$$\frac{P_r}{P_{cy}}\left(1.5-0.5\frac{P_r}{P_{cy}}\right)+\left(\frac{M_{rx}}{C_b M_{cx(C_b=1)}}\right)^2 \le 1.0$$

(2010 *Spec*. Eq. H1-2)

For the design of singly symmetric I-section members, the axial stress effects, P_r/A, can counteract the bending stress effects, M_r/S_x, at a critical location. The reader is referred to Kaehler et al. (2010) for the handling of these cases, including the design of web-tapered I-section members.

Preliminary and final strength design of beam-columns can be handled using the tabular procedure developed by Aminmansour (2000) and contained in AISC *Manual* Part 6. In this section, coefficients are tabulated for all wide-flange shapes with various column lengths (KL with respect to r_y) and unbraced lengths for bending (L_b for strong axis bending).

An even simpler preliminary design procedure for beam-columns is described in the following. Here, depending on whether axial load or flexure is dominant, either the moment (in the case of axial load dominating) is converted into an "equivalent" axial load or the axial load (in the case of flexure dominating) is converted into an "equivalent" moment. The equations (Salmon and Johnson, 1996) are as follows.

If axial load predominates, $P_r/P_c \ge 0.2$

$$P_{EQV} = P_r + 2M_{rx}/d + 7.5M_{ry}/b_f \qquad \text{(A-39)}$$

where
- d = nominal depth of column, in.
- b_f = nominal flange width of column, in.
- M_{rx} = required second-order moment about the x-axis, kip-in.
- M_{ry} = required second-order moment about the y-axis, kip-in.

If moment predominates and $P_r/P_c \ge 0.2$

$$M_{eqv} = (8/9)\left(M_{rx} + 4dM_{ry}/b_f\right) + 0.7P_r d \qquad \text{(A-40)}$$

If moment predominates and $P_r/P_c < 0.2$

$$M_{eqv} = M_{rx} + 4dM_{ry}/b_f + 0.35P_r d \qquad \text{(A-41)}$$

After conversion to an equivalent axial load or moment, the member can be designed as either an axially loaded column or a beam using the AISC *Manual* tables or the appropriate strength equations.

Neither of the above preliminary design approaches include the strength benefits associated with the use of AISC *Specification* Equation H1-2.

A.7.1 Summary of Design Recommendations

Following is a summary of the design recommendations provided in this section:

1. Use AISC *Specification* interaction Equations H1-1a and H1-1b for the design of wide-flange beam-columns subjected to biaxial effects.
2. Consider using interaction Equation H1-2 to check the out-of-plane strength for cases where compact-element wide-flange members are subjected to uniaxial bending and axial compression. Use Equations H1-1a and H1-1b for the in-plane strength checks. This results in additional design economy. It is conservative to use Equations H1-1a and H1-1b for both checks.
3. For preliminary beam-column design, use the coefficient tables in AISC *Manual* Part 6 or use Equations A-39, A-40 and A-41 as appropriate.

A.8 OUT-OF-PLUMBNESS

As discussed earlier under Section A.2, frame out-of-plumbness and column out-of-straightness can have a significant effect on the behavior of steel frames. It has long been recognized that these effects need to be included in the design process. In prior AISC Specifications, these effects were not explicit in the ELM via the column strength determined from the equations in Chapter E. In the context of the ELM, the AISC *Specification* column strength equations, which are the same as prior LRFD Specification (AISC, 1999) column formulas, imply a mean out-of-straightness of the equivalent simply supported column of $L/1{,}470$ (Ziemian, 2010). Specifically, this may be compared to the column out-of-straightness tolerance of $L/1{,}000$ and the maximum out-of-plumbness tolerance of $H/500$ specified in the AISC *Code of Standard Practice* (AISC, 2005d) for columns and frames, where L is the column length between brace or framing points and H is the story height with some absolute limits based on building heights. Without the inclusion of a minimum notional lateral load, the ELM does not consider the influence of geometric imperfections on the forces and moments in the beams and beam-to-column connections of the lateral load resisting system. The primary reason for limiting the application of the ELM to a ratio of second-order to first-order story displacements of $\Delta_{2nd}/\Delta_{1st} < 1.5$ is the error incurred from the manner in which this method handles the combined effects of out-of-plumbness and reduction in stiffness as the ultimate strength condition is approached. Also, as the second-order effects become large, the solution is sensitive to the accuracy of the critical load, P_c, itself (i.e., the denominator of the first term of the interaction equation).

In the DM, a nominal out-of-plumbness is considered explicitly in the design procedure, where a minimum or additive lateral load (depending on the degree of second-order effects—see Chapter 3) is included to account for the destabilizing effects of this tolerance. The required lateral load is based on an out-of-plumbness of $L/500$ or $0.002L$ with an option given to the designer to assume a different value if justified by the design. If preferred, the specific out-of-plumbness on which the above notional load is based can be modeled directly into the geometry of the frame. Depending on the height of the building and its geometry, different geometries may need to be considered to capture the different out-of-plumbness effects that can occur. This is because the AISC *Code of Standard Practice* has envelope limits on the out-of-plumbness as a function of the height of the building. For instance, for exterior columns a $1/500$ out-of-plumbness away from the building line is permitted up to a maximum of two inches in the first twenty stories and an additional $^1\!/_{16}$ inch per story up to a maximum of three inches. Engineering judgment is required in the determination of the critical case(s) to consider in the analysis, based on the actual building geometry.

The requirement for plumbness ($1/500 = 0.002$) in steel frames can be traced as far back as the first edition of the AISC *Code of Standard Practice* (AISC, 1924). The plumbness discussed here is the final plumbness of the steel frame after the erector has made any adjustments required to initial erection caused by anchor rod placement problems (a common cause of plumbness problems) and member misfits. In response to a demand by the design community for more specific and expanded tolerances for steel buildings, the present tolerances for steel construction were initiated in the 1959 edition of the *Code of Standard Practice* (AISC, 1959). Such tolerances are common in specifications around the world but there have been relatively few studies done to justify their values. Experience with surveys undertaken as part of project specifications for high-rise construction in the U.S. since the 1960s indicates that the present tolerances for out-of-plumbness can be easily and economically achieved. Indeed, it is fairly well known in tall building circles that the actual plumbness limits attained are sometimes considerably smaller that what is permitted by the code.

MacGregor (1979) summarized several studies done around the world on measurements of cast-in-place and precast concrete buildings as well as steel buildings. For steel buildings, MacGregor used the work of Beaulieu et al. (1976) and Beaulieu and Adams (1977a and b) in Canada where out-of-plumbness measurements were taken on a total of 3,200 steel columns in three multistory steel buildings. This work consisted of statistical and structural analyses of the out-of-plumb measurements and resulted in the following suggested design equation for out-of-plumbness:

$$\frac{\Delta_d}{L} = \left(\frac{\Delta}{L}\right)_m + \frac{\beta\sigma}{N^{1/2.2}} \tag{A-42}$$

where

$\left(\frac{\Delta}{L}\right)_m$ = mean column lean

N = total number of columns on all floors constructed in the building

σ = standard deviation of the out-of-plumb angles

β = "safety index" related to the probability of exceedence of the design value Δ_d/L

A value of $\beta = 3.5$ was recommended and corresponds to a probability of exceedence of $2\text{x}10^{-4}$.

Beaulieu and Adams checked the validity of this equation using 50 Monte Carlo simulations of the out-of-plumbness forces in three frames, five to ten stories in height with randomly selected column leans. In each case the sway displacements based on the above equation resulted in a reasonable upper bound to the deflections caused by the P-Δ forces from the random column leans. Using the mean and standard deviation values from the steel column study, the following equation was recommended:

$$\frac{\Delta}{L} = 0.000026 + \frac{0.00588}{N^{1/2.2}} \quad \text{(A-43)}$$

This equation gives a relatively small out-of-plumbness for buildings containing a large number of columns. For instance, for a two-story building with 24 columns in plan, $N = 48$ and $\Delta_d/L = 0.00104$. For a 20-story building with 24 columns in plan, $\Delta_d/L = 0.000381$, only about 19% of the AISC 0.002 value. This equation may capture the overall out-of-plumbness for the entire building height, but it may completely miss the local out-of-plumbness in a critical story. This could be a key attribute associated with the strength of shear frame-type structures. Therefore, its use is not recommended.

Chen and Tong (1994) recommended an average out-of-plumbness as follows:

$$\frac{\Delta}{L} = \frac{0.002}{\sqrt{N}} \quad \text{(A-44)}$$

where N is the total number of columns, each with a random Δ, that are stabilized by a bracing system. In this equation, N appears to include only the number of columns in plan without the effect of multiple levels (N=24). Assuming this to be the case, the out-of-plumbness values for the two-story and 20-story case mentioned previously results in Δ_d/L values of 0.000408 in both cases, again smaller than the AISC number of 0.002.

Bridge (1998) reported on out-of-plumb measurements made on a 47-story high-rise steel frame building rectangular in plan with dimensions 150 ft by 98 ft and floor heights of 13.4 ft. The data were provided in the form of global deviations (e_x and e_y) of the top and bottom of each column from the column centerline as defined by the intersection of the reference grid lines. The survey concentrated mainly on the central core of the structure consisting of 15 internal columns (18 columns at the lower levels, less on the upper levels). The majority of the data was presented for each floor from levels one to 22 inclusive, with complete data for levels one to six. For levels 22 to 42, measurements were made at every second floor for selected columns since the columns were fabricated in two-story lifts. The out-of-plumbness of a column was considered as the angle, $\psi = \Delta_o/L$, where Δ_o is the relative displacement of the top relative to the bottom of the column with length L. Based on a total of 368 column measurements, for all the columns in the x- and y-direction, the variation of the real out-of-plumbness (with the directional sign included) was normally distributed about the mean. The mean value was close to zero indicating that out-of-plumbness in one direction was compensated by out-of-plumbness in the opposite direction. For the x- and y-directions combined, 41% of the columns were outside the AISC tolerance of 0.002. The maximum out-of-plumbness for a column was $\psi = +0.00957$. Bridge considered the out-of-plumbness for any given story as the mean value for all the columns measured in the story. Using the measured data, the mean translational story out-of-plumbness was determined in the x- and y-directions. The data showed that the mean out-of-plumbness can vary considerably, alternating between positive and negative such that the mean for all levels was close to zero. This indicated that the construction procedure was compensating for the out-of-plumbness at any given story. Considering the absolute values for all floors, it was found that the value with a probability of 5% exceedence was 0.00220 or about 10% greater than the AISC limit. Thus, the AISC limit appears to be reasonable based on data measured for this building. The data collected also allowed a rotational out-of-plumbness to be calculated. Generally, the rotational values were similar in magnitude to the translational values.

Today's modern fabrication techniques are based on computerized numerical control (CNC) drilling lines. The impact of these fabrication techniques and current erection methods on the out-of-plumbness in steel building frames fabricated and erected in the U.S. has not been well quantified. It is recommended that designers follow the current AISC plumbness tolerance limits specified in the *Code of Standard Practice*, unless it can be shown that smaller tolerances can be attained in the construction.

A.8.1 Summary of Design Recommendations:

Following is a summary of design recommendations discussed in this section:

1. When designing for out-of-plumbness effects, a Δ/L of 0.002 should be applied in the minimum lateral load requirements of the ELM and the DM defined in the AISC *Specification*, unless lower values can be justified in the design and construction quality control of the building. In the latter case, only the DM (as permitted in AISC *Specification* Appendix 7, Section 7.3) [2010 AISC *Specification* Section C2.2] can be used to account for the smaller tolerances. For multi-story buildings, calculate the notional loads based on the maximum tolerance envelope limits specified in the AISC *Code of Standard Practice*.

2. The DM may also be used by modeling the out-of-plumb frame geometry directly, in lieu of using notional loads. This approach not only eliminates the work of calculating the notional loads but is advantageous for determining the appropriate values for notional loads explicitly in cases involving complex loadings and/or geometries. As discussed in Chapter 3, this approach can actually be more straightforward in cases where the nominal geometry can easily be modified in the software to define the out-of-plumb geometry.

APPENDIX B
Development of the First-Order Analysis Method (FOM)

The AISC *Specification* Section C2 requires that a second-order analysis must be performed when either the effective length method (ELM) or the direct analysis method (DM) is used for the design. Alternatively, where the first-order analysis method (FOM) is used, first-order analysis may be employed with only a few checks on the magnitude of the second-order effects. The designer is given considerable latitude in the selection of a second-order analysis approach as long as both P-Δ and P-δ effects are considered (see AISC *Specification* Section C2.1a [2010 AISC *Specification* Section C1]). The AISC *Specification* provides one method of second-order analysis. This method is referred to as the amplified first-order elastic analysis method in Section C2.1b [2010 AISC *Specification* Appendix 8] and has been traditionally called the "B_1-B_2 method." As the names imply, B_1 and B_2 amplifiers are applied to the first-order elastic analysis results to determine the second-order forces and displacements in this method. The B_1-B_2 method is discussed first, followed by an overview of an alternative application of the B_1 and B_2 amplification factors. The latter amplifier-based second-order analysis approach, when combined with the DM for the design calculations, and after introducing a number of conservative simplifying approximations, leads to the FOM of Chapter 4. The corresponding steps in the development of the FOM are explained in this Appendix. One can discern a continuum of various more general (and more accurate) forms of the underlying second-order analysis employed in the FOM based on these developments.

Update Note: The 2010 AISC *Specification* identifies the first-order analysis method by name (it is not so identified in the 2005 edition) and presents its limitations and details in Appendix 7, Alternative Methods of Design for Stability. The B_1-B_2 procedure for second-order analysis is presented in Appendix 8, Approximate Second-Order Analysis.

In an important simplification of analysis requirements, the need to consider the influence of P-δ effects on the overall response of the structure has been relaxed in the 2010 AISC *Specification*. This is explained further in an Update Note in Appendix D.

B.1 AMPLIFIED FIRST-ORDER ELASTIC ANALYSIS METHOD (B_1-B_2 METHOD)

The amplified first-order elastic analysis method has its roots in the 1963 AISC *Specification* (AISC, 1963). One of the beam-column interaction equations, then expressed in terms of stresses, contained the amplification factor, $C_m/(1-f_a/F_e')$, on the bending stress term. This amplifier was thus effectively applied to the total moment on the member, including sway and nonsway moments. The B_1-B_2 method first appeared in the first LRFD *Specification* in 1986 (AISC, 1986). In this method, the first-order analysis is divided into two parts—a no-sway or no-translation (nt) analysis and a sidesway or lateral-translation (lt) analysis. Most (but not all) frames produce little or no sidesway under gravity loads so the method is normally broken down into a gravity load no-sway analysis and a sidesway lateral load analysis. Moments from the two analyses (M_{nt} for no-translation and M_{lt} for lateral translation) are multiplied by amplification factors (B_1 for the no-translation case and B_2 for the lateral-translation case). They are then combined to obtain the member maximum second-order elastic moment to be used in the beam-column interaction equations.

The AISC *Specification* interaction equations (Equations H1-1a, H1-1b and H1-2) are set up to accept the maximum member second-order elastic moment determined by this or any other method of second-order elastic analysis. The specific equations provided in the AISC *Specification* for the approximate calculation of the maximum second-order elastic internal member moment as well as the second-order internal axial force are:

$$M_r = B_1 M_{nt} + B_2 M_{lt} \qquad \text{(}\textit{Spec.}\text{ Eq. C2-1a)}$$

$$P_r = P_{nt} + B_2 P_{lt} \qquad \text{(}\textit{Spec.}\text{ Eq. C2-1b)}$$

where

$$B_1 = \frac{C_m}{1-\alpha P_r/P_{e1}} \geq 1.0 \qquad \text{(}\textit{Spec.}\text{ Eq. C2-2)}$$

$$B_2 = \frac{1}{1-\dfrac{\alpha \Sigma P_{nt}}{\Sigma P_{e2}}} \geq 1.0 \qquad \text{(}\textit{Spec.}\text{ Eq. C2-3)}$$

$$P_{e1} = \frac{\pi^2 EI}{\left(K_1 L\right)^2} \qquad \text{(}\textit{Spec.}\text{ Eq. C2-5)}$$

$$\Sigma P_{e2} = \Sigma \frac{\pi^2 EI}{\left(K_2 L\right)^2}$$ for moment frames, where sidesway buckling factors, K_2, are determined (*Spec.* Eq. C2-6a)

$$\Sigma P_{e2} = R_M \frac{\Sigma HL}{\Delta_H} \quad \text{for all types of lateral load resisting systems} \qquad (\textit{Spec}.\ \text{Eq. C2-6b})$$

Update Note: The equations for M_r, P_r, B_1 and B_2 have been moved to Appendix 8 in the 2010 AISC *Specification*. ΣP_{nt} and ΣP_{e2} have been renamed P_{story} and $P_{e\ story}$, respectively; the equation for ΣP_{e2} or $P_{e\ story}$ based on column stiffnesses (Equation C2-6a in the 2005 AISC *Specification*) has been eliminated.

The term C_m is an equivalent uniform moment factor whose value is:

1. For beam-columns with no transverse loading between supports in the plane of bending:

$$C_m = 0.6 - 0.4\left(M_1/M_2\right) \qquad (\textit{Spec}.\ \text{Eq. C2-4})$$

where M_1 is the smaller and M_2 the larger moment determined from a first-order analysis, at the member ends unbraced in the plane of bending under consideration.

2. For beam-columns with transverse loading, C_m may be taken as 1.0 or may be determined by analysis.

The reader is referred to the developments elsewhere in this guide as well as to the AISC *Specification* Section C2.1b [2010 AISC *Specification* Appendix 8] for other definitions.

Note that in AISC *Specification* Equation C2-6a, which is applicable only for moment frames, the ΣP_{e2} term represents the elastic story sidesway buckling capacity including all columns in the story that contribute to the lateral resistance. Leaning columns are not included in the lateral resistance but their gravity load must be included in the B_2 amplifier via the ΣP_{nt} term. K_2 may be determined from equations in the AISC *Specification* Commentary to Chapter C [2010 AISC *Specification* Commentary to Appendix 7] or from a sidesway buckling analysis of the frame. Alternatively, ΣP_{e2} may be determined directly without the need for calculation of member K_2 values.

Note that B_1 accounts for the P-δ effects in the absence of any sidesway and B_2 accounts for the P-Δ effects associated with the story sidesway displacements. Furthermore, the 2005 AISC B_2 amplifier includes the P-δ effects on the sidesway, Δ, either implicitly through K_2 in AISC *Specification* Equation C2-6a or through R_M in AISC *Specification* Equation C2-6b (see Appendix A). In most frames where the axial loads in members are relatively small ($B_1 \leq 1.05$), it is convenient and conservative to amplify the total moment by B_2 (i.e., $M_r = B_2\left(M_{nt} + M_{lt}\right)$). This may be very conservative in cases where the moments are dominated by the no-translation (nt) loading. Designers should note that the B_2 amplifier can be set in advance by assuming a maximum drift limit in AISC *Specification* Equations C2-3 and C2-6b.

In most moment frames with horizontal floor beams, the effect of any axial load in reducing the flexural stiffness, *EI,* of the beams participating in the moment frame is small and can be neglected. In cases where axial loads are large, such as in heavily loaded sloping girders of single story frames with transverse loads on the girders, the reduction in *EI* of the girder can increase the story drift and second-order effects and thus impact the overall frame design. The designer is cautioned to be alert to these cases and modify the girder stiffness, *EI*, in the analysis by the factor $\left(1 - Q/Q_{cr}\right)$, where Q is the axial load in the girder when the system buckles and Q_{cr} is the in-plane buckling load in the girder using $K = 1$. This approach requires iteration in the analysis procedure and is only important in cases where $Q/Q_{cr} > 0.05$.

In addition, the designer is cautioned about the proper use of the B_1-B_2 method for frames with highly unsymmetric framing and/or loading. In these cases, the designer must assess the sidesway effect under gravity loads and apply the resulting "no-translation" sidesway reaction forces as part of the "lateral-translation" analysis (as strictly required by this method). Many designers ignore the potential sidesway under gravity loads. This can lead to significant errors in the second-order moments. The precise analysis for moments caused by the sidesway under gravity loads is very cumbersome (these moments theoretically should be multiplied by B_2 along with the lateral load moments) and is rarely done in practice. Past experience has shown that the resulting calculations can be either significantly conservative or unconservative depending on the structural characteristics and the specifics of the simplified calculations.

An alternative amplified first-order analysis approach that includes second-order effects and avoids some of the problems inherent with the B_1-B_2 method is presented in the next section. The FOM of Chapter 4 is obtained by combining this second-order analysis approach with the DM for the design and then invoking a number of simplifying assumptions.

B.2 ALTERNATIVE APPLICATION OF THE B_1 AND B_2 AMPLIFICATION FACTORS

White et al. (2007a and b) show that the total second-order story drifts, $\overline{\Delta}_{2tot}$, may be determined in a general fashion as:

$$\overline{\Delta}_{2tot} = \overline{B}_2\left(\Delta_o + \overline{\Delta}_1\right) = \overline{B}_2\overline{\Delta}_{1tot} \qquad \text{(B-1)}$$

where an over-bar is used to highlight the terms that are influenced by the nominal stiffness reduction (e.g., $EA^* = 0.8EA$ and $EI^* = 0.8\tau_b EI$) employed in the DM. These terms are calculated using the nominal elastic properties when the analysis is based on the ELM. The individual terms in Equation B-1 are defined as follows:

$\overline{B}_2$ = 2005 AISC *Specification* sidesway amplification factor given by Equation C2-3 with ΣP_{e2} determined from Equation C2-6b

Δ_o = nominal initial out-of-plumbness, in.
$\bar{\Delta}_1$ = first-order interstory sidesway displacement due to the applied loads, in.

Once the second-order drifts, $\bar{\Delta}_{2tot}$, are known, the overall P-Δ effects within any rectangular frame structure may be included simply by applying equal and opposite P-Δ shear forces at the top and bottom of each story in a separate first-order analysis, where the shear forces are determined as follows:

$$\bar{H}_{P\Delta} = \frac{\alpha \Sigma P_{nt} \bar{\Delta}_{2tot}}{L} \tag{B-2}$$

where

α = 1.0 (LRFD); 1.6 (ASD)
ΣP_{nt} = total vertical load supported at a given story, obtained from the LRFD or the ASD load combinations, kips
L = story height, in.

The results of the first-order analysis are then added to the internal forces determined by other first-order analyses.

As noted previously, the influence of P-δ effects on the sidesway displacements is addressed via the term R_M in AISC *Specification* Equation C2-6b. The influence of these effects at the individual member level may be approximated by applying $\bar{B}_1$, given by Equations C2-2 and C2-5, to the total member moments obtained from the procedure discussed here.

The steps of the alternative amplified first-order elastic analysis may be summarized as follows:

1. Perform a first-order analysis of the geometrically perfect structure for each of the nominal loadings (dead, live, snow, wind, etc.). Combine these analysis results with the appropriate load factors for each ASCE/SEI 7 load combination under consideration.
2. Determine the total first-order story lateral displacements, $\bar{\Delta}_1$, for each load combination.
3. Calculate the story sidesway amplification factors, $\bar{B}_2$, for each load combination.
4. For each load combination, calculate and apply the P-Δ shear forces, $\bar{H}_{P\Delta}$, in each story in a separate first-order elastic analysis of the structure.
5. Add the second-order internal forces obtained from Step 4 to the first-order forces determined in Step 1.
6. Apply the "no-translation" amplifier, $\bar{B}_1$, to the total member moments obtained in steps 1 through 5.

B.3 DEVELOPMENT OF THE FIRST-ORDER ANALYSIS METHOD

The background and derivation of the first-order analysis method is explained in this section. In understanding the derivation, the focus is on deriving a simple and conservative method of stability analysis and design based on the following conditions:

1. Use of a first-order analysis only
2. Use of the nominal geometry (frame perfectly straight and plumb)
3. Use of the nominal properties of the members (e.g., gross *EA, EI*, both unreduced)

It should be recognized that the development of the FOM is based on a combination of the second-order analysis procedure outlined in the previous section with the DM of AISC *Specification* Appendix 7 [2010 AISC *Specification* Chapter C], modified to meet the conditions specified in the above goal. Since the FOM is based on the DM, it is helpful to focus on the question: What notional load must be applied to the structure using a first-order analysis with nominal geometry and nominal member properties to allow a simple yet reasonably conservative determination of the internal forces obtained by a second-order analysis for design by the DM?

The following development is conducted at the ultimate load level and begins by determining the relationship between $\bar{B}_2$ (the sidesway amplification factor including the effects of the reduced stiffnesses, e.g., $EA^* = 0.8EA$ and $EI^* = 0.8\tau_b\, EI$) and B_2 (the sidesway amplification factor determined using the nominal stiffness properties, e.g., *EA* and *EI*). If αP_r is less than $0.5P_y$ in all of the frame members, such that $\tau_b = 1.0$, this relationship may be obtained by recognizing that $\bar{B}_2$ and B_2 may be written as:

$$B_2 = \frac{1}{1-Q} \tag{B-3}$$

and

$$\bar{B}_2 = \frac{1}{1-Q/0.8} \tag{B-4}$$

where

$$Q = \frac{\alpha \Sigma P_{nt}}{R_M \Sigma HL/\Delta_H} \tag{B-5}$$

The term Q with $R_M = 1.0$ is used often in the literature. It is referenced in the ACI 318 Code and is frequently labeled the "stability index." It is identified as θ in the ASCE/SEI 7 expression for P-Δ (see Equation 5-1 in Chapter 5 of this Design Guide). AISC *Specification* Equation C2-1a [2010

AISC *Specification* Equation A-8-1] may be solved for Q to obtain:

$$Q = \frac{B_2 - 1}{B_2} \tag{B-6}$$

If this expression is then substituted back into Equation B-4:

$$\bar{B}_2 = \frac{0.8B_2}{1 - 0.2B_2} \tag{B-7}$$

Equation B-7 relates the amplifier for the reduced stiffness model to the amplifier for the nominal stiffness model. The next step of the development requires an estimate of the relationship between the total first-order interstory drift from each of these models. Using the nominal stiffness model, the total first-order interstory displacement may be expressed as

$$\Delta_{1tot} = \Delta_o + \Delta_1 \tag{B-8}$$

where

Δ_o = story out-of-plumbness, in.

Δ_1 = first-order story drift due to the applied loads, in.

Similarly, assuming $\tau_b = 1.0$, the reduced stiffness model gives a total first-order interstory displacement of:

$$\bar{\Delta}_{1tot} = \Delta_o + \bar{\Delta}_1 = \Delta_o + \Delta_1/0.8 \tag{B-9a}$$

A conservative assumption with respect to the handling of the initial out-of-plumbness effect is to also divide it by 0.8. Thus, Equation B-9a is simplified by showing both Δ_1 and Δ_o divided by 0.8 and then recognizing that $\bar{\Delta}_{1tot}$ becomes Δ_{1tot} divided by the factor 0.8, i.e.:

$$\bar{\Delta}_{1tot} = \left(\Delta_o + \Delta_1\right)/0.8 = \Delta_{1tot}/0.8 \tag{B-9b}$$

This is a conservative simplification with respect to the handling of the initial out-of-plumbness effect, because Δ_o does not need to be divided by 0.8 in a strict interpretation of the DM. Substituting Equations B-7 and B-9b into the equation for P-Δ shears (White et al., 2007a and b) results in the following shear forces:

$$\bar{H}_{P\Delta} = \frac{\alpha \Sigma P_{nt} \dfrac{\bar{\Delta}_{2tot}}{0.8}}{L} \tag{B-10a}$$

$$\begin{aligned}\bar{H}_{P\Delta} &= \frac{\alpha \Sigma P_{nt} \left(\dfrac{0.8B_2}{1-0.2B_2}\right)\dfrac{\Delta_{1tot}}{0.8}}{L} \\ &= \alpha \Sigma P_{nt} \left(\frac{\Delta_{1tot}}{L}\right)\left(\frac{B_2}{1-0.2B_2}\right)\end{aligned} \tag{B-10b}$$

This equation gives an estimate of the P-Δ shears applied at each story level in the reduced stiffness analysis model for the DM, but written entirely in terms of the results from the first-order analysis model using the nominal geometry and nominal stiffnesses. Finally, in a multi-story structure:

- if Δ_{1tot} in Equation B-10b is replaced with

  Δ_{1max} = maximum Δ_{1tot} from all of the stories

  where Δ_{1tot} is calculated at the ultimate load level (α = 1.00 in LRFD and α = 1.60 in ASD), and
- if it is recognized that the notional load that incorporates the second-order P-Δ effects at a given level of the structure, N_i, is the difference between the $\bar{H}_{P\Delta}$ forces in the stories below and above this level, defined at the ultimate load level as Y_i, where Y_i is the gravity load from the LRFD load combination or 1.60 times the ASD load combination applied at each level of the structure, and
- if the overall result is divided by α, such that all of the force quantities can be written at the allowable stress level in the use of these equations for ASD, then the notional load is defined as follows:

$$\begin{aligned}N_i &= \frac{\dfrac{Y_i}{\alpha}\left(\dfrac{0.8B_2}{1-0.2B_2}\right)\dfrac{\Delta_{1max}}{0.8}}{L} \\ &= \left(\frac{B_2}{1-0.2B_2}\right)\frac{\Delta_{1max}}{L}\frac{Y_i}{\alpha}\end{aligned} \tag{B-11}$$

Equation B-11 is a general form of AISC *Specification* Equation C2-8 discussed in Chapter 4. The only restriction required for its use is that αP_r must be less than $0.5P_y$ for all the frame members whose bending rigidity, EI, contributes to the structure's sidesway stiffness (so that the assumption of τ_b = 1.0 is valid). In order to obtain a valid conservative estimate of the internal forces for design by the DM:

1. Conduct a first-order elastic analysis in which the loads, N_i, are applied at each level of the structure.
2. Add the forces resulting from step 1 to the first-order internal forces obtained from the other first-order analyses of the structure under the various applied loads.
3. Apply the no-translation amplifier, B_1, to the total member forces obtained from steps 1 and 2.

It should be noted that this procedure is just a conservative application of the general amplified first-order analysis approach outlined in the previous section, with the exception that the nominal stiffness reduction is not applied in the calculation of the no-translation amplifier, B_1. This exception is considered as a minor effect in the AISC *Specification* because its influence on the differences between the results from the DM and the ELM is generally neglected. The use of $\overline{B}_1$ gives a more accurate calibration with the results from refined benchmark solutions.

As discussed in Chapter 3, when $\Delta_{2nd}/\Delta_{1st} < 1.5$ based on the nominal (unreduced) structure stiffness (or put more simply, when $B_2 < 1.5$), the DM specifies:

1. The out-of-plumbness effect may be neglected in lateral-load combinations, and
2. The out-of-plumbness effect may be applied simply as a minimum lateral load in the gravity-only load combinations.

Therefore, Equation B-11 may be simplified to AISC *Specification* Equation C2-8 by assuming $B_2 \le 1.5$, such that the out-of-plumbness effect may be handled in this way. Also, $B_2 = 1.5$ may be substituted conservatively to further simplify Equation B-11 based on the assumption, $B_2 \le 1.5$. If these two adjustments are applied to Equation B-11, along with the base value of $\Delta_o/L = 0.002$, the following equation is obtained:

$$
\begin{aligned}
N_i &= \left[\frac{1.5}{1-0.2(1.5)}\right]\frac{\Delta_o}{L}\frac{Y_i}{\alpha} \\
&= 2.1(0.002)\frac{Y_i}{\alpha} \\
&= 0.0042\frac{Y_i}{\alpha} \qquad \text{(B-12a)}
\end{aligned}
$$

when the out-of-plumbness effect is handled as a minimum lateral load, and

$$
\begin{aligned}
N_i &= \left[\frac{1.5}{1-0.2(1.5)}\right]\frac{\Delta}{L}Y_i \\
&= 2.1(\Delta/L)Y_i \qquad \text{(B-12b)}
\end{aligned}
$$

when the out-of-plumbness (i.e., Δ_o) effect is neglected and Δ is taken as the drift under the LRFD or ASD loads so that $\Delta = \Delta_{1max}/\alpha$. These two equations are combined to obtain AISC *Specification* Equation C2-8. Obviously, their application must be restricted to frames with $B_2 \le 1.5$.

Similar to the discussions in Chapter 4, when used in the context of ASD, the deflection Δ given by Equation B-1 may be determined at the ASD load level (i.e., using α = 1.00), and only the term Y_i need be calculated at the ultimate strength load level (i.e., using α = 1.60). Since all of the analyses are linear elastic, superposition is valid. Therefore, there is no need to multiply all the ASD loads by α = 1.60, conduct the analyses at the ultimate load level, and then divide the results by 1.60. The multiplication and division by 1.60 is handled implicitly in Equation B-11 simply by using the Δ determined at the ASD load level (i.e., using α = 1.00). However, the term $0.0042Y_i$ in Equation B-12a must be divided by α because Y_i is defined at the ultimate strength load level.

The astute reader will notice that all of the calculations in this appendix are simply an approximate second-order analysis conducted for design by the DM, effectively with 0.8 as the reduction for all the elastic stiffness contributions. However, the final equations hide the handling of the out-of-plumbness effect and the stiffness reductions within the calculation of an appropriate conservative notional load, N_i. It is wise for the designer using the FOM to understand the background of its development to avoid potential misapplication of the FOM. Designers should always be cautious when applying methods such as the FOM where a large number of simplifying assumptions are embedded.

APPENDIX C
Modeling Out-of-Plumbness in the Direct Analysis Method or the Effective Length Method for Taller Building Structures

Appendix 7 of the AISC *Specification* [Chapter C of the 2010 AISC *Specification*] contains the requirements for the direct analysis method (DM) and the stability design of steel structures. In lieu of calculating notional loads to account for the effects of geometric out-of-plumbness, the AISC *Specification* Appendix 7, Section 7.3 states that "For all cases, it is permissible to use the assumed out-of-plumbness geometry in the analysis of the structure in lieu of applying a notional load or a minimum lateral load as defined above."

Update Note: The 2010 AISC *Specification* presents direct modeling in Section C2.2a and use of notional loads in Section C2.2b as alternative techniques for including the effects of initial imperfections in the analysis. The limitations and details of both approaches are presented more clearly and transparently than in the 2005 edition. The following discussion is equally applicable to both *Specification* editions. The sections of the AISC *Code of Standard Practice* referred to are essentially unchanged between 2005 and 2010.

It is reasonable to assume that modeling an assumed out-of-plumbness directly in the analysis model may be easier than calculating and tracking the applied notional loads and load cases, particularly in complex frames and frames that do not contain modular stories. It is believed that software developers will eventually provide capabilities to generate the corresponding out-of-plumb geometry, making it very convenient for designers to apply this approach. This appendix is intended to provide guidance for modeling the out-of-plumb geometry directly in a DM model in taller building structures.

The AISC *Code of Standard Practice* (AISC, 2005d) contains erection tolerances for steel buildings that are often referenced in project specifications. Out-of-plumbness limits for steel columns are specified in Section 7.13.1.1, where the angular variation of the working line from the plumb line should be equal to or less than $1/500$ of the distance between working points within absolute limits specified in subparagraph (b) for the case of exterior columns. This case provides the most liberal limits, where it is specified that for an exterior column shipping piece, displacement of the working points in the first 20 stories cannot exceed 2 in. away from the building line. Above 20 stories, the limit is increased by $\frac{1}{16}$ in. per story up to a maximum value of 3 in. away from the building line. Note that a 2 in. limit with a $1/500$ out-of-plumbness requires a building segment height of 83 ft, i.e., $(2\text{ in.})/(1/500)(12\text{ in./ft}) = 83$ ft. Limits for columns adjacent to elevator shafts are specified in Section 7.13.1.1(a) of the *Code of Standard Practice* and are more stringent.

For analysis, the out-of-plumbness is considered on a story basis taken as the mean of all the columns in the story. Recent measurements of a 42-story high-rise building (Bridge, 1998) indicate the AISC out-of-plumbness limit ($1/500$) is a reasonable tolerance for multistory buildings and it is suggested that the limits specified in AISC be used in design (see Appendix A, Section A.8, Out-of-Plumbness).

Figure C-1 shows the recommended procedure for a 24-story steel building. Five cases are required to be considered in order to be sure the maximum affect is included for each story of the building. Case 1 out-of-plumbness is calculated as follows: 2 in. (maximum permitted for 20 stories) + $\frac{1}{16}$ in. × 4 stories above 20 stories = 2.25 in. total out-of-plumbness at the roof level. Note that the columns are spliced in two-story increments and the splice locations are considered in the assignment of the vertical segments. Out-of-plumbness leans (plus and minus) should be considered separately in each orthogonal building direction.

For buildings that may be sensitive to torsional effects, rotational leans should also be considered. Given the code requirements for wind and seismic torsion and the normal translational leans applied as specified above, it is believed that a rotational lean is not necessary for most average buildings.

Summary of Design Recommendations:

The following steps are recommended for explicitly modeling out-of-plumbness geometry in the DM (all load cases where notional loads are otherwise specified):

1. Determine the ratio of second-order to first-order story drift ($\Delta_{2nd}/\Delta_{1st}$) for each level of the building. Note the floors that have large ratios in relation to other stories. Particular attention should be paid to these stories to be sure that the maximum out-of-plumbness of $1/500$ (or more at the judgment of the designer) is applied to these stories.

2. The first case of out-of-plumbness to be considered should be the maximum overall limit as specified by the AISC *Code of Standard Practice* for the given building height considered as a continuous lean (see Figure C-1, Case 1).

3. For each 83-ft vertical segment of the building (rounded to the nearest convenient column splice point), input an out-of-plumbness in each story within that vertical segment equal to $1/500$. Out-of-plumbness outside of the segment considered should be taken as zero. (See Cases 2 through 5 in Figure C-1. For example, in Case 3, $\Delta_o = (1/500)(78\text{ ft})(12\text{ in./ft}) = 1.87\text{ in.}$) For buildings shorter in height than about 83 ft, use a $1/500$ foot lean for the entire building height. The height of 83 ft is the value at which an out-of-plumbness of $1/500$ produces a total lateral displacement of approximately 2 in., the maximum limit of the envelope given by Figure C-7.5 of the *Code of Standard Practice* for frames up to 20 stories.

The location of column splice points and actual shipping piece column lengths should be considered in setting the vertical out-of-plumb segments. It is suggested that the actual lean angle be maintained at $1/500$ and the vertical segment ends be located so that the 2 in. displacement limit is equaled or exceeded in each case. Case 1, the maximum out-of-plumb lean for the given building height, will probably not control in most cases but could produce the maximum overturning effect on the building for use in design of the columns and foundations.

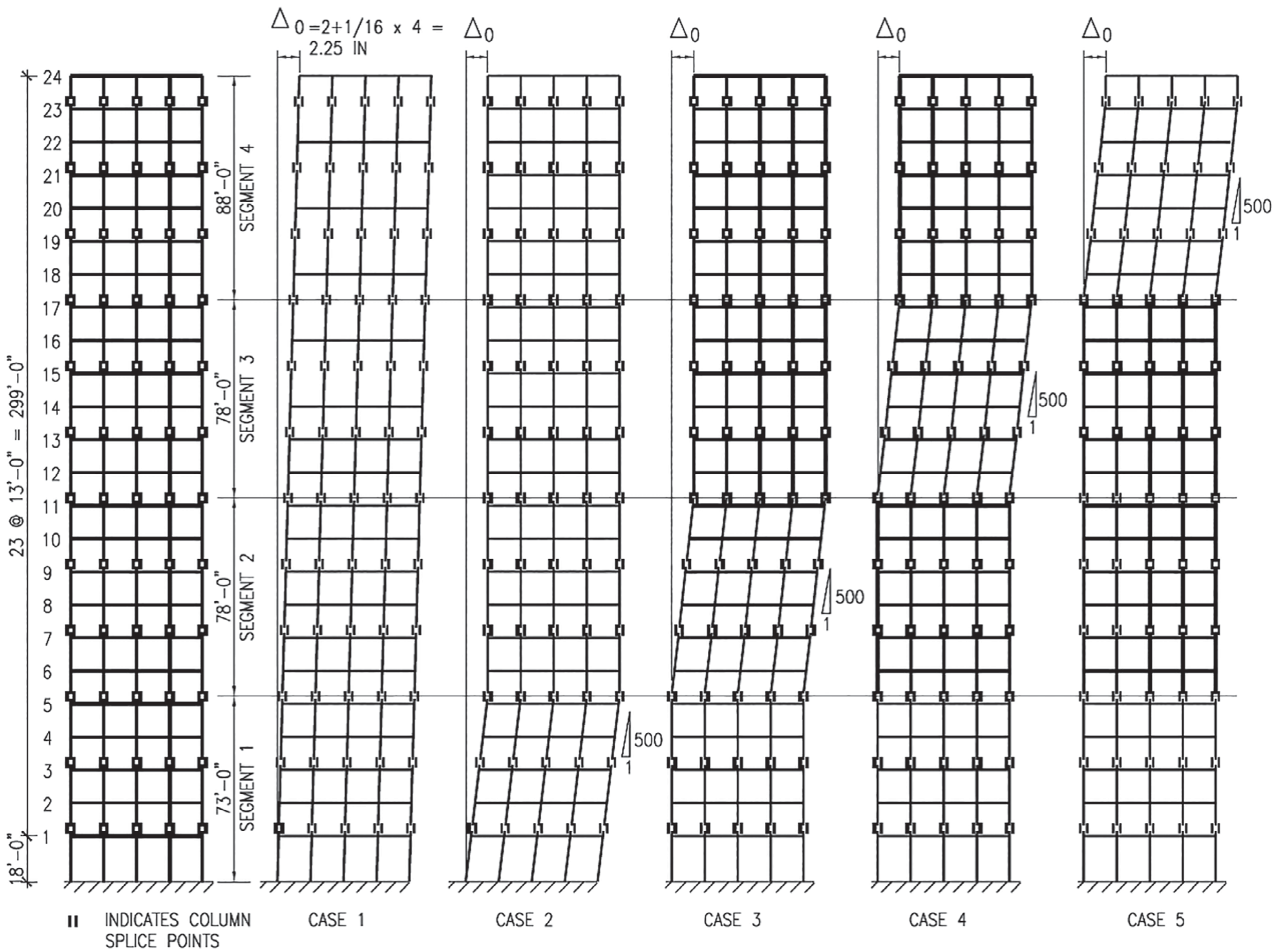


Fig. C-1. Out-of-plumbness cases—24-story building.

APPENDIX D
Practical Benchmarking and Application of Second-Order Analysis Software

It should be clear from the discussion in Appendix A that second-order effects can have a pronounced effect on the design and behavior of steel frames. Proper use of the direct analysis method (DM), particularly in the case of solving nodal or lean-on bracing problems, requires that an accurate frame analysis be undertaken.

Update Note: As indicated in an Update Note in Section 1.4, an important change in the 2010 AISC *Specification* is that it allows use of a P-Δ-only analysis (that is, one that neglects the influence of P-δ effects on the response of the structure) under certain conditions, specified in 2010 AISC *Specification* Section C2.1(2). These conditions are, first, that the ratio of second-order drift to first-order drift (of which B_2 is a good approximation) be less than 1.7 in an analysis using reduced stiffnesses and, second, that no more than one-third of the total gravity load on the structure be supported on columns that are part of moment-resisting frames in the direction being considered.

The conditions that allow use of a P-Δ-only analysis will be found to apply to most buildings. This represents an important simplification of analysis requirements. Much of the following discussion in this Appendix is related to whether the analysis adequately considers the influence of P-δ effects on the response of the structure; but this is now irrelevant to the design of most buildings. Most commercial second-order analysis software packages do an adequate P-Δ analysis. Note that P-δ effects in individual compression members must still be considered in all cases (as typically done using B_1 multipliers).

The number of elements required for each unbraced segment varies significantly among the many different second-order analysis software packages commonly used in current design practice. A review of the many different combinations and permutations of formulations and implementations of frame elements and solution procedures for second-order frame analysis is beyond the scope of this design guide. This appendix emphasizes the basic benchmarking of any type of second-order analysis, the selection of an appropriate number of elements along the length of members in typical methods of second-order analysis commonly implemented in software, and the appropriate calculation of second-order moments between element nodal locations.

Unfortunately, some of the professional software packages used by many designers do not account accurately for P-δ effects, or they include only P-Δ analysis formulations and implementations. P-δ effects in particular are generally a very important attribute of the behavior for bracing problems where the influence of the column flexural rigidities, EI, is included in the analysis solution. P-Δ analysis formulations can still be used to obtain accurate P-δ moments, but generally a large number of elements per member may be required. P-Δ implementations that do not allow for subdivision of members into multiple elements cannot be used in problems where P-δ effects are significant.

Furthermore, the accuracy of second-order analysis software depends on more than just the underlying frame element formulation. The accuracy also can depend on the type of nonlinear analysis solution procedures implemented within the software and on the manner in which the frame elements are programmed in these solution procedures. It is not uncommon for software solutions to diverge or to converge to incorrect results for certain problems.

Designers are urged to carefully benchmark their analysis software using test problems, such as those discussed in this appendix. This is the only way that one can be certain that a sufficient number of elements is used to ensure acceptable accuracy for a given software package. Careful benchmarking is essential. Once the designer has conducted careful benchmarks to determine the number of elements per member necessary for worst-case member end conditions (see the following), an appropriate discretization (number of elements for each unbraced segment) can be used with confidence for general analysis. It is always wise for the designer to re-run a selected suite of benchmark problems before adopting any new release of a given software package.

Approaches to obtain good accuracy with one or only a few elements per unbraced length are reasonably standardized at the present time and are described in textbooks such as McGuire et al. (2000). The McGuire et al. (2000) textbook is worth mentioning in particular because it is coupled with an easy to use educational software package, Mastan. Mastan provides a useful vehicle for the designer to understand many of the key attributes of second-order frame analysis methods.

This discussion is posed entirely in a planar (two-dimensional) context. It is important to understand how these calculations relate to the three-dimensional analysis of general structures. In three-dimensional formulations, the large delta and small delta displacements, Δ and δ, are typically considered at the element level as displacements along either of the principal axes of the element cross section, orthogonal to the longitudinal axis of the element. These displacements are coupled with the rotations about each of the element cross-section principal axes. That is, there are two sets of Δ and δ displacements.

To understand the relationship of the two-dimensional *P*-Δ and *P*-δ calculations to the three-dimensional analysis of general structures, it is useful to consider the sketch shown in Figure D-1. This sketch shows a common three-dimensional frame element having two end nodes with six degrees of freedom (dof) at each of these nodes—three translational displacement dof and three rotational dof. The translational dofs are numbered 1 through 3 at each of the end nodes, and the rotational dofs are numbered 4 through 6. Depending on the type of element formulation and the nature and significance of the second-order effects, it may be sufficient to represent the response of an entire member unbraced length by this single element. However, in general, a given member segment will need to be subdivided into multiple elements to capture the responses within the physical structure with sufficient accuracy. Therefore, the element shown in Figure D-1 may represent an entire member, just one unbraced segment of a member, or only a partial length within a given member unbraced segment.

Figure A-1 of Appendix A provides a general definition of the member (or member segment) *P*-Δ and *P*-δ effects. In the context of the element shown in Figure D-1, the *P*-Δ effect is the additional moment due to the element axial force acting through the relative transverse displacement of the element ends. This is the definition provided in Figure A-1, but where the element length is the member segment length. Also, the *P*-δ effect is specifically the additional moment due to the element axial force acting through the transverse displacement of the cross section relative to a chord between the element ends, paralleling the general definition shown in Figure A-1. The prior and subsequent discussions apply to any element displacements in the *x*-*z* or the *y*-*z* planes as shown in Figure D-1. Assuming that the *x*- and *y*-axes correspond to the principal axes of the member cross section, the displacements in these planes are tied to member axial forces as well as bending moments about the member major and minor principal axes. The ability of three-dimensional software to capture the *P*-Δ and *P*-δ effects can be tested by separate planar analyses that exercise the dof 1, 3 and 5 and the dof 2, 3 and 4. Analyses exercising each of these dof sets must be conducted as part of validating a three-dimensional program's handling of these effects.

The torsional dofs (dof 6 in Figure D-1) are often given relatively little consideration in discussions of second-order (and even first-order) analysis of frame structures. Substantial complexities are associated with the true physical coupling between the element torsional rotations and the other element dof. This is particularly the case if the members are singly symmetric, in which case the cross-section shear center is not located at the same position as the cross-section centroid. Furthermore, for open-section members, such as members with I-shaped cross sections, a major portion of the torsional resistance comes from warping torsion rather than from uniform (St. Venant) torsion. The Mastan software (McGuire et al., 2000) is representative of a number of high-end general purpose software packages in that it captures the warping torsion effects by including a seventh warping displacement dof at each of the element nodes. To the knowledge of the authors, none of the current commercial software packages commonly used for design of steel frame structures include this type of capability (one software package addresses the contribution of warping in determining element torsional stiffness, but assumes that the warping displacements are fully restrained (i.e., zero) at the joints of the structure, but not at intermediate nodes along the member lengths within the structural analysis). Therefore, the authors believe that it is fair to say that none of the professional software packages presently used for overall steel frame design provide any capability for accurate analysis of either first- or second-order torsional responses. To complicate matters further, the orientations of the cross-section *x*-*y* principal axes vary along the length of any member subjected to torsion.

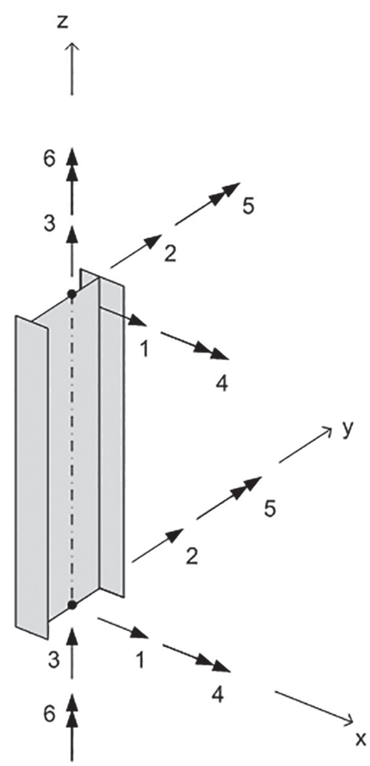


Fig. D-1. Typical three-dimensional frame element with six dof per node.

Fortunately, the contributions of member torsional stiffnesses to the overall stiffnesses in resisting design loads are rather inconsequential for most frame structures. Furthermore, the overall torsional rotations are often restrained such that they are small enough to be neglected in terms of their effect on the overall primary structural responses. Any overall torsional displacements of the structure are resisted predominantly by member axial stiffnesses, major-axis bending stiffnesses, and in some cases, minor-axis bending stiffnesses. As a result, very crude representations of the physical member torsional stiffnesses are sufficient for practical purposes. The designer must be aware of situations where members are called upon to transfer loads primarily by torsion. These cases generally must be addressed by separate solutions such as those described in AISC Design Guide 9, *Torsional Analysis of Structural Steel Members* (Seaburg and Carter, 1997) or AISC Design Guide 22, *Façade Attachments to Steel-Framed Buildings* (Parker, 2008), for example. The second-order three-dimensional analysis of members exhibiting large torsional displacements is possible using a number of research programs documented in the technical literature; however, to the knowledge of the authors, short of discretizing frame members using second-order shell finite elements, these types of tools are not available in software commonly utilized for design of steel frames at the present time.

In some situations involving relatively long torsionally weak and torsionally flexible members, it is possible that simplified three-dimensional analysis solutions, where, for example, the element torsional stiffnesses may be calculated as only GJ/L with the St. Venant torsional rigidity, GJ, being a relatively small number, may predict false member elastic torsional instabilities in the structural analysis solution. These types of instabilities may cause substantial difficulties in the global nonlinear solution algorithms and can be difficult to detect since the software may not be set up to display element or member torsional displacements. These types of situations can be anticipated by considering the beam-column resistance equations of Section H1 of the AISC *Specification* along with the column torsional or flexural-torsional elastic buckling resistance equations of AISC *Specification* Section E4 and the appropriate elastic lateral-torsional buckling resistance equations from AISC *Specification* Chapter F, but with the warping rigidity, EC_w, taken equal to zero. For doubly symmetric I-section members, the corresponding elastic buckling moment is $C_b \pi \sqrt{EI_y GJ}/L$.

D.1 APPROPRIATE NUMBER OF ELEMENTS PER MEMBER IN P-δ FORMULATIONS

The first measure of success of a second-order analysis is the accuracy of the calculated nodal displacements and element end forces. All numerical methods of calculating second-order displacements and forces become increasingly inaccurate at higher levels of axial loading. For structures with relatively high levels of axial load relative to the buckling load of the structure or member, the influence of P-δ and/or P-Δ effects is significant and must be included by some means to obtain accurate nodal displacements and element end forces. Depending on the type of second-order calculations being used, this can be done either by subdividing the members into a sufficient number of smaller-length elements, or by the inclusion of P-δ effects in the element formulation.

The following recommendations are based on the cubic displacement-based frame element formulation and implementation in Mastan and other software with similar formulations. This type of formulation provides an approximate representation of the P-δ effects at the element level. These recommendations are provided here because of the importance of calculating both the P-Δ and P-δ effects accurately when solving stability bracing problems, as well as in some cases, where analyzing the basic sidesway response of moment frames using a second-order analysis (see Appendix A).

For problems in which the column flexural rigidities (i.e., $0.8\tau_b EI$ in the DM) are included in the analysis model, two elements are generally sufficient to give second-order nodal displacements that are accurate to within 3% for the internal forces and 5% for the nodal displacements relative to the exact analytical results for load levels of approximately ⅔ of the elastic buckling load in all possible practical cases (Guney and White, 2008a). For cases where the problem is dominated by sidesway behavior and P-Δ effects, one element per member is sufficient. When using this recommended discretization, it is important to determine the internal P-δ moments along the element lengths (P times the estimated displacement δ relative to the chord between the element ends). Mastan does not include this feature within its analysis calculations; however, a number of the commonly used commercially available analysis programs do calculate the internal P-δ moments explicitly along the element lengths. This calculation is relatively simple for prismatic member geometries and is best implemented explicitly within, rather than outside of, the analysis solution procedures. Section D.3 provides a more in-depth explanation of this calculation. Guney and White (2008b) provide an overview and demonstrate the relative accuracy of various ways of estimating the P-δ moments along the element lengths for a wide range of element loading conditions.

D.2 APPROPRIATE NUMBER OF ELEMENTS PER MEMBER IN P-Δ FORMULATIONS

In the most extreme cases, P-Δ formulations only (without P-δ considered) may require up to 11 elements per unbraced segment to obtain the aforementioned required accuracy of the nodal displacements and internal forces. However, these extreme cases typically involve significant rotational and translational end restraints, or in the context of the effective length method, design using $K << 1$. For DM or ELM designs based on $K \geq 1$, i.e., where substantial rotational end restraint is not relied upon to develop applied member axial loads larger than the elastic buckling load based on $K = 1$, six P-Δ elements per member are generally sufficient (Guney and White, 2008a). Guney and White (2008a) provide several tables that specify smaller numbers of elements per member for general accuracy provided a given limit on $\alpha P_r / \bar{P}_{eL}$, where $\bar{P}_{eL}$ is the member elastic flexural buckling load based on $K = 1$ (including reductions in the elastic stiffness where required, i.e., as required by the DM) and P_r is the required axial compressive strength using LRFD or ASD load combinations. These P-Δ recommendations are conservative (i.e., they result in the use of more elements than absolutely necessary) in analysis formulations that have intermediate levels of accuracy between that of the Mastan (McGuire et al., 2000) element and a basic P-Δ element formulation and implementation. The reader is cautioned that in certain cases, P-δ effects on the sidesway displacements, Δ, must be included (see Appendix A). This requires multiple P-Δ elements per member.

In many situations where the lateral load resistance is provided by moment frames, the second-order sidesway response of steel frame structures is still captured sufficiently by the use of a P-Δ analysis without any discretization of the members into multiple elements. This fact can be understood in simple terms by considering the sidesway amplification given by Equation A-12 with R_M defined by Equation A-13. It should be noted that Equation A-12 with R_M taken equal to 1.0 is equivalent to a basic P-Δ sidesway analysis. The factor R_M is just a very simple way of describing a practical upper bound influence of the P-δ effects on the "large delta" story drift displacements.

In most steel framing systems, a relatively large fraction of the story gravity load will be supported by gravity only (i.e., leaning) columns. One can show, using Equations A-12 through A-14 that if $R_L = \Sigma P$ leaning columns, and braced frames$/\Sigma P$ all columns (Equation A-14) is greater than 1/3 and the P-Δ sidesway amplification is less than or equal to 1.5, the error in the second-order sidesway amplification obtained from a P-Δ analysis (equivalent to Equation A-12 with R_M taken equal to 1.0) is never larger than about 5% relative to the second-order sidesway displacement determined including P-δ effects (approximated by Equation A-12 with $R_M = 0.85 + 0.15R_L = 0.9$). The true sidesway moment amplification tends to be smaller than estimated using Equation A-12 (see Figure A-5); hence the error in P-Δ sidesway moments compared to exact calculations including P-δ effects is less than 3% given these constraints.

Most lateral-load resisting members are subjected not only to sidesway bending moments. They are also subjected to "nonsway" bending moments caused by gravity loads. The accurate calculation of nonsway second-order effects using P-Δ type elements generally requires up to six elements per member as explained previously.

D.3 CALCULATION OF INTERNAL P-δ MOMENTS ALONG ELEMENT LENGTHS

One approach to calculate the internal P-δ moments along element lengths is as follows:

1. Given the end rotations relative to the element chord plus any transverse loads applied to the member along its length, determine the transverse displacements, δ, relative to the element chord based on a first-order approximation. For elements that do not have internal transverse loads, the transverse displacement varies as a cubic function along the element length; for elements subjected to uniformly distributed transverse loads, this displacement is a quartic function. It is recommended that the displacements, δ, and the corresponding P-δ moments discussed in the following should be determined at a spacing, s, along the element length such that the following inequality is satisfied.

   When using the DM:

   $$\alpha P_r / \bar{P}_{es} < 0.02 \tag{D-1a}$$

   where

   $\bar{P}_{es} = \pi^2 \left(0.8\tau_b EI\right) / s^2$, kips

   When using the ELM:

   $$\alpha P_r / P_{es} < 0.02 \tag{D-1b}$$

   where

   $P_{es} = \pi^2 EI / s^2$, kips

   For both methods:

   P_r = required axial compressive strength using LRFD or ASD load combinations, kips

   α = 1.0 for LRFD or 1.6 for ASD

2. Given the displacements at each of the points determined in step 1, along with the applied loads at the ends and internally within the element, calculate the internal second-order moments at the corresponding cross sections based on a free-body diagram of a portion of the deformed element in which a cut is made at the desired cross section.

This procedure is recommended for use with both the cubic displacement based type of element such as in Mastan as well as with elements that account only for P-Δ effects in the analysis formulation and implementation. This solution gives acceptable accuracy, but if desired, a generalized B_1 amplification factor can be applied in the calculation of the above internal element moments to gain additional accuracy. The reader is referred to Guney and White (2008b) for a recommended calculation procedure of this type. Additional discussions pertaining to web-tapered members are provided in Kaehler et al. (2010).

Given the above calculation of the internal moments between the element ends, the element internal moments are obtained generally with better accuracy than the nodal displacements. The beam-column interaction unity checks have even greater accuracy, since both the axial force and the internal moment contribute to the unity check value of the interaction equations. Therefore, a targeted accuracy of at least 5% in the calculated nodal displacements is considered acceptable (this implies that 5% error is acceptable in the calculated bracing force values for basic bracing problems, since if the bracing is represented by a linear spring, the bracing force is proportional to the brace-point displacement). The reader is referred to Guney and White (2008a) for a more detailed discussion of this recommendation.

D.4 BASIC TEST PROBLEMS FOR EVALUATION OF SECOND-ORDER ANALYSIS SOFTWARE

The Commentary to AISC *Specification* Appendix 7 contains two basic benchmark problems that can be used to confirm the accuracy of software programs and other methods of second-order analysis. The engineer is encouraged to consider other solutions as well; however, these two simple benchmark problems are often successful in quickly identifying programs that do not give acceptable solutions. Many programs available today capture the overall frame deflection increase from P-Δ effects, but do not consider the effects of P-δ moments (individual member stiffness reduction for members in compression) on lateral frame deflections. This effect was examined in Appendix A and is the reason for the inclusion of $R_M = 0.85$ in the B_2 amplification factor of Equations C2-3 and C2-6b in AISC *Specification* Chapter C for moment frame structures. Where $B_2 = 2.5$, calculated without the influence of R_M, the upper bound influence of this stiffness reduction factor causes an additional amplification of 36% (i.e., B_2 increases from 2.5 to 3.4 when $R_M = 0.85$ is considered). At a more modest and realistic value of $B_2 = 1.5$, calculated without the influence of $R_M = 0.85$, the influence is 10% (i.e., B_2 increases from 1.5 to 1.64 when R_M is considered) (see Figure A-9 in Appendix A of this Design Guide).

Update Note: The benchmark problems are different in the 2010 AISC *Specification* and they are located in the Chapter C Commentary not the Appendix 7 Commentary. Furthermore, as noted previously, the analysis requirements have been simplified in the 2010 edition.

It is suggested that that the following procedure be undertaken as a bare minimum to evaluate the accuracy of second-order analysis software:

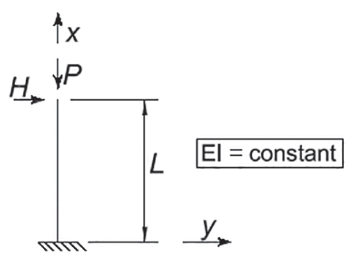
1. Run the flagpole problem, Case 2 in AISC *Specification* Commentary Appendix 7 (shown here as Figure D-2), using nodes at the base and top only. Apply the lateral and vertical load proportionally up to a load level that produces at least a moment and deflection amplification value of 2.5. Any reasonable ratio of lateral load, H, to vertical load, P, may be used. However, small ratios of lateral to vertical load (say 0.01 to 0.12) produce the most realistic cases that have large amplification but also satisfy the AISC member design requirements. If the results match the plot shown in AISC *Specification* Commentary Appendix 7, Figure C-A-7.3, Case 2 within 3%, the program properly considers the P-Δ and P-δ effects for this case (the value of 3% is based on the AISC *Specification* Commentary Appendix 7, but as discussed previously, 5% error on the nodal displacements is considered to be acceptable by Guney and White (2008a)).

   Note that a P-Δ analysis conducted at $P/P_{eL} = 0.05$ and an exact sidesway amplification of 1.25 violates this limit of 3% unconservative error for this problem (the error is approximately 4%). For $P/P_{eL} = 0.15$, giving an exact sidesway amplification of 2.51, the moment error is approximately 21% unconservative. Refer to Appendix A, Figure A-5 in this Design Guide and compare the curve based on Equation A-12 with the curve for Equation A-7. The influence on the beam-column interaction equation value is smaller than the error described here because both axial load and moment influence the interaction value. However, the base moment in this problem, which is representative of the moment transferred to adjacent beams and connections in a general frame analysis, must also be accurate to within the target 3%. Proceed to step 2.

2. Run Case 1 in AISC *Specification* Commentary Appendix 7 (included herein as Figure D-3) with nodes only at the supports. If the moments and midspan deflections match within 3% for a proportional increase in axial and lateral load up to a magnification of at least 2.5, the program accounts for P-δ effects properly for this case. It should be noted that it is difficult for a transversely loaded member with a second-order P-δ amplification of 2.5 to meet service transverse deflection and beam-column strength requirements unless the transverse load is unusually small. Also, as in the Case 2 problem, the error in the beam-column interaction equations is generally smaller than the internal moment error. Nevertheless, this level of accuracy can be important in certain cases, such as the analysis of bracing problems.

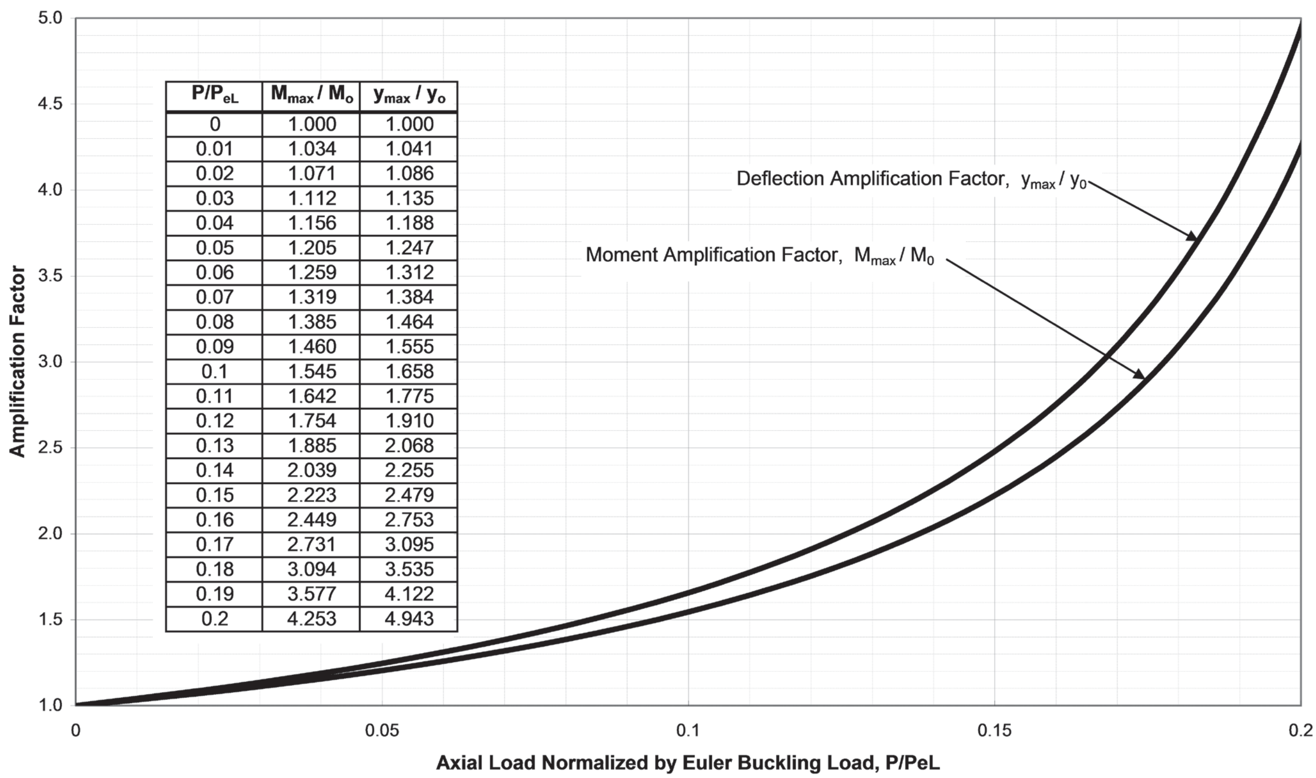
If a computer program fails to pass these tests, it may still be possible to use it by adding additional nodes along the member. For programs that capture P-δ effects accurately at the element level, one additional node at the middle of the unsupported length is typically sufficient. For P-Δ analysis programs, as many as five additional nodes (six segments) may be needed along the member length for each compression member where the flexural rigidity contributes to resistance to the beam-column joint and/or brace-point displacements. However, this requirement can vary from one program to another. It is suggested to re-run the above problems with this change and check to see if the results match within the 3% target. Many programs will calculate the P-Δ effect exactly but will fail to capture the P-δ effect. Adding the additional nodes, using a minimum of five intermediate nodes (six elements), should always provide sufficient accuracy in capturing the member P-δ effects.

Other benchmark problems can be found in Chen and Lui (1987), which contains rigorous solutions for a simply supported beam-column subjected to compression and applied end moments and also a solution for a fixed-ended beam-column subjected to compression and uniformly distributed loads. Typically, the calculation of accurate internal moments is more difficult in problems where member load and/or displacement boundary conditions are not symmetrical. The reader may also consult Guney and White (2008a)



$$M_{max}\left(@\,x=0\right)=HL\left(\frac{\tan\alpha}{\alpha}\right),\quad \text{where } \alpha=\sqrt{\frac{PL^2}{EI}},\quad M_o=HL$$

$$y_{max}\left(@\,x=L\right)=\frac{HL^3}{3EI}\left(\frac{3\left(\tan\alpha-\alpha\right)}{\alpha^3}\right),\quad \text{where } y_o=\frac{HL^3}{3EI}$$

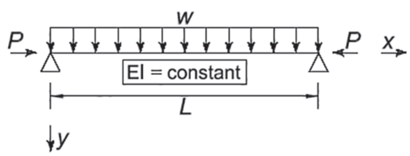


P/P_{eL}	M_{max}/M_o	y_{max}/y_o
0	1.000	1.000
0.01	1.034	1.041
0.02	1.071	1.086
0.03	1.112	1.135
0.04	1.156	1.188
0.05	1.205	1.247
0.06	1.259	1.312
0.07	1.319	1.384
0.08	1.385	1.464
0.09	1.460	1.555
0.1	1.545	1.658
0.11	1.642	1.775
0.12	1.754	1.910
0.13	1.885	2.068
0.14	2.039	2.255
0.15	2.223	2.479
0.16	2.449	2.753
0.17	2.731	3.095
0.18	3.094	3.535
0.19	3.577	4.122
0.2	4.253	4.943

Fig. D-2. AISC Specification *Commentary Appendix 7—Case 2.*

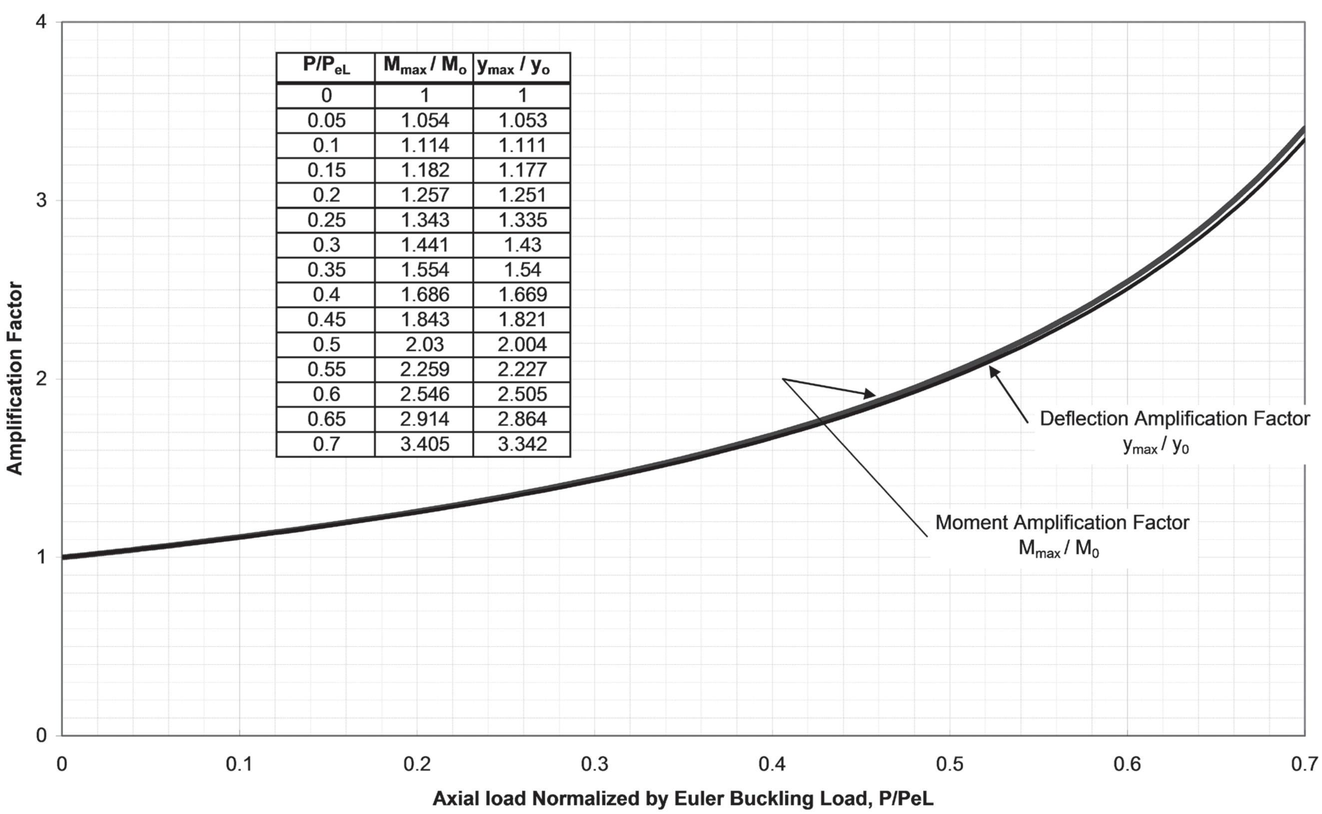
for examples of more comprehensive evaluations of program capabilities for a wide range of boundary conditions. In addition, Kaehler et al. (2010) provide several additional prismatic member benchmark problems as well as several benchmark problems for tapered members.

Figure D-4 provides a specific version of the cantilever column of Figure D-2. It should be helpful for understanding the potential errors associated with a P-Δ analysis using a single element per member. A P-Δ analysis generally requires the use of multiple elements per member to properly capture P-δ effects. For example, consider the evaluation of the fixed-base cantilever column shown in Figure D-4 using the direct analysis method. The exact sidesway displacement amplification factor is 3.97 and the exact base moment amplifier is 3.45, giving M_u = 1,394 kip-in. For the specified applied loads, the AISC *Specification* beam-column strength interaction Equation H1-1a is equal to 1.0. The sidesway displacement and base moment amplification determined by a single element P-Δ analysis is 2.61, giving an estimated M_u = 1,070 kip-in., an error of 24.6% relative to the exact M_u and a beam-column interaction value of 0.912. For this example, three equal-length P-Δ analysis elements are required to reduce the errors in the second-order base moment and sidesway displacement to less than 3% and 5%, respectively. These errors are due to the lack of consideration of P-δ effects on the overall sidesway displacement, Δ, in the single-element-per-member P-Δ analysis. Interestingly, the use of R_M = 0.85 in Equation A-12 gives a B_2 amplifier of 3.66 in this example, which would yield a moment of 1,420 kip-in. and an interaction equation result of 1.02.



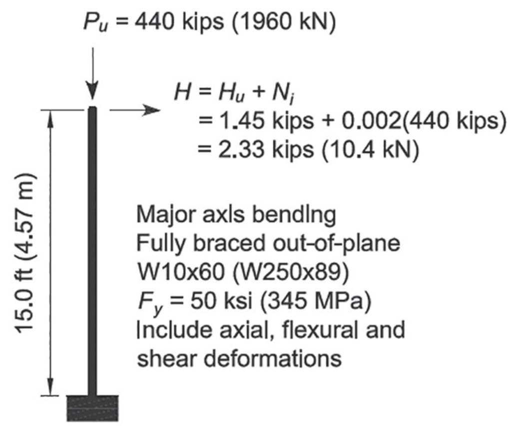
$$M_{max}\left(@\, x = \frac{L}{2}\right) = \frac{wL^2}{8}\left[\frac{2\left(\sec u - 1\right)}{u^2}\right], \quad \text{where } u = \sqrt{\frac{PL^2}{4EI}}, \quad M_o = \frac{wL^2}{8}$$

$$y_{max}\left(@\, x = \frac{L}{2}\right) = \frac{5wL^4}{384EI}\left[\frac{12(2\sec u - u^2 - 2)}{5u^4}\right], \quad \text{where } y_o = \frac{5wL^4}{384EI}$$



P/P_{eL}	M_{max} / M_o	y_{max} / y_o
0	1	1
0.05	1.054	1.053
0.1	1.114	1.111
0.15	1.182	1.177
0.2	1.257	1.251
0.25	1.343	1.335
0.3	1.441	1.43
0.35	1.554	1.54
0.4	1.686	1.669
0.45	1.843	1.821
0.5	2.03	2.004
0.55	2.259	2.227
0.6	2.546	2.505
0.65	2.914	2.864
0.7	3.405	3.342

Fig. D-3. AISC Specification *Commentary Appendix 7—Case 1.*



$P_u/P_y = 0.50$, $\tau = 1.0$
$E = 0.8 \times \tau \times 29{,}000$ ksi = 23,200 ksi (160 GPa)
$G = E / 2(1 + \nu)$ = 8,920 ksi (61.5 GPa)

<u>Rigorous P-Δ and P-δ analysis:</u>
Δ_{2nd} = 2.22 in. (56.5 mm)
M_u = 1,394 kip-in. (158 kN-m)
$P_u / \phi_c P_n + \frac{8}{9}(M_u / \phi_b M_n) = 1.00$

<u>Single-element P-Δ analysis:</u>
Δ_{1st} = 0.580 in. (15 mm)
$M_{1st} = HL$ = 419 kip-in. (48 kN-m)
$1 / [1 - P_u / (HL / \Delta_{1st})] = 2.55$
$M_u^{P\text{-}\Delta} = 2.55 M_{1st}$ = 1,070 kip-in. (121 kN-m)
$P_u / \phi_c P_n + \frac{8}{9}(M_u^{P\text{-}\Delta} / \phi_b M_n) = 0.910$

Fig. D-4. Illustration of potential errors associated with the use of a single-element-per-member P-Δ analysis.

APPENDIX E
Bracing Requirements for Columns and Frames Using Second-Order Analysis

E.1 INTRODUCTION

Requirements for bracing of columns and beams can be found in Appendix 6 of the AISC *Specification*. Stability bracing for frames is covered in Chapter C and Appendix 7. The provisions of Appendix 6 address bracing intended to stabilize individual beams and columns where the bracing is not a part of the overall lateral load resisting system. A discussion of all types of bracing for different situations can be found in Yura (1993, 1995) and Ziemian (2010, Chapter 12). This appendix specifically addresses bracing to prevent flexural buckling of individual columns and overall side-sway buckling of framing systems. It focuses on the use of second-order analysis, as permitted in AISC *Specification* Appendix 6, Section 6.1, to solve practical bracing problems as an alternative to and an extension of the bracing formulas provided in Appendix 6.

E.2 TYPES OF COLUMN BRACING

Four general types of column bracing can be identified. They are relative bracing, nodal bracing, continuous bracing, and lean-on bracing. These bracing types are illustrated in Figure E-1. Only the first two are addressed explicitly in Appendix 6. For a brace to be effective in preventing buckling of a column and to serve as a brace point, it must satisfy both a strength and a stiffness requirement. If either of these requirements is not met, the column can fail prematurely.

Many designers struggle with the concept of a brace stiffness requirement and think only of the required strength of the brace. A common traditional practice for many years was to design or check column braces for 2% of the column force without considering the brace stiffness. This approach may suffice in many situations found in practice, but not in all situations. Reduced bracing stiffness allows greater deformation; this in turn causes larger forces in the bracing members. If the bracing stiffness is too small, the required bracing forces can be excessive. The AISC *Specification* strength and stiffness requirements are summarized below for relative and nodal bracing problems. This is followed by a brief discussion of the behavior and information sources pertaining to continuous and lean-on bracing problems. These types of bracing are not addressed specifically in the AISC *Specification*.

E.2.1 Relative Bracing

Relative bracing is defined as bracing that controls the movement of a brace point with respect to adjacent brace points along the column. All types of lateral bracing such as diagonal or cross bracing, V bracing, and inverted-V bracing are classified as relative bracing. If a section taken through the cross section of the column intersects the brace itself, then the brace is classified as a relative brace (Yura, 1995). The base requirements for the strength and stiffness of a column relative brace are found in AISC *Specification* Appendix 6, Section 6.2.1 and are summarized in the following.

The required brace strength is:

$$P_{br} = 0.004\, P_r \qquad (\textit{Spec}.\ \text{Eq. A-6-1})$$

where

P_r = required axial compressive strength in the column using LRFD or ASD load combinations, kips

The required brace stiffness is:

$$\beta_{br} = \frac{1}{\phi}\left(\frac{2P_r}{L_b}\right) \text{ (LRFD)} \qquad \beta_{br} = \Omega\left(\frac{2P_r}{L_b}\right) \text{ (ASD)} \qquad (\textit{Spec}.\ \text{Eq. A-6-2})$$

where

ϕ = 0.75 (ASD)
Ω = 2.00 (LRFD)
L_b = distance between brace points, in.

E.2.2 Nodal Bracing

Nodal bracing controls the movement at the brace point without any direct interaction between adjacent brace points. A cross section taken through the braced member does not intersect the brace itself (Yura, 1995). Column nodal bracing requirements are found in AISC *Specification* Appendix 6, Section 6.2.2 and are summarized in the following.

The required brace strength is:

$$P_{br} = 0.01 P_r \qquad (\textit{Spec}.\ \text{Eq. A-6-3})$$

The required brace stiffness is:

$$\beta_{br} = \frac{1}{\phi}\left(\frac{8P_r}{L_b}\right) \text{ (LRFD)} \qquad \beta_{br} = \Omega\left(\frac{8P_r}{L_b}\right) \text{ (ASD)} \qquad (\textit{Spec}.\ \text{Eq. A-6-4})$$

where

 $\phi = 0.75$ (LRFD)
 $\Omega = 2.00$ (ASD)

When the column axial load, P_r, is smaller than P_c based on $KL = L_b$ (i.e., with $K = 1$ for the unsupported length between the brace points), the load P_r can be supported using a smaller nodal bracing stiffness than given by AISC *Specification* Equation A-6-4. This is because the flexural rigidity of the column, EI, is able to help in resisting the deflection at the brace points. In essence the column flexural rigidity is not fully "used up" by buckling between the brace points and thus K can be greater than 1.0. Appendix 6 accounts for this combined resistance of a column and its nodal bracing system by permitting L_q to be substituted in place of L_b in AISC *Specification* Equation A-6-4, where L_q is the maximum unbraced length for which the member can support the required axial load using $K = 1$. The length L_q is determined in general by setting P_r equal to P_c and backsolving for the effective length, $KL = L_q$. This substitution gives a conservative approximation of other more rigorous stiffness requirements summarized by Winter (1960) for "partial bracing" systems that are too flexible to develop $KL = L_b$. The substitution of L_q for L_b in Equation A-6-4 accommodates the common practical situation where the number of brace points is greater than that required to support the column load, P_r (Yura, 1995). In these cases, there is no need to require the larger bracing stiffnesses necessary to develop the available axial compressive strength, P_c, based on $KL = L_b$.

The above substitution of L_q for L_b is not permitted for relative bracing, i.e., it is not permitted for AISC *Specification* Equation A-6-2. The reason is that the relative bracing equations do not consider any contribution from the column flexural rigidity, *EI,* to the resistance of the brace point deflections. The column is idealized as a pin-connected strut between the brace points. This is a practical simplification based on the fact that in many relative bracing systems, the contribution from the column EI values is small compared to the stiffness of the bracing system itself. For example, considering the relative bracing system shown in Figure E-1, moment connections would be necessary between

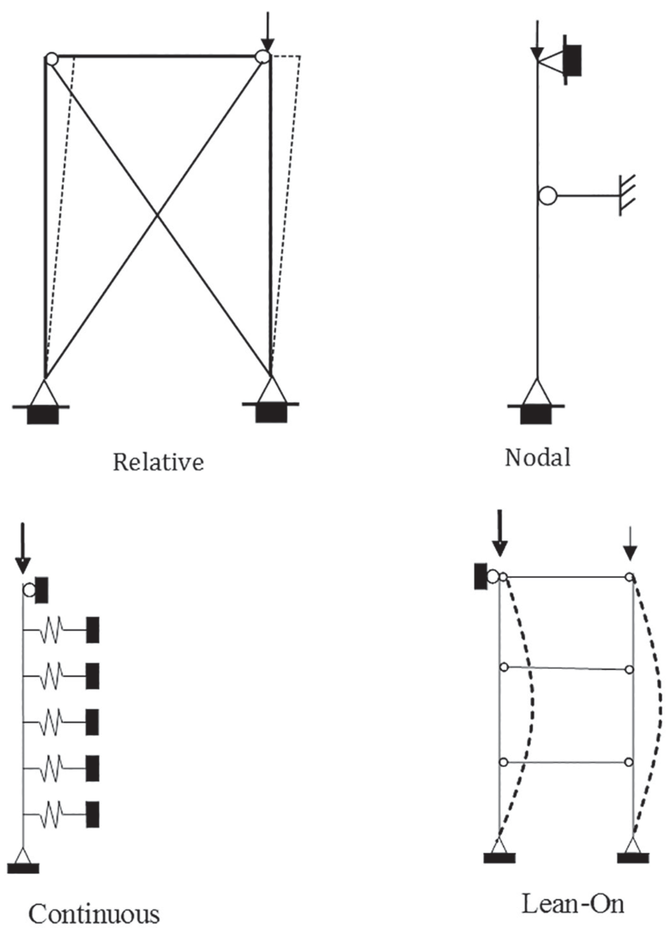


Fig. E-1. Types of column bracing.

the column and the foundation and/or the column and the horizontal beam or braces to develop any contribution from the column flexural rigidity to the resistance of the brace point lateral deflections. The influence of column continuity through the floor levels in braced frames or truss chord continuity through truss panel points is often small relative to the influence of the overall bracing or truss system on the lateral stiffness.

As discussed in the AISC *Specification* Commentary Appendix 6, Section 6.2, for nodal bracing, Equation A-6-4 may be written more generally as:

$$\beta_{br} = \frac{1}{\phi}\left[2(4-2/n)\right]\frac{P_r}{L_q} \quad \text{(LRFD)}$$
$$\beta_{br} = \Omega\left[2(4-2/n)\right]\frac{P_r}{L_q} \quad \text{(ASD)} \tag{E-1a}$$

where n = number of brace points within the column length (not including end points). Therefore, for a single intermediate nodal brace, the requirements are:

$$\beta_{br} = \frac{1}{\phi}\left(4\frac{P_r}{L_q}\right) \quad \text{(LRFD)}$$
$$\beta_{br} = \Omega\left(4\frac{P_r}{L_q}\right) \quad \text{(ASD)} \tag{E-1b}$$

These more liberal bracing stiffness requirements are specified in the AISC *Specification* Commentary Appendix 6, Section 6.2. Equations E-1a imply that the nodal bracing stiffness requirement, β_{br}, is larger (for a given P_r and L_q) as the number of nodal braces is increased. AISC *Specification* Equations A-6-4 are an upper bound to Equations E-1a. In Equations E-1a and E-1b, it should be noted that L_q is independent of n; L_q depends only on P_r and the column cross-section properties.

If the actual provided brace stiffness is different than that required by AISC *Specification* Equations A-6-2 for relative bracing or Equations A-6-4 for nodal bracing, the AISC *Specification* Commentary states that the required brace strength can be multiplied by Commentary Equation C-A-6-1 as in the following:

$$P_{br} = \frac{\left(P_{br} \text{ from Eq. A-6-1 or A-6-3}\right)}{2-\dfrac{\beta_{br}}{\beta_{act}}} \tag{E-2}$$

where β_{act} is the actual brace stiffness provided. Equation A-6-1 is employed for relative brace systems and Equation A-6-3 is used for nodal brace systems. If β_{act} is larger than β_{br}, the required brace strength is reduced. If β_{act} is less than the required stiffness from Equation A-6-2 or Equation A-6-4, as appropriate, the required brace strength is increased. However, in this case, the displacement at the brace point may be excessive under the ultimate load conditions. Therefore, the designer must either satisfy the minimum β_{br} requirements of Appendix 6 or check to ensure that the brace-point displacements are sufficiently small to avoid a significant reduction in the strength of the member being braced. This may be accomplished using the direct analysis method (DM) as discussed in the following section.

E.2.3 General Application of the AISC *Specification* Appendix 6 Relative and Nodal Bracing Force Requirements

The AISC relative and nodal bracing force requirements are based on an assumed $\Delta_{o,total} = 0.002L_b$, where $\Delta_{o,total}$ is taken as:

1. the total relative transverse displacement between adjacent brace points due to initial out-of-plumbness and/or out-of-straightness of the column, assumed within the tolerances specified in the AISC *Code of Standard Practice* (Δ_o), plus
2. any relative transverse displacements due to applied loads, determined from a first-order analysis of the ideal geometrically perfect structure (Δ_1).

The foregoing can be expressed as:

$$\Delta_{o,total} = \Delta_o + \Delta_1 \tag{E-3}$$

The Commentary to AISC *Specification* Appendix 6 uses the term Δ_o to represent $\Delta_{o,total}$. The notation $\Delta_{o,total}$ is used here to clarify the two sources of the relative transverse displacement between the brace points. In this Design Guide, Δ_o always represents solely the initial out-of-plumbness between adjacent points where a column is connected to other framing.

AISC *Specification* Equations A-6-1 and A-6-3, and Equation E-2 do not include the primary (first-order) force induced in the bracing due to the applied loadings on the structure, although they do account for the second-order amplification of the bracing forces necessary for equilibrium on the total deformed geometry of the structure. Thus, the bracing must be designed in general for a total force of:

$$P_{br,total} = P_{br1} + P_{br} \tag{E-4}$$

where

P_{br1} = bracing force determined by a first-order elastic analysis of the ideal geometrically perfect structure under the code required load combination considered in the calculation of P_{br}, and

P_{br} = stability bracing force determined from Equations A-6-1 or A-6-3 as appropriate, plus the use of Equation E-2 if desired to account for the effect of differences between β_{br} and β_{act}.

In some situations, such as when the stability bracing is a major lateral load resisting system, the stability bracing force, P_{br}, may be quite small relative to the forces P_{br1}. Also, a substantial fraction of $\Delta_{o,total}$ may be due to Δ_1. In other cases, such as a column in an isolated indoor structure subjected to little wind or seismic lateral load, or for lightly-braced truss chords, P_{br1} and Δ_1 may be small or zero.

If $\Delta_{o,total}$ is different than $0.002L$, the AISC *Specification* Commentary to Appendix 6, Section 6.1, states that the bracing force requirements are to be scaled in proportion to $\Delta_{o,total}/0.002L_b$. In situations where the bracing resists primary forces, P_{br1}, $\Delta_{o,total}$ will always be larger than $0.002L_b$ if one assumes an initial geometric imperfection of $\Delta_{o,total}/0.002L_b$ based on the tolerances in the AISC *Code of Standard Practice*.

E.2.4 Basis for the Relative Bracing Force Requirements

Given the base formulas for the stability bracing force (AISC *Specification* Commentary Equations A-6-1 and A-6-3), P_{br}, the adjustments to these formulas by Equation E-2 and by the multiplier $\Delta_{o,total}/0.002L_b$, and the final addition of this force to the first-order force, P_{br1}, in Equation E-4, the reader may indeed wonder what analysis problem is actually being solved in this Appendix. For the relative bracing problem illustrated in Figure E-1, the answer is: Equation E-4 is the approach involving the alternative application of the B_2 amplification factor explained in Section B.2, performed using the requirements of the DM. The force, P_{br}, given by the combination of AISC *Specification* Equation A-6-1 and Equation E-2 is the second-order P-Δ shear force, $\bar{H}_{P\Delta}$, obtained from Equations E-6 and E-7 in the following (note that the over bar on this and other quantities indicates that the quantity is affected by a reduction in stiffness such as that employed within the DM).

For example, if the DM is applied to determine the ASD force required to brace the axially loaded column of the relative bracing problem in Figure E-1, the second-order shear force is:

$$\bar{P}_{br} = \bar{H}_{P\Delta} = \frac{P_r}{L}\bar{\Delta}_{2tot} = \frac{P_r}{L}\bar{B}_2\left(\Delta_o + \bar{\Delta}_1\right) = \frac{P_r}{L}\bar{B}_2\left(\bar{\Delta}_{o,total}\right) \tag{E-5}$$

where

L = column length between the brace points, in.

$\bar{\Delta}_{2tot}$ = total second-order lateral displacement between the brace points (at the ultimate load level), including the effects from the nominal stiffness reduction employed in the DM, in.

$\bar{B}_2$ = sidesway amplifier

$$= \frac{1}{1-\dfrac{\alpha P_r}{0.8\Sigma HL/\Delta_H}}$$

$$= \frac{1}{1-\dfrac{1.6P_r/L}{0.8\Sigma H/\Delta_H}} \tag{E-6}$$

Δ_o = initial out-of-plumbness between the brace points, in.

$\bar{\Delta}_1$ = relative transverse displacement between the brace points due to applied loads (at the ultimate load level), determined from a first-order analysis of the ideal geometrically perfect structure and including the effects of the nominal stiffness reduction employed in the DM, in.

$\Delta_{o,total} = \Delta_o + \bar{\Delta}_1$, in.

The factor $\alpha = 1.6$ is included in Equation E-6 so that the second-order amplification is properly determined based on the ultimate load of $1.6P_r$. In addition, the 0.8 factor in this equation reflects the reduction in the elastic stiffness required by the DM. This factor accounts for the loss of some stiffness due to minor yielding and other incidental effects as the ultimate strength condition is approached. By utilizing the fundamental definition of the actual (nominal) bracing system stiffness as a total horizontal force, ΣH, applied at the brace point divided by the corresponding horizontal deflection between the brace points, Δ_H, the actual stiffness is:

$$\beta_{act} = \frac{\Sigma H}{\Delta_H} \tag{E-7}$$

and by utilizing the fundamental definition of the ideal bracing stiffness as the ultimate load corresponding to the sidesway buckling of the bracing system (i.e., the load level at which $\bar{B}_2$ becomes unbounded) divided by the unbraced length, the ideal stiffness is:

$$\beta_i = \frac{\alpha P_r}{L} = \frac{1.6P_r}{L} \tag{E-8}$$

Equation E-6 becomes:

$$\bar{B}_2 = \frac{1}{1-\dfrac{\beta_i}{0.8\beta_{act}}} \tag{E-9}$$

Correspondingly, Equation E-5 becomes:

$$\bar{P}_{br} = P_r\left(\frac{1}{1-\dfrac{\beta_i}{0.8\beta_{act}}}\right)\left(\frac{\bar{\Delta}_{o,total}}{L}\right) \tag{E-10}$$

Finally, if it is recognized that the relationship between the the ideal bracing stiffness, β_i, in Equation E-8 and the required bracing stiffness for ASD, β_{br}, in AISC *Specification* Equation A-6-2 is:

$$\beta_i = \frac{\alpha\beta_{br}}{2\Omega} = \frac{1.6\beta_{br}}{2(2)} \tag{E-11}$$

Equation E-10 may be written as:

$$\bar{P}_{br} = P_r \left(\frac{1}{1 - \dfrac{\beta_{br}}{2\beta_{act}}} \right) \left(\frac{\bar{\Delta}_{o,total}}{L} \right) \tag{E-12}$$

AISC *Specification* Equation A-6-1 is obtained by substituting the assumptions $\bar{\Delta}_{o,total}/L = 0.002$ and $\beta_{act} = \beta_{br}$ into Equation E-12. Also, for relative bracing, Equation E-2 includes the factor given in AISC *Specification* Commentary C-A-6-1, which is the ratio between the bracing force given by Equation E-12 and that given by AISC *Specification* Equation A-6-1, assuming $\bar{\Delta}_{o,total}/L = 0.002$. However, more importantly, Equation E-12 is the fundamental expression for the stability bracing force obtained by applying the DM to the relative bracing problem. The initial form of this equation, Equation E-5, states that the stability bracing force, P_{br}, is equal to the P-Δ shear caused by the axial force, P_r, acting through the total second-order sidesway displacement of the bracing system, $\bar{\Delta}_{2tot} = \bar{B}_2 \left(\Delta_o + \bar{\Delta}_1\right)$. Equation E-4 states that this force must be added to the first-order bracing force, P_{br1}, determined from a first-order analysis of the perfect structure under the same load combination, to obtain the total required bracing force.

If the foregoing analysis is conducted using LRFD loads, the process is the same except that $\alpha = 1.0$ and a stiffness reduction factor of 0.75 rather than 0.8 is needed to arrive exactly at the final Equation E-12. That is, Equation E-12 still applies. The difference in the results using the DM stiffness reduction of 0.8 rather than 0.75 are minor. In the traditional application of Equation E-4, the displacement, $\bar{\Delta}_{o,total} = \Delta_o + \bar{\Delta}_1$, in Equation E-12 is often taken equal to $0.002L$. Also, when the relative brace point displacements are calculated including the influence of the primary bracing forces, P_{br1}, the total displacement is determined traditionally by Equation E-3. That is, the influence of any stiffness reduction at the ultimate load level, caused by minor yielding or other incidental effects, is not considered in the calculation of Δ_1. The DM predicts a larger deflection associated with the primary bracing force of $\bar{\Delta}_1 = \Delta_1/0.8$, which accounts for these effects. In many cases, this in turn gives a slightly larger value of $\bar{P}_{br}$ from Equation E-12 compared to that determined using the traditional approach. An example is provided in Section E.3 to illustrate this behavior.

E.2.5 Basis for the Nodal Bracing Force Requirements

The DM may be applied in a similar fashion to the above for the analysis and design of nodal bracing systems. The only key differences relative to the above discussion are: (1) the contributions from the column bending rigidity, EI, are included in the analysis model, and (2) the selection of the appropriate shape of the initial geometric imperfection typically is more involved. These attributes are discussed in Section E.4 using several examples.

Equation A-6-3 actually includes an increase in the bracing force requirement by a factor of 1.25 from $0.008P_r$ to $0.01P_r$ to account for the influence of column initial curvature. When conducting an explicit second-order analysis of bracing systems by the DM, it is considered acceptable to neglect any initial curvature of the columns associated with the geometric imperfections, Δ_o. In the most extreme cases, an assumed initial sinusoidal geometric imperfection increases the total second-order brace point displacements relative to the initial geometry, as follows:

$$\bar{\Delta} = \overline{DAF}\left(\Delta_o + \bar{\Delta}_1\right) - \Delta_o \tag{E-13}$$

(and correspondingly the stability bracing forces, $\bar{P}_{br}$) by approximately 0.01/0.008 = 1.25 compared to the result using an analysis with a straight line representation of the imperfect geometry between the brace points, where $\overline{DAF}$ is the amplification factor for the brace point displacements. The column curvature associated with the displacements, $\bar{\Delta}$, is automatically included in an explicit second-order analysis in either case. For nodal bracing designed as full bracing (i.e., $L_q = L_b$), the influence of the initial curvature on the bracing forces is typically considered negligible. In these cases, the column is idealized at the ultimate load condition by inserting fictitious pins at the brace points (Winter, 1960; Nair, 1992). Based on this idealization, the additional 1.25 factor does not apply. For relative bracing problems, where the contribution of the column flexural rigidity is neglected entirely, the initial curvature of the column is not a consideration.

E.2.6 Implications of the Appendix 6 Relative and Nodal Bracing Stiffness Requirements on Brace-Point Deflections

For the analysis and design of relative bracing systems by ASD, the second of Equations A-6-2 ensures that the relative deflection, $\bar{\Delta}_{o,total}$, is never amplified by more than a factor of 2.0 based on a second-order analysis by the DM, using 0.8 of the nominal stiffness. This fact is demonstrated fundamentally by Equation E-12. That is, the additional relative deflection between the brace points due to the stability effects is:

$$\bar{\Delta} = \bar{B}_2\left(\Delta_o + \bar{\Delta}_1\right) - \Delta_o \qquad \text{(E-14)}$$

This deflection, determined using the reduced stiffness of the DM, will never be larger than $\bar{\Delta}_{o,total}$. For analysis and design of relative bracing systems by LRFD, the first of Equations A-6-2 results in an amplification of $\bar{\Delta}_{o,total}$ slightly smaller than 2.0 using an explicit second-order analysis conducted according to the DM requirements (due to the small difference between $\phi = 0.75$ and the DM stiffness reduction of 0.8). For nodal bracing problems, Equations E-1a result in an amplification of the brace point displacements approximately equal to 2.0 in an explicit second-order analysis conducted using the DM (under ASD or LRFD). The displacement amplification tends to be slightly smaller in nodal bracing problems versus relative bracing problems though, due to the contributions from the column flexural rigidity and due to the conservatism associated with the practical approximation of substituting L_q for L_b in Equations E-1a.

E.2.7 Recommendations for Applying Second-Order Elastic Analysis or the Appendix 6 Equations for Bracing Design

Based on the preceding discussion, if an explicit second-order analysis by the DM is employed for the analysis and design of relative or nodal bracing (using a straight-line geometric imperfection pattern between the brace locations), the additional displacement at the brace points due to the stability effects, $\bar{\Delta}$, is never larger than $\bar{\Delta}_{o,total}$ when the AISC *Specification* Appendix 6 requirements are met. In problems where the primary bracing forces, P_{br1}, are zero, it is possible to conveniently check that $\bar{\Delta} \leq \Delta_o$ to ensure that the intent of the AISC *Specification* Appendix 6 bracing provisions is satisfied. For nodal bracing problems, the Appendix 6 bracing stiffness requirements tend to limit the brace point displacements conservatively to slightly smaller values, thus ensuring that the bracing force requirements are also adequate. If an accurate second-order analysis is used, configured as defined by the DM, the brace point displacements are computed more accurately and the bracing stiffness requirement from Equation E-1a may be relaxed. This is allowed explicitly in Section 6.2 of the AISC *Specification* Commentary to Appendix 6, which points to Ziemian (2010) and Lutz and Fisher (1985) for more accurate bracing formulas. These more accurate formulas are indeed determined from a rigorous second-order analysis. A rigorous second-order analysis shows in fact that, given a specified column and axial load level, the ideal nodal bracing stiffness, β_i, indeed becomes smaller as the number of nodal braces is increased. Figure E-2 compares the exact solution for the critical load of a column with one or two intermediate nodal braces from Winter (1960) to the β_i approximation associated with Equations

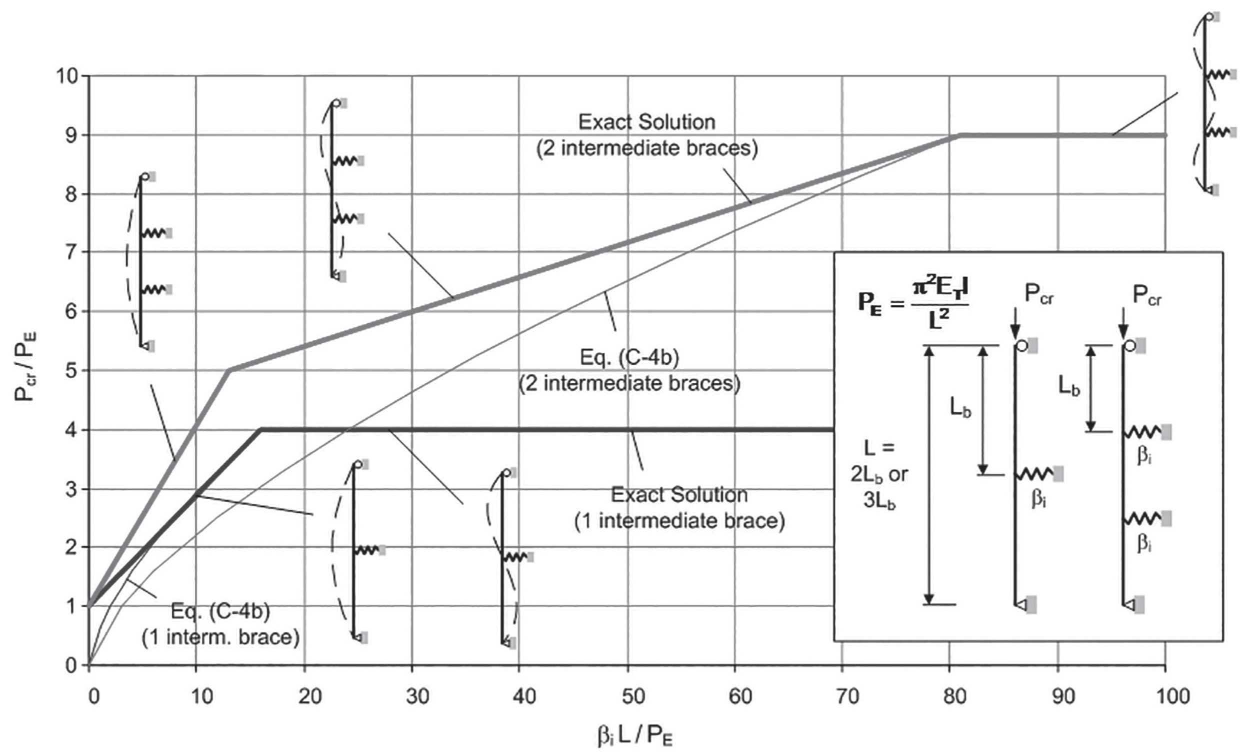


Fig. E-2. Comparison of the exact solution for the system buckling load, P_{cr}*, versus the ideal bracing stiffness,* β_i *(Winter, 1960) to the underlying approximate solution in Equations E-1a for a partially or fully braced column with one or two intermediate nodal braces.*

E-1a. The paradoxical trend of β_{br} increasing with larger n values in Equations E-1a (for a given value of P_r), where $\beta_{br} = 2\beta_i/\phi$ (LRFD) or $2\Omega\beta_i$ (ASD), is due to the simplifications invoked in the development of these practical design expressions.

When applying AISC *Specification* Equations A-6-1 through A-6-4 and E1 through E4, the designer must be aware that these equations are expressed in the context of bracing for a single column. If the bracing system stabilizes multiple columns, AISC *Specification* Equations A-6-1 through A-6-4 and Equations E1 through E4 must be used with:

1. the sum of the axial forces, ΣP_r, from all of the columns, as well as,
2. an appropriate average $\Delta_{o,total}$ weighted in proportion to the vertical load on each of the columns or, alternatively, the maximum $\Delta_{o,total}$.

Also, the designer must keep in mind that the bracing force determined from the equations discussed here is actually a transverse force normal to the axis of the column. This force must be developed within the bracing system. When diagonal bracing is employed, the diagonal bracing force must be resolved from this transverse force based on equilibrium.

One other consideration applies to relative bracing at multiple points along a column. For relative bracing, the brace stiffness and strength requirements are applied on a tier-by-tier (or panel-by-panel) basis. That is, the strength requirements actually give the P-Δ shear force that must be resisted within a given tier or panel. Conversely, the nodal bracing strength equation, AISC *Specification* Equation A-6-3, gives the absolute force normal to the column that must be resisted at a given bracing location.

Lastly, the nodal bracing equations are based on equal brace spacing and constant internal axial force along the length of the column(s). AISC *Specification* Equations A-6-3 and A-6-4 and Equations E-1 through E-4 given in this Design Guide should not be applied to unequally spaced nodal bracing. If intermediate axial loads are applied along the length of the column(s), Equations A-6-1 through A-6-4 and Equations E-1 through E-4 can be employed conservatively by using the largest internal axial force for P_r. They can be applied approximately using the average axial force from the two adjacent unbraced segments.

In many respects, the explicit use of a second-order analysis is much simpler than the application of the AISC *Specification* Appendix 6 equations with all of these caveats. If a second-order analysis is used to evaluate the bracing requirements, the designer only needs to specify Δ_o. The transverse deflections, Δ_1, as well as the second-order amplification of $\Delta_{o,total} = \Delta_o + \Delta_1$ are handled explicitly within the second-order analysis. Also, the handling of aspects such as the stability effects from the ΣP_r on all of the columns, nonuniform brace spacing, or nonuniform column axial loading are handled by properly including these effects in the second-order analysis model. In many cases, the use of an explicit second-order analysis gives results that have better accuracy than the Appendix 6 equations, even when the adjustments to these equations as explained in the Commentary are applied.

On the other hand, the Appendix 6 equations are typically more useful than an explicit second-order analysis for routine preliminary design. This is because estimates of the stiffness of the structure are needed in addition to the applied loadings and the overall structure geometry before a second-order analysis can be conducted. These estimates are built into the implicit brace-point displacement amplification factor of 2.0 in Equation A-6-1 and 2.0(1.25) = 2.5 in Equation A-6-3. The stability bracing forces are fundamentally equal to the bracing stiffness times the amplified brace-point displacements, $\bar{\Delta}$, adjusted by α, i.e.:

$$\bar{P}_{br} = \bar{\beta}_{act}\bar{\Delta}/\alpha \tag{E-15}$$

where

$$\bar{\Delta} = \bar{\Delta}_{2tot} - \Delta_o$$

$$\bar{\Delta}_{2tot} = \overline{DAF}\left(\Delta_o + \bar{\Delta}_1\right) = \overline{DAF}\left(\bar{\Delta}_{o,total}\right)$$

and $\overline{DAF}$ is a general displacement amplification factor applicable for either relative or nodal bracing. The brace stiffness, $\bar{\beta}_{act}$, is implicitly taken equal to $2\alpha P_r/L_b$ and $4\alpha P_r/L_b$ in AISC *Specification* Equations A-6-1 and A-6-3 respectively. Also, the displacement, $\bar{\Delta}_{o,total}$, is implicitly taken as $0.002L_b$ and $\overline{DAF}$ is taken as 2.0 in these equations. For ASD, the analysis is implicitly conducted at α times the ASD load level, where $\alpha = 1.6$, and the internal force results are subsequently divided by α.

Various calculations from the above equations and from an explicit second-order analysis are illustrated for representative relative and nodal bracing problems in Sections E.3 though E.5 of this Design Guide. In addition, White et al. (2007a) show the analysis and design calculations for a specific relative bracing problem using the DM.

E.2.8 Continuous Bracing

Continuous column bracing, as illustrated in Figure E-1, can be depicted as a series of closely spaced independent springs placed uniformly along the span. This problem is solved by Timoshenko and Gere (1961) and is discussed by Winter (1960), Lutz and Fisher (1985), Yura (1995), and Ziemian (2010). Usually, in practical bracing problems where a line of braces occur, the member to be braced is connected to a system in which the braces are not truly independent but are related or tied to each other (such as a compression member braced continuously or at various locations along its length by a beam, a truss or a diaphragm). The classical solutions for continuous bracing can be thought of as a series of closely spaced nodal braces—indeed, the solutions merge into one and the same as the number of braces increases. However, in these practical situations, the effective bracing stiffness provided at the various brace points is not constant along the member length. An explicit second-order analysis of the actual structure accounts for this attribute of the system behavior. Many of these practical situations may be considered as lean-on bracing problems as discussed in the following.

E.2.9 Lean-on Bracing

Lean-on bracing occurs where one or more columns are braced by another system, either a column or frame, such that buckling can not occur until the combined system buckles. This concept (also known as the ΣP concept) was first developed by Yura (1971, 1995) and is illustrated in Figure E-1. However, as may be demonstrated for the above relative, nodal and continuous bracing problems, a buckling analysis (elastic or inelastic) as implied by the lean-on bracing approach does not provide any information about the bracing forces needed to stabilize the individual members. That is, it does not provide any information about the forces in the imperfect structure as the stability limit is approached. The DM provides this information in a consistent fashion that applies across all types of systems, including combinations and hybrid forms of the four bracing system types. Section E.5 presents the solution of a representative lean-on bracing problem using the DM.

E.3 RELATIVE BRACING EXAMPLE

This design example is implemented using the 2005 AISC *Specification* and the 13th Edition AISC *Manual*.

Given:

Figure E-3 is a representation of the most basic relative bracing problem. In fact, this is the simplest type of bracing problem upon which Equations A-6-1 and A-6-2 are based. The 14-ft-long W14×90 column shown in the figure is used to support a gravity load from an LRFD load combination of P_u = 1,000 kips. This column is a "gravity column" stabilized by a separate lateral load resisting system. The lateral load resisting system is subjected to a horizontal load of H = 5 kips from the same LRFD load combination used to calculate P_u. This system is represented by the grounded lateral spring with a nominal stiffness, β_{act}, in the figure. It is assumed that the lateral load resisting system was proportioned such that its stiffness is exactly equal to the requirement from Equations A-6-2. That is, the sidesway stiffness of the lateral load resisting system is:

$$\begin{aligned}\beta_{act} &= \beta_{br}\\ &= \frac{1}{\phi}\left(\frac{2P_r}{L_b}\right) \qquad (\textit{Spec}.\ \text{Eq. A-6-2})\\ &= \frac{1}{0.75}\left[\frac{2\ (1{,}000\ \text{kips})}{(14.0\ \text{ft})(12\ \text{in./ft})}\right]\\ &= 15.9\ \text{kip/in.}\end{aligned}$$

Determine the strength requirements for the lateral load resisting system and for the member tieing the column to this system shown in Figure E-3 using: (a) an explicit second-order analysis conducted according to the DM, and (b) using the 2005 AISC *Specification* Appendix 6 equations. This example provides a useful illustration of the fundamental application of a second-order analysis by the DM and points out the differences between the Appendix 6 approach and the DM approach for solving relative bracing problems.

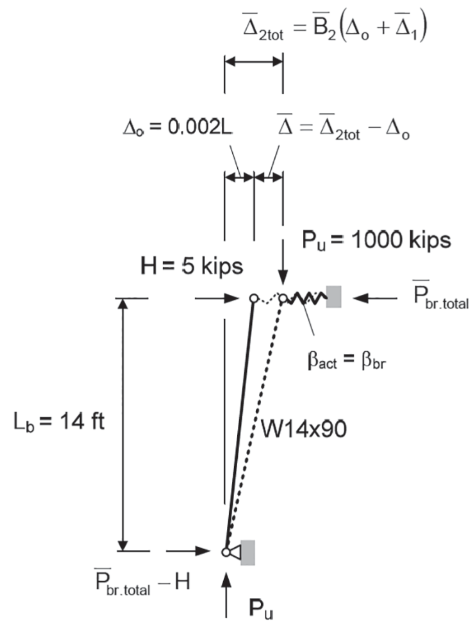


Fig. E-3. Relative bracing example.

Solution:

(a) For the second-order analysis and design solution by the DM, the following specific steps are used:

1. Perform a first-order analysis of the ideal geometrically perfect structure. This analysis gives a first-order bracing system force, P_{br1}, in Equation E-4, equal to 5 kips as well as the column axial force of P_u = 1,000 kips.

2. Determine the total first-order story lateral displacement, $\bar{\Delta}_1$. The reduced sidesway stiffness of the lateral load resisting system is:

$$\begin{aligned}\bar{\beta}_{act} &= 0.8\beta_{act}\\ &= 0.8(15.9 \text{ kip/in.})\\ &= 12.7 \text{ kip/in.}\end{aligned}$$

assuming that none of the members in this system are flexural members with $P_u > 0.5P_y$. Therefore, the first-order lateral displacement of the brace point can be determined from Equation E-7:

$$\begin{aligned}\bar{\Delta}_1 &= \frac{H}{\bar{\beta}_{act}}\\ &= \frac{5 \text{ kips}}{12.7 \text{ kip/in.}}\\ &= 0.394 \text{ in.}\end{aligned}$$

3. Calculate the story sidesway amplification factor, $\bar{B}_2$. The ideal bracing stiffness needed for calculation of $\bar{B}_2$ using Equation E-8 is:

$$\begin{aligned}\beta_i &= \frac{P_u}{L}\\ &= \frac{1{,}000 \text{ kips}}{(14 \text{ ft})(12 \text{ in./ft})}\\ &= 5.95 \text{ kip/in.}\end{aligned}$$

Based on Equation E-9:

$$\begin{aligned}\bar{B}_2 &= \frac{1}{1-\dfrac{\beta_i}{0.8\beta_{act}}}\\ &= \frac{1}{1-\dfrac{5.95 \text{ kip/in.}}{12.7 \text{ kip/in.}}}\\ &= 1.88\end{aligned}$$

4. Calculate and apply the P-Δ shear force, $\bar{H}_{P\Delta}$, in a separate first-order elastic analysis of the structure. The force $\bar{H}_{P\Delta}$ is simply the overturning effect of the axial load, P_u, acting through the total second-order sidesway displacement, $\bar{\Delta}_{2tot}$:

$$\begin{aligned}\bar{\Delta}_{2tot} &= \bar{B}_2\left(\Delta_o + \bar{\Delta}_1\right)\\ &= 1.88\left[0.002(14 \text{ ft})(12 \text{ in./ft}) + 0.394 \text{ in.}\right]\\ &= 1.37 \text{ in.}\end{aligned}$$

Given this total sidesway displacement, the second-order P-Δ shear force is:

$$\bar{H}_{P\Delta} = \frac{P_u \bar{\Delta}_{2tot}}{L} \tag{E-5}$$

$$= \frac{(1{,}000 \text{ kips})(1.37 \text{ in.})}{(14 \text{ ft})(12 \text{ in./ft})}$$

$$= 8.15 \text{ kips}$$

For this example, the application of $\bar{H}_{P\Delta}$ simply produces an equal horizontal force in the lateral load resisting system.

5. Add the second-order internal forces obtained from Step 4 to the first-order forces determined in Step 1. This is Equation E-4 with $\bar{P}_{br}$ from the reduced stiffness model substituted for P_{br}, or

$$\begin{aligned} \bar{P}_{br,total} &= P_{br1} + \bar{P}_{br} \\ &= H + \bar{H}_{P\Delta} \\ &= 5 \text{ kips} + 8.15 \text{ kips} \\ &= 13.2 \text{ kips} \end{aligned}$$

It is important to understand that $\bar{P}_{br,total}$ is simply the force required to maintain equilibrium in the total deflected position of the structure shown in Figure E-3. In fact, if one has an acceptable estimate of the total second-order lateral displacement of the brace point, $\bar{\Delta}_{2tot}$, by any means, the bracing force is obtained simply from equilibrium on the free-body diagram of the deflected structure shown in Figure E-3. The explicit second-order analysis by the DM gives an accurate estimate of the interrelationship between the system lateral stiffness, the displacement, $\bar{\Delta}_{2tot}$, and the system bracing force, $\bar{P}_{br,total}$.

(b) If the AISC *Specification* Appendix 6 LRFD strength requirement in Equation A-6-1 is applied directly, the required brace strength is as follows (after adjusting for the fact that $\Delta_{o,total} = \Delta_o + \Delta_1$ is greater than $0.002L$, where $\Delta_o = 0.002L = 0.336$ in. and $\Delta_1 = 5$ kips/15.9 kip/in. = 0.314 in.):

$$\begin{aligned} P_{br} &= 0.004 P_u \left(\frac{\Delta_{o.total}}{0.002L} \right) \\ &= (0.004)(1{,}000 \text{ kips}) \left(\frac{0.336 \text{ in.} + 0.314 \text{ in.}}{0.336 \text{ in.}} \right) \\ &= 7.74 \text{ kips} \end{aligned}$$

and therefore:

$$\begin{aligned} P_{br,total} &= P_{br1} + P_{br} \\ &= 5 \text{ kips} + 7.74 \text{ kips} \\ &= 12.7 \text{ kips} \end{aligned}$$

The total bracing force calculated by the DM is slightly larger than obtained from the Appendix 6 equations. This is due to the larger estimate of the deflection $\bar{\Delta}_1 = \Delta_1 / 0.8$ in Step 2, which accounts for the influence of minor yielding and other incidental effects at the strength limit. If the applied horizontal load, H, were equal to zero in this problem, the second-order analysis by the DM gives $P_{br,total} = P_{br} = 3.76$ kips versus $P_{br,total} = P_{br} = 4.00$ kips from the Appendix 6 equations. As discussed previously in Section E.2, if ASD loadings were specified for this problem, the DM and the Appendix 6 equations would give identical results for this case. The minor differences between the above calculations are considered acceptable by the AISC *Specification*.

It is interesting to note that the brace in the above example could be a horizontal strut tied to a structural wall, or it could be the combination of the column and diagonal brace as shown for the relative bracing illustration in Figure E-1. In the first case, the fact that the brace is a horizontal strut actually does not make it a nodal brace. The bracing of a single-story simply supported column is always a relative bracing problem.

E.4 NODAL BRACING EXAMPLE

This design example is implemented using the 2005 AISC *Specification* and the 13th Edition AISC *Manual*.

Given:

Figure E-4 shows a representative nodal bracing problem. The wide-flange column shown in the figure is located in the middle of a mechanical shaft and supports a required strength of P_u = 1,400 kips. Horizontal bracing members frame into the column in the weak-axis bending direction at each of the floor levels of the building, which are at 15-ft intervals along the column length. However, clearance is not available to provide any bracing over the entire 60 ft height of the column in the strong-axis bending direction. The efficiency of the design is improved by providing a "flying beam" AB across the shaft to restrain the column major-axis bending lateral deflections at its mid-height as shown in Figure E-4(b). A moment connection is made through the column with the splice being provided in the anticipated lighter weight flying beam member. The flying beam spans 30 ft across the width of the shaft and is simply supported at its ends by a relatively rigid wall system. The flying beam is also a wide-flange section and it is turned such that its strong-axis bending stiffness and strength braces the column.

For purposes of using a planar structural analysis, and for ease of sketching the behavior of both the column and bracing beam, the bracing beam AB is turned such that it spans vertically within the plane of the page and it is connected to the column by a rigid strut for the modeling and analysis. This idealization is shown in Figure E-4(b). The bending moments at the LRFD load level for the final design of the column and the bracing beam are shown in Figure E-4(c).

Using the LRFD method select a W14 ASTM A992 steel column to support the load P_u using KL_y = 15 ft and KL_x = 30 ft. Once a suitable column size is selected, it is desired to then select a W14 ASTM A992 bracing beam based on the stiffness requirement from Equation E-1b, assuming that this requirement governs the bracing beam design. Given the selected bracing beam size, the problem then is to check the bracing force requirements by two approaches:

(a) using the combination of Equation A-6-3 from AISC *Specification* Appendix 6 and Equation E-2, and

(b) using an explicit second-order elastic analysis.

Lastly, the bracing beam design is refined using the more accurate calculations from explicit second-order analyses. The influence of the bracing beam stiffness and strength on the behavior of the combined column and bracing beam system is illustrated and broader design considerations are discussed.

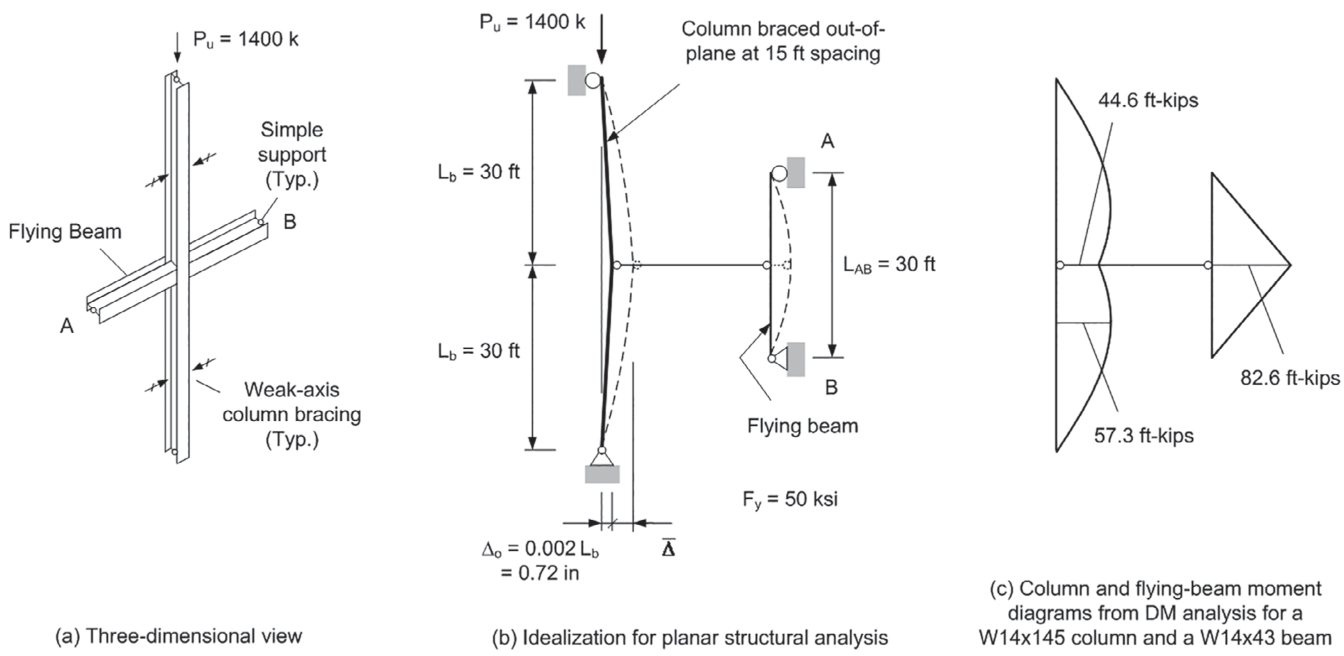


Fig. E-4. Nodal bracing example.

Solution:

From AISC *Manual* Table 2-3, the material properties are as follows:

ASTM A992
$F_y = 50$ ksi
$F_u = 65$ ksi

Column Design

Using AISC *Manual* Table 4-1, select the lightest wide-flange section for the given effective length and loading. Given the unbraced lengths, enter the table with the larger of $KL_y = 15$ ft and an equivalent $(KL)_{yeq} = (KL)_x/(r_x / r_y) = 30 \text{ ft}/1.59 = 18.9$ ft. Select a W14×145 column ($\phi_c P_n = 1{,}510$ kips, $A = 42.7$ in.², $I_x = 1{,}710$ in.⁴) as the lightest wide-flange section.

Bracing Beam Selection Based on Stiffness

Because $\phi_c P_n = 1{,}510$ kips is significantly larger than $P_u = 1{,}400$ kips, it is beneficial to back-calculate the unbraced length $KL_x = L_q$ corresponding to $\phi_c P_n = P_u = 1{,}400$ kips and to use this length in Equation E-1b. $KL_x = L_q$ can be determined by working directly with the AISC *Specification* Equations E3-1, E3-2 and E3-4 and solving for the length KL_x that satisfies the above condition. Alternatively, the effective length corresponding to $\phi_c P_n = 1{,}400$ kips can be determined from Table 4-1 of the AISC *Manual* for the W14×145 column to be 21.9 ft $= L_q/(r_x/r_y)$. Then $L_q = (21.9 \text{ ft})(1.59)(12 \text{ in./ft}) = 418$ in. Using this value for L_q, Equation E-1b gives a nodal bracing stiffness requirement of:

$$\begin{aligned}
\beta_{br} &= \frac{1}{\phi}\left(4\frac{P_r}{L_q}\right) \\
&= \frac{1}{0.75}\left[4\left(\frac{1{,}400 \text{ kips}}{418 \text{ in.}}\right)\right] \\
&= 17.9 \text{ kip/in.}
\end{aligned}$$

The elastic stiffness of the bracing beam is given by:

$$\begin{aligned}
\beta_{act} &= \frac{48EI_{AB}}{L_{AB}^3} \\
&= \frac{48(29{,}000 \text{ ksi})I_{AB}}{\left[(30 \text{ ft})(12 \text{ in./ft})\right]^3} \\
&= 0.0298 I_{AB}
\end{aligned}$$

By relating this stiffness to the required bracing stiffness, the required moment of inertia of the bracing beam is:

$$\begin{aligned}
I_{AB} &> \frac{\beta_{br}}{0.0298} \\
&> \frac{17.9 \text{ kip/in.}}{0.0298 \text{ kip/in.}^5} \\
&> 601 \text{ in.}^4
\end{aligned}$$

From AISC *Manual* Table 3-3, a W14×61 is selected as the lightest weight W14 section that satisfies this stiffness criterion with $I_x = 640$ in.⁴ From Table 3-2, for a W14×61, $\phi_b M_p = (383 \text{ kip-ft})(12 \text{ in./ft}) = 4{,}600$ kip-in. Assuming a fully restrained moment connection through the column, this member provides an actual (nominal) bracing stiffness of $\beta_{act} = (0.0298 \text{ kip/in.}^5)(640 \text{ in.}^4) = 19.1$ kip/in.

Check of Bracing Beam for Strength

The next step of the design solution is to check whether the W14×61 bracing beam has sufficient strength according to the AISC

Specification Appendix 6 bracing strength requirements. The combination of Equations A-6-3 and E-2 gives:

$$
\begin{aligned}
P_{br} &= \frac{0.01P_u}{2-\dfrac{\beta_{br}}{\beta_{act}}} \\
&= \frac{0.01(1,400 \text{ kips})}{2-\dfrac{17.9 \text{ kip/in.}}{19.1 \text{ kip/in.}}} \\
&= 13.2 \text{ kips}
\end{aligned}
$$

Assuming that the column braces the beam AB against lateral-torsional buckling at its mid-length, and using C_b = 1.67 from AISC *Manual* Table 3-1 for a linear moment diagram with lateral bracing at the center load point, the resistance of the W14×61 bracing beam is governed by its plastic moment strength, $\phi_b M_n = \phi_b M_p = 383$ kip-ft. This can be determined from Table 3-10 of the AISC *Manual* by adjusting the available flexural strength given for L_b = 15 ft for C_b = 1.67; then $\phi_b M_n$ = 1.67(335 kip-ft) = 559 kip-ft. The available flexural strength is limited to $\phi_b M_p$ = 383 kip-ft according to AISC *Specification* Equation F2-2.

Therefore, the strength of the bracing beam may be checked as follows:

$$
\frac{P_{br}L_{AB}}{4} \le \phi_b M_p
$$

Solving for P_{br}:

$$
\begin{aligned}
13.2 \text{ kips} &\le \frac{4\phi_b M_p}{L_{AB}} \\
&\le \frac{4(383 \text{ kip-ft})}{30 \text{ ft}} \\
&\le 51.1 \text{ kips}
\end{aligned}
$$

The W14×61 satisfies the AISC *Specification* Appendix 6 strength requirements.

Check of Bracing Beam for Strength Using the DM and an Explicit Second-Order Analysis, Followed by a Refinement of the Design

Figure E-5 shows the column load versus the brace-point deflection amplification factor, $\overline{DAF} = (\bar{\Delta} + \Delta_o)/\Delta_o = 1 + \bar{\Delta}/\Delta_o$ for the column and beam sizes predicted by an explicit second-order analysis configured according to the DM requirements and using the initial out-of-plumb shape shown in Figure E-4. This is done to directly compare the solution for the bracing force by the DM to the solution using the Appendix 6 requirement from Equations A-6-3 and E-2. However, solutions are also shown for the W14×145 column with a W14×53 bracing beam (from AISC *Manual* Tables 3-2 and 3-3: I_x = 541 in.^{4}, $\bar{\beta}_{act} = 0.8\beta_{act} = 12.9$ kip/in., $\phi_b M_p$ = 327 kip-ft), a W14×43 bracing beam (I_x = 428 in.^{4}, $\bar{\beta}_{act} = 0.8\beta_{act} = 10.2$ kip/in., $\phi_b M_p$ = 261 kip-ft), and a bracing stiffness $\bar{\beta}_{act}$ equal to the rigorous ideal bracing stiffness corresponding to P_u = 1,400 kips, $\bar{\beta}_i$ = 5.32 kip/in. (the solution for $\bar{\beta}_i$ = 5.32 kip/in. is discussed subsequently). The first two of these solutions are based on the following alternative design criteria that can be employed with a DM second-order analysis:

1. Limiting the additional brace-point deflection to $\bar{\Delta} < \Delta_o$, and
2. Simply checking that all the components have adequate strength based on the force requirements determined from the DM second-order analysis (with the stiffness requirements defined implicitly within the satisfaction of the member second-order force requirements, i.e., no explicit stiffness check).

The significance of the W14×53 bracing beam is that this lighter member satisfies the intent of the Appendix 6 bracing provisions discussed previously in Section E.2. That is, it limits the brace-point deflection, $\bar{\Delta}$, (calculated using the stiffness reduction specified by the DM) to a value slightly smaller than the initial Δ_o, i.e., the alternative design criterion (1). The significance of the W14×43 bracing beam is that this even lighter member is sufficient to maintain the unity check for the W14×145 column to a value slightly less than 1.0 at P_u = 1,400 kips, i.e., the alternative design criterion (2) in the preceding discussion. The specific calculations for these DM solutions are explained in the following. As discussed previously, the ideal stiffness β_i is the bracing stiffness corresponding to incipient buckling of the combined column and its bracing system at the required strength, P_u. For relative bracing systems, this stiffness is a function of only the required strength, P_u, and the unbraced length, L_b. However, for nodal bracing, the rigorous ideal bracing stiffness depends on the column flexural rigidity; hence, the value $\bar{\beta}_i$ based on the DM reduced stiffness is specified here. The solution with $\bar{\beta}_{act} = \bar{\beta}_i$ shows that the displacements become excessive well before reaching the design load level. If the bracing beam is sized with just enough stiffness to reach P_u at buckling, i.e., if $\bar{\beta}_{act} = \bar{\beta}_i$, the flying beam will fail in lateral bending due to the second-order forces transmitted from the column.

One can observe that the $\overline{DAF}$ is approximately equal to 2.0 and thus $\bar{\Delta} \cong \Delta_o$ at $P = P_u = 1{,}400$ kips when the W14×53 bracing beam is used. However, the $\overline{DAF}$ is only slightly larger than 2.0 ($\overline{DAF} = 2.5$ and $\bar{\Delta} = 1.5\Delta_o$) for the W14×43 bracing beam. The W14×61 selected using the Appendix 6 equations is an acceptable conservative solution relative to the underlying philosophy of the Appendix 6 rules ($\overline{DAF} = 1.76$ and $\bar{\Delta} = 0.76\Delta_o$ at $P = P_u = 1{,}400$ kips). However, substantial savings are possible in the size of the bracing beam by using the more accurate explicit second-order analyses by the DM. For the solution with $\bar{\beta}_{act} = \bar{\beta}_i$, the displacements are excessive well before the design load level is reached. One clearly cannot design bracing systems in general by simply providing just enough stiffness such that P_u is less than or equal to the system buckling load.

The details of the DM analysis model used in developing Figure E-5 are as follows. The yield load in the W14×145 column, $P_y = (42.7\text{ in.}^2)(50\text{ ksi}) = 2{,}140$ kips. Therefore, $P_u/P_y = 1{,}400\text{ kips}/2{,}140\text{ kips} = 0.654$ for the example column, causing the stiffness reduction parameter, τ_b, to be less than 1.0, where τ_b is defined in AISC *Specification* Appendix 7, Section 7.3, as follows:

$$
\begin{aligned}
\tau_b &= 4\frac{\alpha P_u}{P_y}\left(1-\frac{\alpha P_u}{P_y}\right) \\
&= 4(1.0)(0.654)\left[1-1.0(0.654)\right] \\
&= 0.905
\end{aligned}
$$

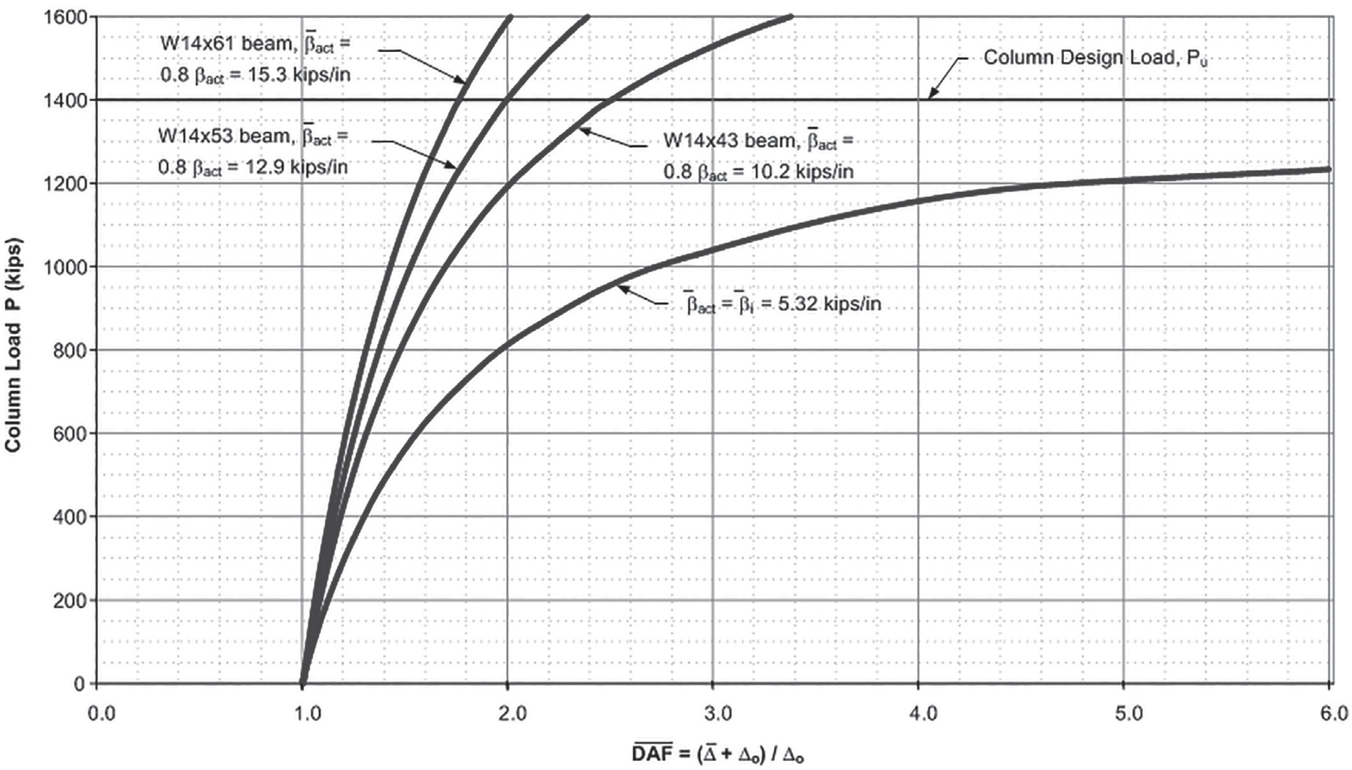


Fig. E-5. Column load versus brace-point deflection amplification factor using a DM based second-order analysis model.

As a result, the effective column flexural rigidity is reduced by $0.8\tau_b = (0.8)(0.905) = 0.724$. All other contributions to the stiffness, including the bracing beam stiffness, are reduced by 0.8 in the DM analysis model. The straight-line or "kinked" column geometric imperfection shown in Figure E-4, with $\Delta_o = (0.002)(360 \text{ in.}) = 0.72$ in., is applied explicitly in the DM model of the structure. The effect of selecting a sinusoidal geometric imperfection with a brace-point amplitude equal to Δ_o is discussed below. The selection of appropriate geometric imperfections for more complex nodal bracing problems is discussed subsequently.

For cases where P_u is exactly equal to $\phi_c P_n$ based on $KL = L_b$, the column resistance may be assumed to be completely "used up" by the buckling between the brace locations. Correspondingly, the ideal brace stiffness may be obtained as (Winter, 1960):

$$\beta_i = \frac{2P_u}{L_b}$$

(note that the member required axial strength, P_u ($\leq \phi_c P_n$), is substituted for the buckling load, P_e, in Winter's analysis). However, for P_u ($< \phi_c P_n$) based on $KL = L_b$, the ideal bracing stiffness (i.e., the bracing stiffness that results in buckling of the column and its bracing system at the load level P_u) is smaller than specified by this equation, since a portion of the column bending rigidity ($0.8\tau_b EI$ in the DM solution) is available to help resist the brace-point deflections. As discussed previously, Equations E-1a and E-1b account for this behavior by using L_q rather than L_b in their denominator. This is a reasonable design approximation in many situations as illustrated in Figure E-2. However, when the explicit second-order analysis approach is used, the "exact" stability behavior of the DM model is captured. The rigorous $\bar{\beta}_i$ for a single nodal brace at the mid-length of a column is approximated well using the following equation from Lutz and Fisher (1985) (the over bar is shown to emphasize that this parameter is affected by the DM stiffness reduction):

$$\begin{aligned}
\bar{\beta}_i &= \frac{16}{3}\left[\frac{P_u - \pi^2(0.8\tau_b EI)/(2L_b)^2}{2L_b}\right] \\
&= \frac{16}{3}\left[\frac{1{,}400 \text{ kips} - \pi^2(0.8)(0.905)(29{,}000 \text{ ksi})(1{,}710 \text{ in.}^4)\big/\left[2(30 \text{ ft})(12 \text{ in./ft})\right]^2}{2(30 \text{ ft})(12 \text{ in./ft})}\right] \\
&= 5.31 \text{ kip/in.}
\end{aligned}$$

Figure E-6 shows the variation in the load transferred to the bracing beam, $\bar{P}_{br}$, for the four designs considered in Figure E-5. The solutions in Figure E-6 illustrate that even for the W14×43 bracing beam, where $\bar{\beta}_{act} = 10.2$ kip/in. is approximately equal to $2\bar{\beta}_i$, the bracing force $P_{br} = 11.0$ kips at $P = P_u = 1{,}400$ kips is well below the design resistance of the flying beam of 34.8 kips.

Figures E-7 through E-9, respectively, show how the amplification of the brace point lateral deflection, $(\bar{\Delta} + \Delta_o)/\Delta_o$, the bracing force, $\bar{P}_{br}$, and the column strength unity check by the AISC beam-column interaction Equation H1-1a vary as a function of the reduced brace stiffness, $\bar{\beta}_{act} = 0.8\beta_{act}$, for the example problem. Figure E-7 shows that as $\bar{\beta}_{act}$ is reduced below the value of approximately $2\bar{\beta}_i$, the brace point displacements start to increase dramatically with only small changes in the bracing stiffness. However, at $\bar{\beta}_{act} = 2\bar{\beta}_i$, $(\bar{\Delta} + \Delta_o)/\Delta_o$ is only slightly larger than 2.0. For relative bracing problems (i.e., solutions in which the contribution of the column EI to the resistance of the brace-point displacements is neglected), the DAF is exactly equal to 2.0 when $\bar{\beta}_{act} = 2\bar{\beta}_i$. The relationship between the DAF and $\bar{\beta}_{act}$ is more complicated for nodal bracing problems due to the interaction between the bracing stiffness and the column flexural rigidity.

The dramatic increases in the brace point lateral displacement as $\bar{\beta}_{act}$ approaches $\bar{\beta}_i$ translate into the dramatic increases in the bracing force requirement, $\bar{P}_{br}$, shown in Figure E-8. However, for the W14×53 flying beam, the calculated force, $\bar{P}_{br}$, is smaller than given by the combination of Equations A-6-3 and E-2, where $(\bar{\Delta} + \Delta_o)/\Delta_o = 2.0$ at $P = P_u = 1{,}400$ kips. The bracing force, $\bar{P}_{br}$, is equal to 9.2 kips at this displacement level versus the estimate of 15.7 kips from the earlier equation given for P_{br} (using $\beta_{act} =$ 12.9 kip/in. / 0.8 = 16.1 kip/in. for the W14×53 beam). There are two reasons for this difference between the explicit second-order analysis solution and the result from Equations A-6-3 and E-2:

1. The column is providing some of the resistance to the brace point deflection, and
2. The explicit second-order analysis solution shown here is based on a straight-line geometric imperfection between the brace points.

Since the brace point deflection is partially resisted by the column in flexure, the amount of force transferred to the bracing for a given brace point deflection is smaller than if the column were in effect pinned at the brace point in this problem. Also, the solution neglecting any initial curvature of the column leads to smaller internal moments along the column length and smaller bending deformations of the column.

For the case with the W14×43 bracing beam ($\bar{\beta}_{act}$ = 10.2 kip/in.), $\bar{P}_{br}$ is increased from 11.0 kips to 13.8 kips, $(\bar{\Delta}+\Delta_o)/\Delta_o$ is increased from 2.50 to 2.88, $M_u/\phi_b M_p$ in the column is increased from 0.077 to 0.107, and the beam-column strength unity check is increased from 0.992 to 1.02 when the initial geometric imperfection is modeled as a sine curve with an amplitude of $\Delta_o = 0.002L_b$ rather than a straight-line pattern between the brace points. Interestingly, the ratio between the $\bar{P}_{br}$ values with and without initial curvature included is about the same (approximately 1.25) for all the practical $\bar{\beta}_{act}$ values considered in Figures E-7 through E-9. The ratio, $M_u/\phi_b M_p$, in the column, with and without initial curvature included, decreases with decreasing $\bar{\beta}_{act}$. Also the ratio, $(\bar{\Delta}+\Delta_o)/\Delta_o$, with and without initial curvature included increases with decreasing $\bar{\beta}_{act}$.

It can be concluded that the W14×43 bracing beam is adequate for the example problem based on an explicit second-order analysis and the member design strength checks by the DM. No explicit stiffness check is necessary. The fact that the total brace-point displacement is $\bar{\Delta}_{2tot} = (0.002)(360 \text{ in.})(2.50) = 1.80$ in. over the 60 ft length of the column at the ultimate strength condition, assuming a straight-line geometric imperfection pattern between the brace points, or $\bar{\Delta}_{2tot} = (0.002)(360 \text{ in.})(2.88) = 2.07$ in. assuming the more severe sinusoidal imperfection pattern should not be of any concern unless there are unusual clearance requirements within the mechanical shaft. If the transverse deflections are of a concern, they should be checked at an appropriate service load level.

Broader Design Considerations

Design based on full bracing. It should be noted that the initial solution in this example using the W14×61 bracing beam gives a stiffness very close to the AISC Appendix 6 requirement obtained from Equation E-1b for the case of full lateral bracing (i.e., using L_b rather than L_q in Equation E-1b):

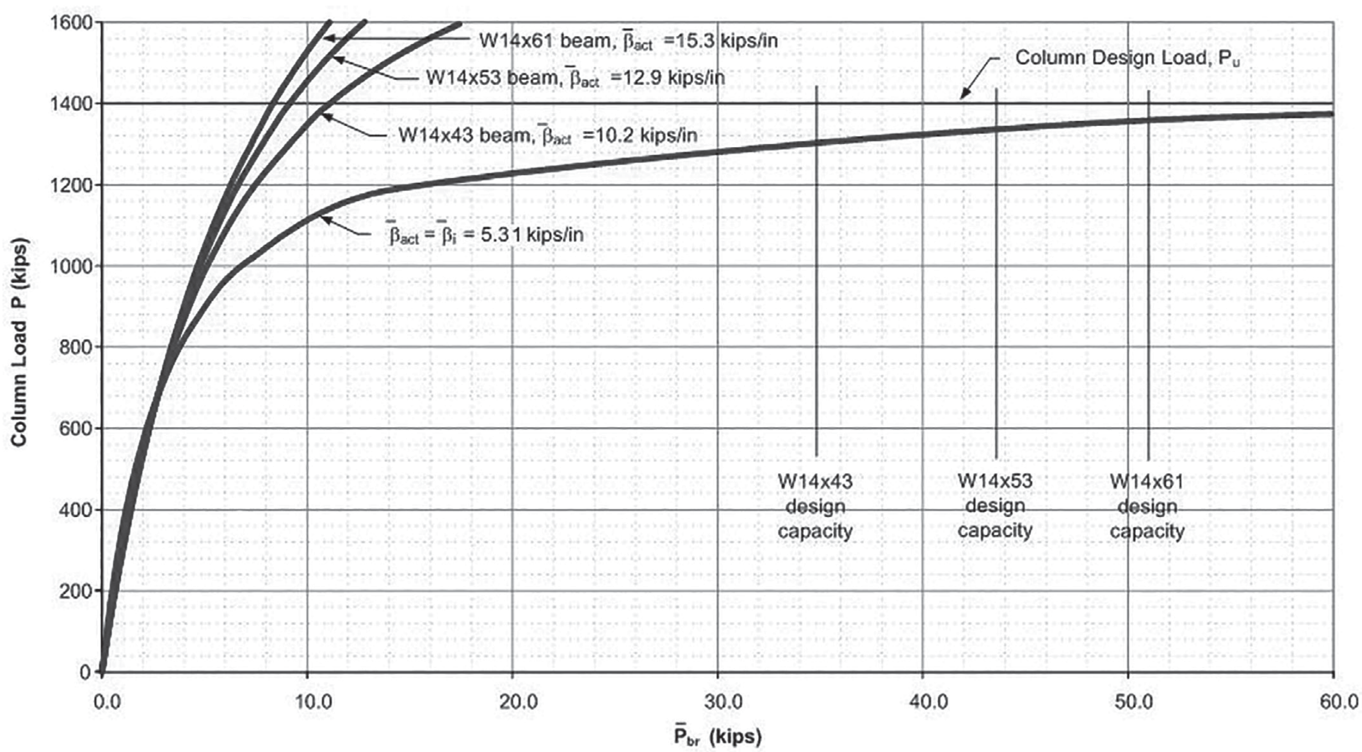


Fig. E-6. Column load versus stability bracing force, $\bar{P}_{br}$.

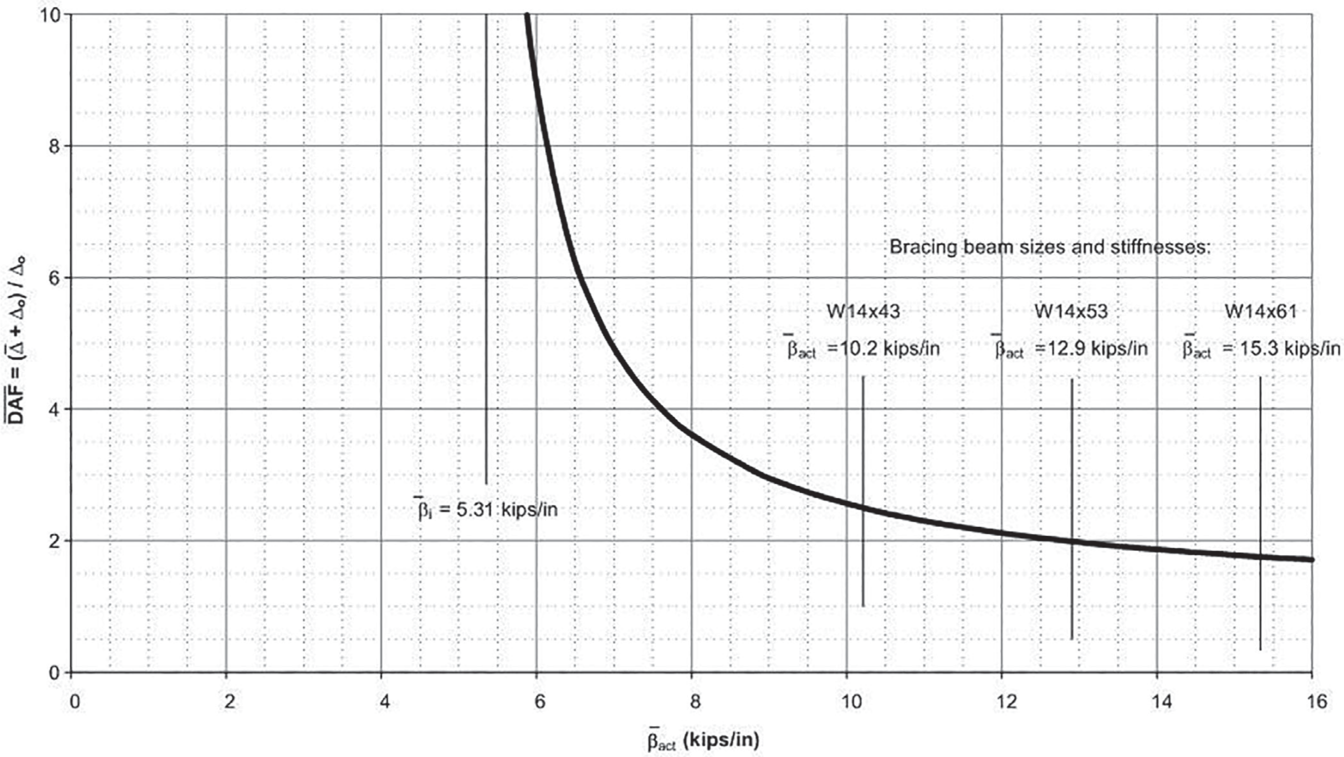


Fig. E-7. Displacement amplification factor versus brace stiffness, $\overline{\beta}_{act}$.

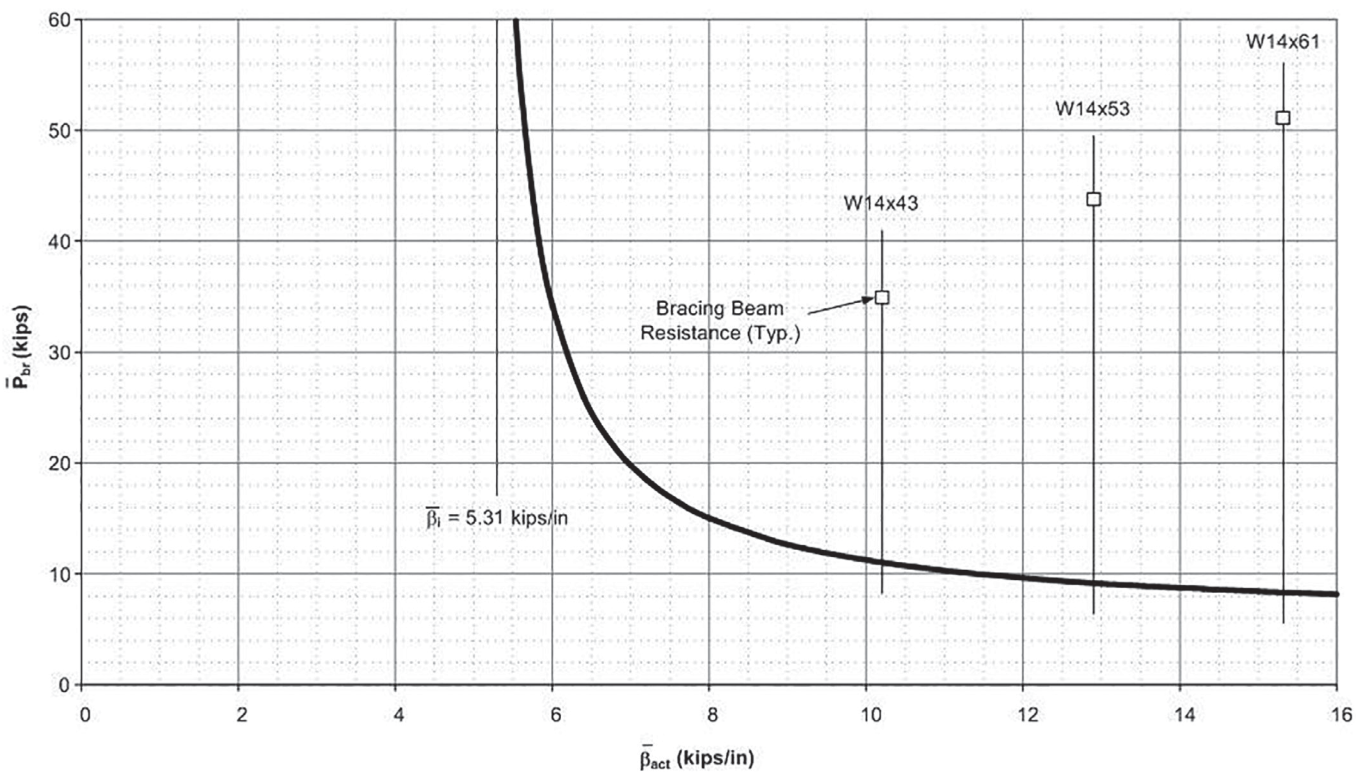


Fig. E-8. Stability bracing force, $\overline{P}_{br}$, *versus brace stiffness,* $\overline{\beta}_{act}$.

$$\left[\beta_{act} = 19.1 \text{ kip/in.}\right] \cong \left[\beta_{br} = \frac{1}{\phi}\left(4\frac{P_u}{L_b}\right) = \frac{1}{0.75}\left(\frac{4(1{,}400 \text{ kips})}{360 \text{ in.}}\right)\right] = 20.7 \text{ kip/in.}$$

However, strictly speaking, a W14×68 flying beam (from AISC *Manual* Tables 3-2 and 3-3, I_x = 722 in.^{4}, β_{act} = 21.5 kip/in., $\phi_b M_p$ = 431 kip-ft) would be required to satisfy this full bracing requirement—a 58% increase in the beam weight relative to the W14×43. For cases approaching full bracing, it is common practice to design the bracing system by assuming a pin at the brace point as shown in the free-body diagram of Figure E-10 (Winter, 1960; Nair, 1992). Using this idealization for the design with the W14×68 bracing beam, the bracing force can be obtained by evaluating the statics of a few basic free-body diagrams of the deflected structure as shown in the figure. The designer still needs to estimate $\overline{\Delta}_{2tot}$, but this can be accomplished using the same type of pin-connected model used in the development of Equation E-12. The result for the required bracing force is:

$$\begin{aligned}
\overline{P}_{br} &= P_r\left(\frac{2}{1-\dfrac{\beta_{br}}{2\beta_{act}}}\right)\left(\frac{\overline{\Delta}_{o,total}}{L}\right) \\
&= 1{,}400 \text{ kips}\left(\frac{2}{1-\dfrac{20.7 \text{ kip/in.}}{2(21.5 \text{ kip/in.})}}\right)(0.002) \\
&= (1{,}400 \text{ kips})(0.00771) \\
&= 10.8 \text{ kips}
\end{aligned}$$

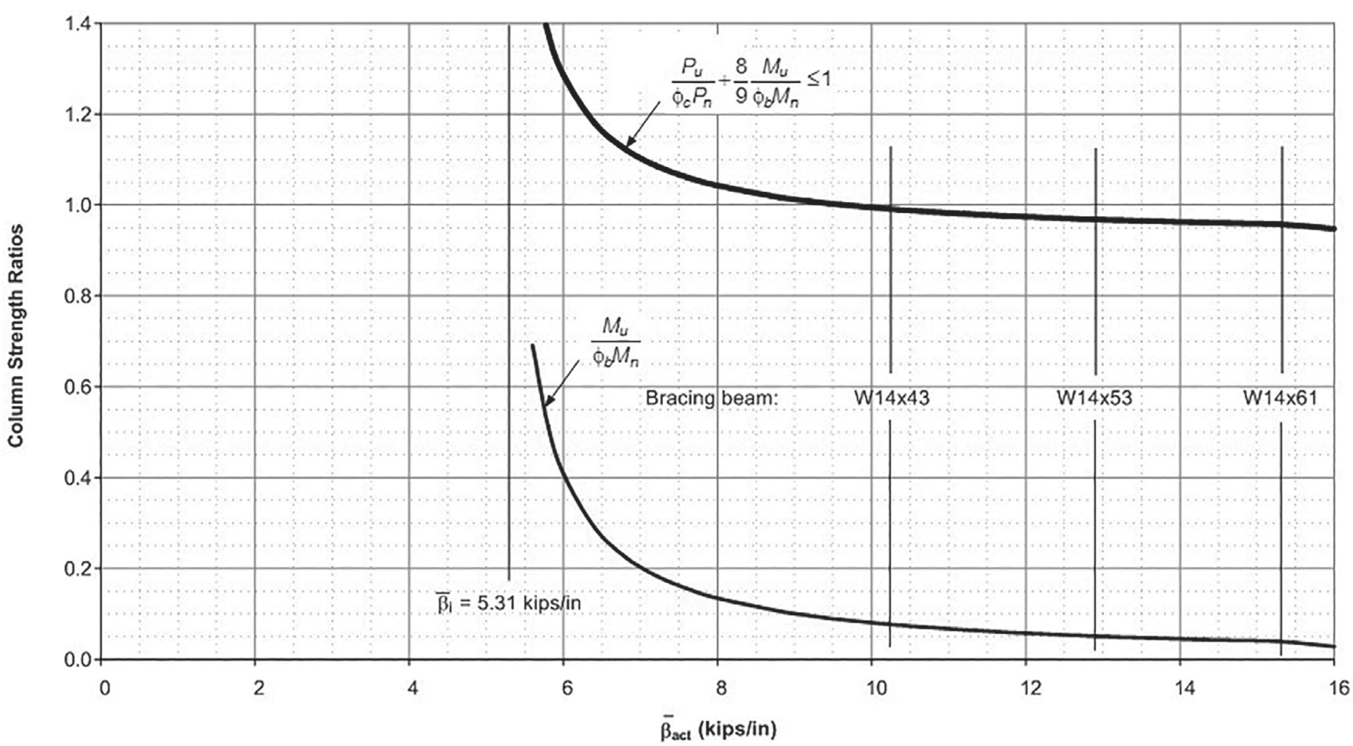


Fig. E-9. Column strength ratios versus brace stiffness, $\overline{\beta}_{act}$.

where the factor of 2 in the numerator of the first fraction comes from the fact that, in this problem, we are considering a brace at the column mid-height, with an unbraced length above and below the brace, whereas the development of Equation E-12 corresponds to the relative bracing requirements for a single unbraced length.

The W14×68 beam member can withstand a point load at midspan of 50.8 kips, which is almost five times this force requirement. The full bracing solution is obviously acceptable; however, the partial bracing solution arrived at by explicit second-order analysis using the DM allows a much lighter weight bracing beam. This design is achieved by recognizing that the column flexural rigidity, *EI*, can still provide a substantial contribution to the resistance of the brace-point deflection, and by understanding and applying an appropriate geometric imperfection within the second-order analysis by the DM.

E.5 ADDITIONAL NODAL AND RELATIVE BRACING CONSIDERATIONS

E.5.1 Implications of Partial Bracing on the Column Resistance

The preceding discussion and example shows that in cases where P_u is only slightly smaller than $\phi_c P_n$, substantial economies potentially can be attained by recognizing the contribution to the resistance of the brace-point displacements from the flexural rigidity of the column and by designing for less than full lateral bracing. This is accomplished practically and conservatively in the context of Appendix 6 by calculating and using L_q in the nodal bracing stiffness equations. However, by using an explicit second-order analysis configured according to the DM requirements, a more accurate assessment is realized. This can in turn result in significant savings in the bracing system requirements.

Nevertheless, it is very important to recognize that the use of partial bracing based on either of the methods presented implies that the column's capacity to resist a concentrically applied axial force is smaller than $\phi_c P_n$ calculated with $KL = L_b$. In the DM, the beam-column strength interaction equations are applied to the column to assess its complete capacity, including axial force and bending effects. Conversely, if using the effective length method (ELM) for the design along with Equations E-1a or E-1b and L_q for the nodal bracing stiffness requirement, K is in effect greater than 1.0 and $\phi_c P_n$ is approximately equal to P_u if the bracing is sized such that $\beta_{act} = \beta_{br}$. Therefore, there is effectively zero resistance left in the column to resist any applied bending moments.

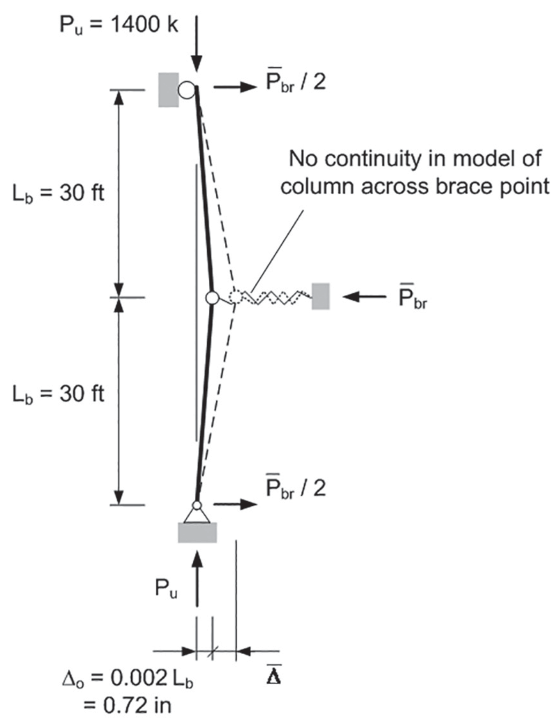


Fig. E-10. Full-bracing idealization of column from nodal bracing example.

For a beam-column subjected to large axial compression plus bending, a second-order analysis should be conducted to determine the influence of the partial bracing on the column moments. In these cases, the DM is a fully consistent approach for assessment of both the bracing system as well as the beam-column that is being braced.

E.5.2 Appropriate Selection of Geometric Imperfections

The selection of the appropriate geometric imperfection pattern for second-order analysis of nodal bracing problems is more complicated in cases involving more than one intermediate nodal brace. However, if one considers the fundamental illustration in Figure E-11, adapted from Nair (1992), the selection can be simplified. Figure E-11 shows a representative column deflected geometry for a case involving multiple intermediate braces along the member length. As demonstrated by Nair, if pins are assumed in the column at the brace points, the nodal bracing force between segments a and b is:

$$\bar{P}_{br} = P_a\theta_a - P_b\theta_b$$

where P_a, P_b, θ_a and θ_b are the axial compression forces and the skew angles in the deflected configuration for the member unbraced lengths, L_a and L_b, adjacent to the brace point. If P_a and P_b are approximately equal and are both denoted by the force P, then the above equation simplifies to

$$\bar{P}_{br} = P(\theta_a - \theta_b)$$

where $(\theta_a - \theta_b)$ is the difference in the skew angles between the segments at the brace point. Hence, if the goal is to determine the maximum potential force that needs to be resisted at the brace point, $(\theta_a - \theta_b)$ needs to be maximized.

The process of rigorously determining the initial imperfection pattern that maximizes θ_a and θ_b (for a member that does not have any transverse load, that is, a member in which the first-order brace forces in the geometrically perfect system are zero) generally requires that one consider: (1) the affinity of various potential imperfections with the lowest eigenvalue buckling modes of the model, as well as (2) the equivalent lateral loadings associated with the different imperfections, along with influence lines for the force at the brace under consideration. For problems in which the first-order brace forces are nonzero, the deflection of the structure due to the loads also generally needs to be considered. Simplifications are possible for certain cases. The reader is referred to Tran and White (2008) for discussions of these types of calculations.

It is important to note that when using the AISC *Specification* Appendix 6 equations, the calculations are typically performed for one or a few bracing locations where the bracing force and stiffness requirements are judged to be most critical. The same bracing sizes are then repeated at other less critical locations. The use of an explicit second-order analysis does not change this practice. The engineer should use his or her judgment to limit the number of analysis and design calculations. Generally there is no need to conduct a large number of analyses to select different brace sizes at every brace location. One only needs to identify and design the most critical brace out of a number of braces. The same brace size is then repeated at other less critically loaded brace locations for purposes of design economy.

In addition to these nodal bracing considerations, the system that the nodal braces in Figure E-11 are attached to needs to be able to withstand a shear within the tiers or panels a and b of:

$$\bar{P}_{br,a} = P_a\theta_a \quad \text{and} \quad \bar{P}_{br,b} = P_b\theta_b$$

For determining appropriate maximum values of the stability bracing P-Δ shear forces within the tiers or panels, generally it is considered acceptable to specify an overall out-of-plumbness that gives a maximum overall total out-of-plumb lateral imperfection of the system within the limits permitted by the AISC *Code of Standard Practice*. The lean-on bracing example in Section E.6 illustrates this consideration in the context of a system that is laterally restrained at its top and bottom (or at its ends). Appendix C addressed this consideration in the context of general multi-story building structures.

It should be noted that, in some cases, the idealization of the member(s) as "pinned" at each of the brace locations can underestimate the brace forces determined by a model in which the continuity of the member(s) through the brace

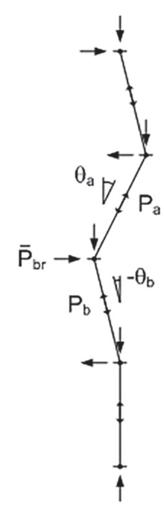


Fig. E-11. Forces on displaced configuration of a braced compression member from Nair (1992).

point is included in the analysis. For instance, the stability effects in columns with one relatively long unbraced length connected to adjacent short unbraced lengths can develop a lever type action in the short unbraced lengths. This is due to the flexural end restraint that the short lengths provide to the longer length. Nevertheless, Winter's model (i.e., the use of idealized pins at each of the brace points) still provides an exact calculation of the ideal bracing stiffness requirements (Ziemian, 2010; Tran and White, 2008).

E.5.3 Development of the Load Path at a Brace Point

A slight modification to the problem in Figure E-4 results in a case that emphasizes an important aspect of the relative versus the nodal bracing equations of the AISC *Specification* Appendix 6. Suppose that the bracing beam in Figure E-4 is replaced by the truss shown in Figure E-12. The question to consider is the following: What are the corresponding bracing force and stiffness requirements? A second-order structural analysis using the same geometric imperfection as shown previously in Figure E-4 shows that the appropriate force requirement in the horizontal strut is approximately $P_{br} = 0.008P_r$, close to the base nodal brace requirement, given a stiffness provided by the truss against lateral displacement at its midspan of $\beta_{br} = 4P_r/(\phi L_q)$ for LRFD. However, the lateral force and stiffness requirements provided by each panel of the truss are one-half of the above values, i.e., approximately the relative brace requirements, $P_{br} = 0.004P_r$ and $\beta_{br} = 2P_r/(\phi L_b)$. In the limit that the horizontal strut is a very short element, say a gusset plate connection to the truss, this problem reduces to a typical illustration of relative bracing. Nevertheless, the connection element at the middle of the column is governed by the nodal bracing requirements, and strictly speaking, the force and stiffness requirements for each panel of the truss are one-half of the nodal bracing requirements. Conversely, the stiffness and strength requirements at the column ends are represented sufficiently by the relative bracing equations. The engineer must understand these aspects of the structural analysis and the structural behavior to properly apply the Appendix 6 relative and nodal bracing equations in general. Alternatively, solving general bracing problems using the DM approach does not require the designer to distinguish whether the problem is classified as relative or nodal bracing.

In general, the main differences between the AISC *Specification* Appendix 6 relative and nodal bracing requirements are:

1. The relative bracing requirements address the shear force that must be resolved in a given panel of a bracing system.
2. The relative bracing requirements neglect the help from the *EI* of the column(s). (This is a simplification, the influence of which is small in many but not all cases.)
3. The nodal bracing requirements address the absolute or direct force that needs to be transferred from the brace point to the bracing system.
4. The nodal bracing requirements include the help from the *EI* of the column(s) via the L_q parameter.

When designing bracing systems based on a second-order analysis according to the DM, the designer may elect to include or not include the help from the column *EI* values in determining both the shear force requirements within a given panel of the bracing system as well as in determining the direct force that must be transferred to the bracing system at a given brace point.

E.5.4 General Analysis Modeling Considerations

The analysis solutions illustrated for the nodal bracing example in Section E.5.3 were obtained from a second-order elastic analysis program that accounts for P-δ effects using a cubic approximation for the transverse displacements along the element lengths. Column shear deformations were neglected. Accurate solutions were obtained using two elements for each unbraced length of the column for the cases with straight-line geometric imperfections between the nodes. However, for the cases with a sinusoidal geometric imperfection, four elements were needed within each unbraced length (since the program employed did not have explicit capabilities for modeling column initial curvature). Four elements are needed within each unbraced length to achieve a comparable accuracy for either case if an analysis program is employed that has only P-Δ capabilities. In general, up to six P-Δ elements are necessary to ensure that the second-order internal moments are accurate to within 3% of the converged elastic solution for problems with $\alpha P_r < \left[\bar{P}_{e\tau L} = \pi^2\left(0.8\tau EI\right)/L^2\right]$. For problems where αP_r exceeds $\bar{P}_{e\tau L}$ (and thus the column segment is designed

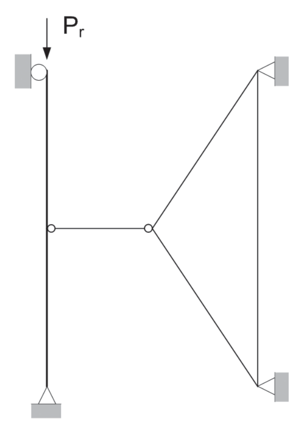


Fig. E-12. Problem involving both nodal and relative bracing considerations.

in essence using $K \leq 1$), more than six P-Δ elements per member are generally required (Guney and White, 2008a; Kaehler et al., 2010).

If the column in this example were designed using full nodal bracing, the second-order analysis may be conducted neglecting the contribution of the column rigidity to the resistance of the brace-point deflections. A pin may be inserted in the model of the column at the brace point and the influence of the brace-point deflections on the column bending moments may be neglected. The AISC *Specification* Section C1.3a provisions allow this idealization in the context of braced frame systems via the statement, "In braced-frame systems, it is permitted to design the columns, beams, and diagonal members as a vertically cantilevered, simply connected truss." In this case, a P-Δ analysis is sufficient to capture all of the second-order effects.

E.6 LEAN-ON BRACING EXAMPLE

This design example is implemented using the 2005 AISC *Specification* and the 13th Edition AISC *Manual*.

Given:

Many times, designers seek to brace a column to an adjacent column or frame without giving sufficient attention to the strength and stiffness of what it is being braced to. Figure E-13 shows an example where two exterior ASTM A992 W14×257 columns support a required load of 2,290 kips (LRFD) at the bottom of a multi-story high-rise building. These columns are located in an open atrium lobby with a height of four stories. This situation is common in high-rise office buildings. The engineer recognized the problem with the large unbraced length in-plane and placed a bracing beam across the open lobby at the mid-height to the building elevator core where interior core columns (W14×426) supporting a required load of 4,260 kips are located. There was no core bracing in-plane so the engineer had previously decided to increase the elevator floor beam sizes to W21×44 and install moment connections in these beams to develop sufficient lateral stiffness for the elevator core columns to support their loads. Interestingly, simply connected W18×40 members are sufficient to support LRFD gravity loads of 500 lb/ft over the 30 ft span of the unbraced interior beams (not shown in the figure). However, the larger W21×44 beam stiffnesses and FR moment connections are necessary for the W14×426 columns to adequately support their directly applied axial loads.

This is a classic example of lean-on bracing. The question is whether the moment frame in the elevator core is sufficiently strong and stiff to brace (or partially brace) the exterior columns at their mid-height. Is the overall system adequate to resist buckling as the engineer presumed, with the exterior columns braced or partially braced in-plane at their mid-height, and with the stability of the interior columns reduced by the P-Δ forces from the exterior columns? The assumption is made that everything is braced adequately in the out-of-plane direction. Both the interior and exterior columns are bent in-plane about their strong axis.

The interior columns in Figure E-13 are supporting stairs and elevators at each floor level. However, the directly applied gravity loads on the interior beams, which are relatively small in comparison to the loads transferred to the columns at their tops, are neglected to simplify the following example calculations. Due to the four-story atrium, the four columns in the figure are connected to the building lateral load resisting system only at their tops. The building lateral load resisting system is an exoskeleton braced frame system. Therefore, all the columns are assumed to be laterally braced at their tops. The exterior columns are connected to the interior columns by bracing beams framing across the atrium. These beams are modeled as rigid links pinned at each end for the preliminary analysis. They are assumed to be the same size as the larger of the interior beams in the final design. All the columns are laterally supported out-of-plane at every 13 ft.

The LRFD required gravity loads for all the columns are shown in Figure E-13. The problem here is to first determine if the combined system has sufficient strength, and if not, to determine an appropriate increase in the interior floor beam sizes. Given a selected interior floor beam size, the complete framing system is checked using a rigorous second-order analysis. Finally, the broader implications of the example calculations are discussed. All of the following design calculations are performed using the DM.

Solution:

From AISC *Manual* Table 2-3, the material properties are:

ASTM A992
$F_y = 50$ ksi
$F_u = 65$ ksi

From AISC *Manual* Table 1-1, the geometric properties are:

W14×257
$A_g = 75.6 \text{ in.}^2$
$I_x\ = 3{,}400 \text{ in.}^4$

W14×426
$A_g = 125 \text{ in.}^2$
$I_x\ = 6{,}600 \text{ in.}^4$

Strength Check using W21×44 Interior Floor Beams

The initial geometric imperfection pattern shown by the bold solid lines in Figure E-13 is specified for the second-order analysis. The maximum out-of-plumbness, located at the middle floor, is $\Delta_o = (0.002)(26 \text{ ft}) = 0.624$ in. Based on the concepts illustrated in Figure E-11, it is clear that this out-of-plumbness induces a maximum destabilizing effect from the exterior columns on the interior core framing system.

Note that there is no need to model the S-shape out-of-straightness imperfection of the exterior columns shown in Figure E-14a. The in-plane axial resistance of these columns, $\phi_c P_{nx}$, is calculated using the unbraced length $KL_x = 26$ ft. This accounts for the potential imperfection effects in Figure E-14a. Similarly, there is no need to model the comparable "double-S"-shape out-of-straightness shown on the interior columns in Figure E-14a. The influence of these potential imperfections is accounted for by calculating the in-plane axial resistance, $\phi_c P_{nx}$, of the interior columns using $KL_x = 13$ ft. Finally, there is no need to consider the single- or double S-shape geometric imperfection patterns spanning over the two stories illustrated in Figure E-14b. The buckling modes that these imperfections have the greatest affinity to are associated with much larger buckling load levels than the lowest eigenvalue buckling mode, which is similar in shape to the imperfection shown in Figure E-13. The specific buckling modes for the final design of this frame are discussed later in the discussion of broader design considerations. It is simply noted here that, for general stability bracing analysis problems, one may need to determine the lowest eigenvalue buckling modes of the structure or subassembly in order to establish the appropriate geometric imperfections for the second-order load-deflection analysis. In addition, it is important to note that eigenvalue buckling solutions can be obtained readily from many of the structural

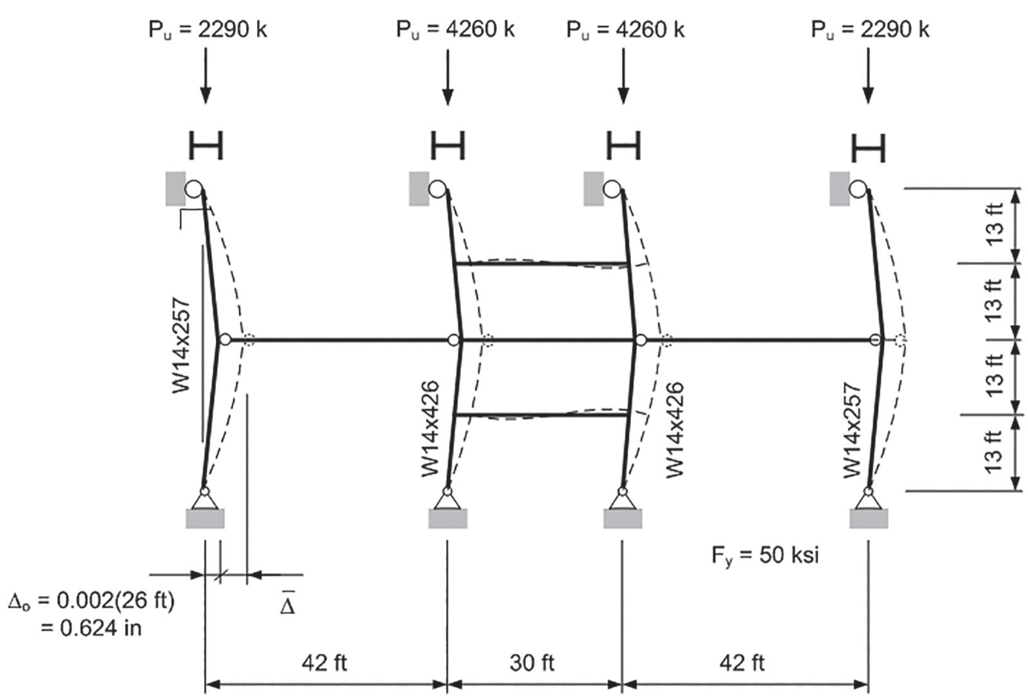


Fig. E-13. Lean-on bracing example.

analysis and design software packages used in current practice. The reader is referred to Tran and White (2008) for more detailed discussions.

Based on the above in-plane KL_x values and on the out-of-plane KL_y = 13 ft for all the columns, the design axial compressive strength of the W14×257 exterior columns is governed by in-plane flexural buckling ($\phi_c P_n$ = 2,900 kips) while the axial resistance for the W14×426 interior columns is governed by out-of-plane flexural buckling ($\phi_c P_n$ = 5,120 kips). Given the applied loads, $P_u/P_y = P_u/F_y A_g = 0.606$ and 0.682 for the exterior and interior columns, respectively. This in turn results in a stiffness reduction of τ_a = 0.955 and 0.868 for these columns according to the equation given in AISC *Specification* Appendix 7, Section 7.3: $\tau_b = 4\left(\alpha P_u/P_y\right)\left[1-\left(\alpha P_u/P_y\right)\right]$, where α = 1.0 for LRFD. Therefore, reduced column flexural rigidities of $0.8\tau_b EI$ are used for the DM analysis, while all other contributions to the elastic stiffness are reduced by the factor 0.8.

Step 3 of the procedure outlined in Section E.3 involves the calculation of the story sidesway amplification factor, $\overline{B}_2$. For this calculation, the general story-stiffness based equations are used and the bottom two stories of the interior frame are treated as a single equivalent "story." This approach is acceptable for this problem since all the stories support essentially the same gravity load, the exterior columns span two stories of the interior frame, and the key stiffness in this problem is the one related to the sidesway at the middle floor level. In addition, the symmetry of the responses about the middle floor level is recognized so that only the bottom two stories need be considered in the structural analysis. All the column and beam rotations are zero at the middle floor level. That is, due to the symmetry about the middle floor, the columns all behave flexurally as if they were attached to a rigid girder at this location. Given these idealizations, and based on AISC *Specification* Equation C2-3, $\overline{B}_2$ can be expressed as:

$$\overline{B}_2 = \frac{1}{1-\dfrac{\Sigma P_u/L}{R_M\ \Sigma H/\overline{\Delta}_H}}$$

$$= \frac{1}{1-\dfrac{\Sigma P_u}{0.85\ \Sigma\overline{\beta}L}}$$

$$= \frac{1}{1-\dfrac{\Sigma P_u}{\Sigma\overline{P}_{e2}}}$$

where

ΣP_u = 13,100 kips = total vertical load supported by all the columns
L = 26 ft = 312 in. = height of the effective story being considered in the analysis
ΣH = arbitrary lateral load applied at the middle floor level, kips
$\overline{\Delta}_H$ = first-order elastic sidesway deflection at the middle floor level due to ΣH, calculated using the DM reduced stiffnesses, in.
R_M = reduction factor that accounts for the influence of P-δ effects on the second-order sidesway displacements
= 0.85 for moment-frame and combined systems
$\Sigma\overline{\beta}$ = total (reduced) first-order sidesway stiffness of the effective story, kip/in.
$\Sigma\overline{P}_{e2}$ = total sidesway buckling capacity for the reduced stiffness DM model, kips

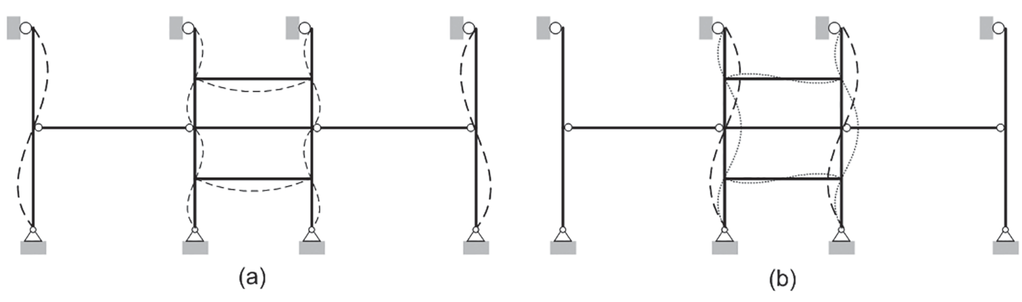


Fig. E-14. Framing imperfections that need not be considered in DM second-order analysis of the lean-on bracing example.

For each of the exterior columns, the contribution to $\Sigma\bar{\beta}$ is:

$$\begin{aligned}\bar{\beta}_{ext} &= \frac{3\,(0.8\tau_b EI)}{L^3}\\ &= \frac{3(0.8)(0.955)(29{,}000 \text{ ksi})\left(3{,}400 \text{ in.}^4\right)}{\left[(26 \text{ ft})(12 \text{ in./ft})\right]^3}\\ &= 7.44 \text{ kip/in.}\end{aligned}$$

However, the $\bar{\beta}$ contribution for the interior frame is most easily determined by subjecting this frame to an arbitrary horizontal force of $\Sigma H = 1$ kip in a first-order elastic analysis. This gives:

$$\bar{\Delta}_H = 0.0292 \text{ in.}$$

and

$$\begin{aligned}\bar{\beta}_{int.\,frame} &= \frac{\Sigma H}{\bar{\Delta}_H} && \text{(E-7)}\\ &= \frac{1 \text{ kip}}{0.0292 \text{ in.}}\\ &= 34.2 \text{ kip/in.}\end{aligned}$$

Based on these two stiffness contributions, the total sidesway buckling load can be estimated as:

$$\begin{aligned}\Sigma\bar{P}_{e2} &= 0.85\left(2\bar{\beta}_{ext} + \bar{\beta}_{int.\,frame}\right)L\\ &= 0.85\left[2(7.44 \text{ kip/in.}) + 34.2 \text{ kip/in.}\right](26 \text{ ft})(12 \text{ in./ft})\\ &= 13{,}000 \text{ kips}\end{aligned}$$

This load is smaller than ΣP_u= 13,100 kips. Therefore, the larger internal beams will be needed such that the overall system will have sufficient sidesway stiffness to support the gravity loads.

Selection of Interior Floor Beam Size

The foregoing analysis indicates that significantly larger internal floor beams will likely be needed if the overall structure is to be stiff enough to support the total gravity loads ΣP_u = 13,100 kips. However, it is also apparent from the analysis that the interior beam at the middle floor doesn't provide any contribution to the overall lateral load resistance. Hence, a W18×40 floor beam with simple end connections is selected at the middle floor location. This beam size is sufficient to support an LRFD factored gravity load of 500 lb/ft as discussed previously. For the selection of the other floor beams at the first and third levels, it is assumed that the beam-column strength interaction equations for the interior columns will govern the overall design. This is because the column axial strength ratio for these members is $P_u/\phi_c P_n$ = 4,260 kips/5,120 kips = 0.832, and the stiffness of an adequately designed interior frame will tend to resist a large fraction of the second-order moments caused by the stability effects. Based on this assumption, the stiffness of the first and third-floor beams will need to be increased to a sufficient extent such that the following is satisfied for the interior columns (where $P_u/\phi_c P_n > 0.2$):

$$\frac{P_u}{\phi_c P_n} + \frac{8}{9}\left(\frac{M_u}{\phi_b M_n}\right) \le 1 \qquad \text{(from \textit{Spec}. Eq. H1-1a)}$$

It should be noted that AISC *Specification* Equation H1-2 could be employed to obtain a more liberal estimate of the interior column resistances; however, Equation H1-1a is used to simplify the calculations, avoiding the need for separate in-plane and out-of-plane resistance checks. Given the unbraced length, L_b = 13 ft, for lateral-torsional buckling, the design flexural strength of the W14×426 columns from AISC *Manual* Table 3-2 is $\phi_b M_n = \phi_b M_p$ = 3,260 kip-ft, because $L_b < L_p$. Therefore, one can solve Equation H1-1a for the maximum value of M_u that can be tolerated in the columns, M_u = 616 kip-ft.

Given the selected critical geometric imperfection pattern with Δ_o = 0.002(26 ft)(12 in./ft) = 0.624 in. at the middle floor level, the total P-Δ force at the middle floor caused by the initial imperfect geometry is determined from Equation B-2:

$$\begin{aligned} H_{P\Delta_o} &= 2\ (0.002\ \Sigma P_u) \\ &= 2(0.002)(13{,}100 \text{ kips}) \\ &= 52.4 \text{ kips} \end{aligned}$$

Half of this load is distributed as a shear force to the framing above and half to the framing below this level. Based on a first-order analysis using the DM reduced stiffnesses and the initial W21×44 beams, this load causes a maximum moment of M_{lt} = 188 kip-ft in the interior columns. Therefore, assuming that the shape of the moment diagram is not influenced significantly by increases in the interior beam sizes, the sidesway amplification factor, $\overline{B}_2$, can be estimated by limiting it to M_u/M_{lt} = 616 kip-ft/188 kip-ft = 3.28. Given this limit for $\overline{B}_2$, solve for the required lateral stiffness of the interior frame using the following equation (as derived earlier in this example):

$$\begin{aligned} \left[\overline{B}_2 = 3.28\right] &\le \left[\frac{1}{1-\dfrac{\Sigma P_u}{0.85\ (2\overline{\beta}_{ext} + \overline{\beta}_{int.\,frame})L}}\right] \\ &= \left[\frac{1}{1-\dfrac{13{,}100 \text{ kips}}{0.85\left[2(7.44 \text{ kip/in.}) + \overline{\beta}_{int.\,frame}\right](26 \text{ ft})(12 \text{ in./ft})}}\right] \end{aligned}$$

This solution gives

$$\overline{\beta}_{int.\,frame} \ge 56.2 \text{ kip/in.}$$

A W30×90 is the lightest section from Table 3-3 of the AISC *Manual* that approximately satisfies this requirement, i.e.:

$$\begin{aligned} \overline{\beta}_{int.\,frame} &= \frac{\Sigma H}{\overline{\Delta}_H} \\ &= \frac{1 \text{ kip}}{0.0182 \text{ in.}} \\ &= 54.9 \text{ kip/in.} \end{aligned}$$

This observation is reached by progressively selecting bolded cross sections larger than the original W21×44 in Table 3-3, then analyzing the interior frames for their deflection under unit lateral load.

The next section demonstrates that this section is adequate based on a final design check using a rigorous second-order analysis.

Frame Design Check using a Rigorous Second-Order Analysis

Given the above preliminary design, a rigorous second-order analysis is conducted using the DM approach to verify the column and beam size selections. Figure E-15(a) shows the deflected shape and Figure E-15(b) shows the moment diagram for the bottom two stories due to the applied loads. For the W14×426 interior columns, the governing M_u = 583 kip-ft is located at the top of the first story. This is slightly smaller than the maximum value of 611 kip-ft allowed by Equation H1-1a. The interior columns pass the interaction check as follows:

$$\frac{P_u}{\phi_c P_n}+\frac{8}{9}\frac{M_u}{\phi_b M_n}=\frac{4{,}260\text{ kips}}{5{,}120\text{ kips}}+\frac{8}{9}\left(\frac{583\text{ kip-ft}}{3{,}260\text{ kip-ft}}\right)$$
$$=0.832+\frac{8}{9}(0.179)$$
$$=0.991\le 1.0 \quad \textbf{o.k.}$$

where $\phi_c P_n = 5{,}120$ kips is based on $KL_y = (1.0)(13\text{ ft})$, and $\phi_b M_n = \phi_b M_p = 3{,}260$ kip-ft is easily achieved by the interior columns based on an unbraced length for LTB of $L_b = 13$ ft as discussed previously in the context of Equation H1-1a. Similarly, with $\phi_b M_n = \phi_b M_p = 1{,}830$ kip-ft from AISC *Manual* Table 3-2 for $L_b = 13$ ft, check the interaction Equation H1-1a ($P_u/\phi_c P_n > 0.2$) for the W14×257 exterior columns:

$$\frac{P_u}{\phi_c P_n}+\frac{8}{9}\frac{M_u}{\phi_b M_n}=\frac{2{,}290\text{ kips}}{2{,}900\text{ kips}}+\frac{8}{9}\left(\frac{228\text{ kip-ft}}{1{,}830\text{ kip-ft}}\right)$$
$$=0.790+\frac{8}{9}(0.125)$$
$$=0.900\le 1.0 \quad \textbf{o.k.}$$

The W30×90 interior beams are subjected to a governing sidesway moment of $M_u = 476$ kip-ft due to the stability effects from the gravity load, as shown in Figure E-15(b). With an unbraced length of $L_b = 30$ ft and the double-curvature bending shown in Figure E-15(b), the design flexural strength can be determined from AISC *Specification* Section F2 with $C_b = 2.00$, as $\phi_b M_n =$ 788 kip-ft. This results in a strength ratio of 0.604. Furthermore, the W30×90 beams are adequate to support the gravity load of $w_u = 500$ lb/ft, causing an additional end moment of $w_u L_b^2/12 = 38$ kip-ft.

An interesting question related to the interaction equation check for the interior columns is the following. Since all the beams are simply connected to these members at the middle story (the interior beam doesn't actually see any bending in this problem even if it were moment connected, due to the symmetry about the middle floor), and since the exterior columns are in fact leaning on the interior columns at the middle floor, should the interior columns be checked using an in-plane unbraced length of KL_x = 26 ft rather than 13 ft? The length $KL_x = 13$ ft is the appropriate length for the DM strength checks of the interior columns in this problem for the following reasons:

1. The exterior columns provide some resistance to lateral displacement of the middle floor. The situation where certain weaker columns can lean on other stronger columns can occur in many different frame configurations. Generally if some resistance is provided to transverse displacement at a joint from somewhere else within the framing system, it is considered acceptable to use the joint as an end point in defining KL for the calculation of $\phi_c P_n$.

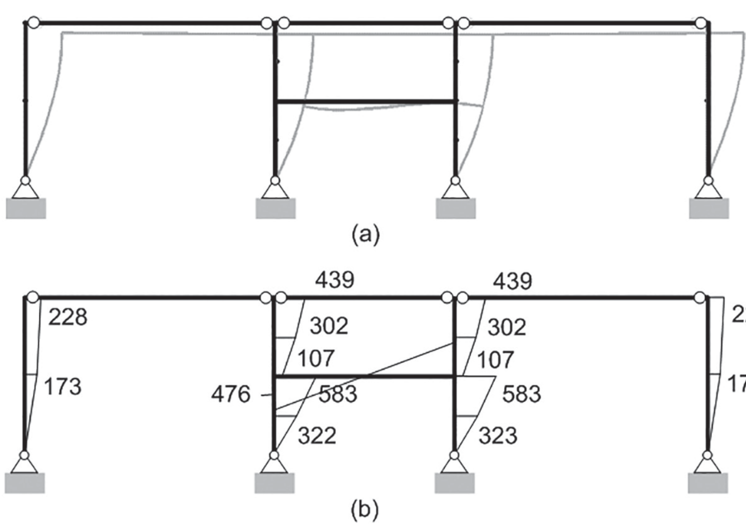


Fig. E-15. DM model analysis results on the framing below the middle level for the lean-on frame design example: (a) deflected shape magnified 50x and (b) internal bending moments (kip-ft).

2. The behavior shown in Figure E-15 is identical to that of a two-story sway frame with 13-ft-high stories and a rigid girder at the top story (due to the symmetry of the responses in this problem). For such a problem, one would use $KL_x = KL_y = 13$ ft for checking the interior columns.

3. The buckling mode affine to the geometric imperfection represented by the thin dotted lines in Figure E-14b is associated with a much higher load level than the governing sidesway buckling mode, which is similar to the sidesway imperfection shown by the bold solid lines in Figure E-13 (see the subsequent ELM design check using an exact inelastic buckling analysis for a discussion of the various buckling modes for this frame).

In general, if in doubt, the designer should use the larger KL in a situation such as the above. If the larger value of $KL_x = 26$ ft is used for the interior columns in this example, $\phi_c P_n = \phi_c P_{nx} = 4{,}920$ kips and Equation H1-1a results in a value of 1.02.

The DM second-order analysis predicts an additional sidesway deflection of the middle floor due to the applied gravity loads at the strength load level of $\bar{\Delta} = 1.49$ in., giving a total second-order sidesway displacement of $\bar{\Delta}_{2tot} = \Delta_o + \bar{\Delta} = 2.11$ in., a ratio, $\bar{\Delta}/\Delta_o = 2.39$, and a sidesway amplification factor of $\bar{\Delta}_{2tot}/\Delta_o = 3.38$. The corresponding drift ratio of the columns in the first and fourth stories is $\bar{\Delta}/L = 1.10 \text{ in.}/156 \text{ in.} = 1/142$. Given the large sidesway amplification, it is prudent to check the sidesway of the columns under the service gravity load level of $1.0D + 1.0L$. Based on a strength-to-service load ratio of 1.5, the service load axial forces are 1,530 kips and 2,840 kips for the exterior and interior columns, respectively. These loads are placed on the nominally imperfect structure but using the nominal elastic stiffness of the members to estimate the service load racking of the first and fourth stories. The maximum service load drift of these stories is calculated as $\Delta/L = 0.210 \text{ in.}/156 \text{ in.} = 1/743$, which is well within the limits for cracking of the interior partitions.

Broader Design Considerations

General analysis modeling considerations. The analysis solutions for this problem were obtained using the same type of second-order analysis (cubic displacement-based element including both P-Δ and P-δ effects at the element level) and member discretization (two elements within each unbraced length) discussed for the previous nodal bracing example. The use of two elements per length between the member intersections in the plane of the frame gives displacements and internal forces in this problem that are within two percent of the exact solution. Errors larger than five percent relative to the exact solution are obtained for the first and third level interior beam moments if a single element is used for each of the above lengths. In this example, four elements are required for each length using programs that have only P-Δ capabilities.

ELM design check using an exact inelastic buckling analysis. The engineer conducting the design in this problem, being less accustomed to the use of the new DM and more accustomed to design using a system buckling analysis with the ELM, decided to check the design also by conducting a rigorous inelastic buckling analysis using the procedure described in Section A.5 (see Equations A-34). The reader is reminded here that the use of effective length factors, K, is simply a convenient way of quantifying the member buckling load obtained from any appropriate buckling analysis. If desired, the member buckling load may always be calculated directly without necessarily determining K. Based on $P_u/\phi_c P_y = 0.758$ for the interior columns and 0.673 for the exterior columns, respectively, the inelastic stiffness reduction factors for the inelastic buckling analysis are:

$$\begin{aligned}\tau_a &= -2.724\frac{P_u}{\phi_c P_y}\ln\left(\frac{P_u}{\phi_c P_y}\right)\\ &= 0.572 \text{ and } 0.726\end{aligned}$$

Given these inelastic stiffness reduction factors, the total stiffness reduction applied to the column flexural rigidities is:

$$\begin{aligned}\phi_c\,(0.877\tau_a) &= 0.9(0.877\tau_a)\\ &= 0.789\tau_a\\ &= 0.451 \text{ and } 0.573\end{aligned}$$

for the interior and exterior columns respectively, based on Equation A-30 and the AISC resistance factor for column axial compression, $\phi_c = 0.90$. It is desired to determine the fraction of the total applied LRFD loads at which the overall buckling of the structural system will occur, assuming that the column flexural rigidities are reduced by the above factors. As explained in Sections A.4 and A.5, the flexural rigidities $0.789\tau_a EI$ are the equivalent values corresponding "exactly" to the AISC column design strength curves, assuming that the structure buckles at a load fraction of 1.0, i.e., at $P_u = \phi_c P_n$ in all the columns. These equivalent flexural rigidities can be inserted into any legitimate buckling analysis model constructed using the nominal (perfect) geometry of the frame. In this example, they are substituted into the eigenvalue buckling analysis function of the computer software utilized for the previous second-order load-deflection solutions. The complete structure is loaded with the applied axial forces shown in Figure E-13 and the smallest eigenvalue returned by the software is determined (i.e., symmetry of the buckling mode about the middle story is not assumed, in order to ensure that the potential mode illustrated previously in Figure E-14(b) is not missed). This eigenvalue is the fraction of the total strength loads at which buckling occurs for the reduced stiffness model.

The software predicts that the buckling of the equivalent reduced stiffness model occurs at 0.998 of the total strength load, with a buckling mode very similar to the sidesway deflected shape in Figure E-15a (but on the complete structure). This means that the DM and the ELM solutions predict very similar capacities for this frame. The DM solution gives a maximum unity check value for the interior columns of 0.991. The ELM solution, based on the above "exact" inelastic buckling analysis, predicts that the frame fails in sidesway at approximately 0.998 of the total applied load (the actual AISC *Specification* ELM strength is slightly smaller due to the notional lateral load that must be included for the gravity-only load combination considered here). The column strength ratio for the interior columns in the DM is based on out-of-plane flexural buckling. However, the interior columns are governed by in-plane flexural buckling in the ELM calculations, with $\phi_c P_{nx} = 0.998(4{,}260 \text{ kips}) = 4{,}250$ kips compared to $\phi_c P_{ny}$ = 5,120 kips based on $KL_y = 13$ ft in the DM solution. The exterior columns are governed by in-plane flexural buckling for both the ELM and the DM calculations.

There may be concern about other potential buckling modes that may have been neglected in selecting the geometric imperfection pattern used for the DM second-order analysis. Therefore, higher eigenvalue buckling modes and their corresponding eigenvalues were also computed. The next highest buckling mode for the example frame, using the above inelastic stiffness reductions, occurs at 2.05 of the LRFD load level. This buckling mode is similar to the mode illustrated by the dark dashed lines in Figure E-14(b). The eigenvalues corresponding to other buckling modes are substantially higher. Figure E-16 shows the first through

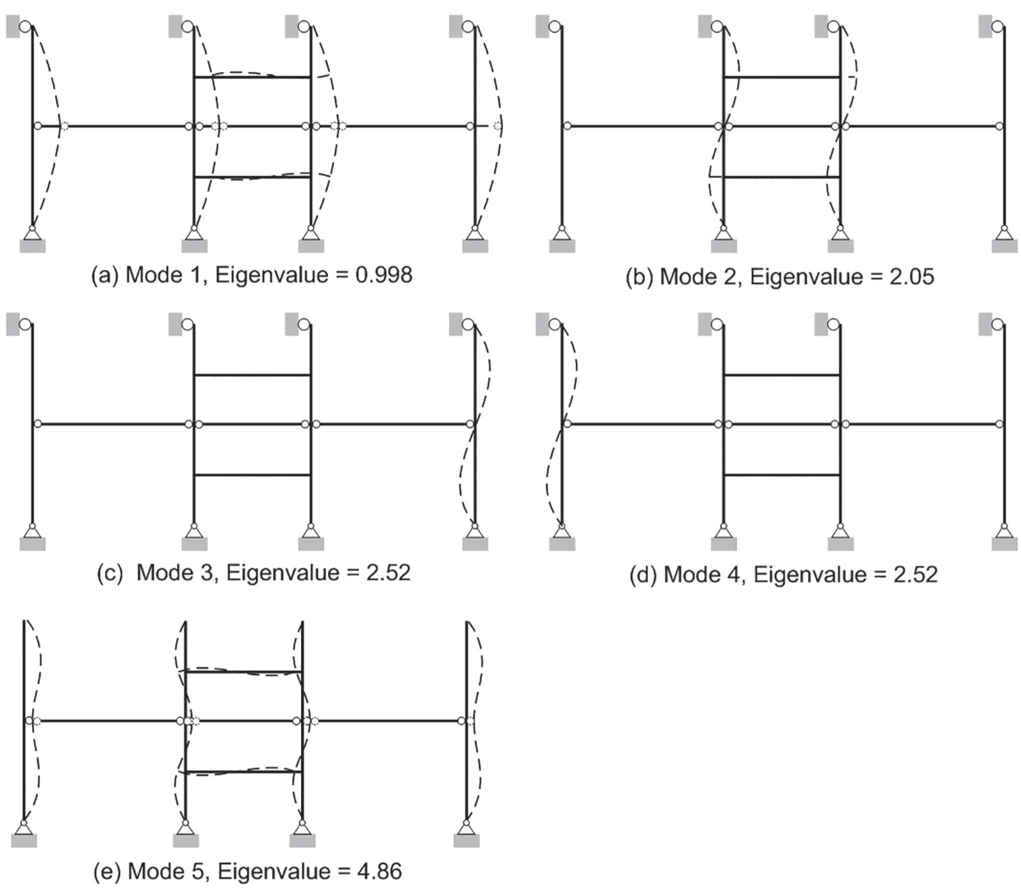


Fig. E-16. Five lowest eigenvalue buckling modes and their corresponding eigenvalues for the final design of the lean-on frame design example.

the fifth buckling modes and the corresponding eigenvalues (i.e., the multiple of the design loads corresponding to each mode) for the final design of the example frame. These buckling modes are determined using a model of the complete framing system, to accommodate the calculation of higher eigenvalue buckling modes that are not necessarily symmetric about the middle level of the structure.

Implications with respect to design by the ELM. The ELM, conducted using an "exact" eigenvalue buckling analysis in the foregoing, or conducted using any other legitimate buckling analysis model in general, has been accepted traditionally as a legitimate procedure for checking the resistance of frames subjected to concentric axial compression. This lean-on bracing problem is one example of such a frame. However, it is important to recognize that none of the traditional ELM procedures give any estimate of the internal moments induced within the structure as it approaches its stability limit. The ELM procedures from prior *Specifications* do not predict any moments in the interior beams and their connections to the W14×426 columns, nor do they predict any moments in the columns themselves. The column moments induced by the stability effects can be important in certain cases, particularly when the member out-of-plane lateral-torsional bucking resistance under bending and axial compression is critical. Also, the stability bracing forces induced in the beams and beam-to-column connections should be checked in general. The AISC *Specification* ELM provisions require the inclusion of a notional load of $0.002Y_i$ for gravity-only load combinations to compensate for these shortcomings of the ELM calculations. However, in cases such as this example, where the second-order sidesway amplification is large, the internal forces calculated using the ELM can be significantly different from the DM results and the physical stability behavior. To control these potential errors, the AISC *Specification* limits the application of the ELM to cases in which the second-order sidesway amplification is less than or equal to 1.5 using the nominal structure stiffness (or 1.71 using the reduced structure stiffness of the DM).

It is interesting to note that if the interior frame in this problem were a braced frame, it would fall under the requirements of Appendix 6 of the AISC *Specification*. Appendix 6 requires that geometric imperfections and the corresponding bracing forces must be considered in the stability design of bracing systems. The ELM permits the design of moment frames without the direct consideration of these stability bracing forces, but is limited to frames with the limit on second-order sidesway amplification discussed here. The DM provides a fully consistent approach to the design of all framing systems that explicitly addresses the sidesway effects from geometric imperfections within the limits specified by the AISC *Code of Standard Practice*.

E.7 SUMMARY OF DESIGN RECOMMENDATIONS FOR STABILITY BRACING PROBLEMS USING THE DM

Any column or frame bracing problem can be solved using a second-order analysis. This is specifically allowed by Appendix 6 of the AISC *Specification*. Section 6.1 states, "A second-order analysis that includes an initial out-of-straightness of the member to obtain brace strength and stiffness is permitted in lieu of the requirements of this appendix." The direct analysis method (DM) gives consistent requirements for the analysis and design of all types of structural systems. Suggestions for solving stability bracing problems using the DM are as follows:

1. For bracing problems in which the column flexural rigidities are assumed to contribute to the resistance of the brace-point deflections, one must conduct a second-order analysis that accurately includes both P-Δ and P-δ effects. For bracing problems where the column(s) and bracing are modeled as a simply connected truss, neglecting the contribution of the column flexural rigidity to the resistance of the brace-point deflections, a P-Δ analysis is sufficient. In the former case, most second-order analysis formulations that include both P-Δ and P-δ effects at the element level give solutions for the nodal displacements, using two elements per unbraced segment, that are within five percent of the exact analytical results. P-Δ analysis software can be used for these types of problems, but a larger number of elements is required to obtain comparable accuracy. In general, a B_1 amplification factor or an explicit estimate of the P-δ moments must be included in calculating the maximum column second-order moments along the element lengths. For the analysis approaches recommended in this Appendix, the accuracy of the calculated internal forces is typically equal to or better than the accuracy of the nodal displacements.
2. Multiply the nominal flexural rigidity, *EI,* of all members subjected to axial forces $\alpha P_r > 0.5P_y$ by $0.8\tau_b$, where τ_b is the column inelastic stiffness reduction factor used with the DM. Multiply all other elastic stiffness contributions by 0.8.
3. Use a nominal initial geometric imperfection of $\Delta_o = 0.002L_b$ where L_b is the length between adjacent brace points and Δ_o is the deflection of a brace point relative to adjacent brace points in the direction perpendicular to the axis of the column(s). In some problems, several imperfection shapes may need to be considered. The imperfection patterns must be selected to induce the

largest second-order internal force(s) in the component under consideration. In general, the critical imperfect geometry is different for different components (much like the critical LRFD or ASD load combinations are different for different components in general analysis and design problems); the maximum forces on different components generally do not occur concomitantly. Initial out-of-straightness of the column(s) between brace points need not be considered; this geometric imperfection effect is accounted for in the DM strength checks by the use of $KL = L_b$ in determining the column axial resistance.

4. Typically, the engineer need only select, analyze and check a few of the bracing components in a given problem. Engineering judgment may be used to select the potentially critical components. Other bracing components can be sized using the same sections as the critical components.

5. Check the resistance of all the system components based on the internal forces obtained from the DM second-order analysis. No explicit stiffness check is required.

The DM is useful to arrive at the most efficient bracing solutions, particularly in cases where the flexural rigidity of the component or components being braced provide(s) a substantial contribution to the resistance of the brace-point deflections, or in cases where a column or moment frame is partially braced by another column or moment frame. Several examples of these types of designs have been provided in this Appendix.

SYMBOLS

(Based on the 2005 AISC *Specification*.)

A	Cross-sectional area, in.^{2}
A_e	Effective net area, in.^{2}
A_g	Gross area of member, in.^{2}
B_1	Multiplier to account for P-δ effects
B_2	Multiplier to account for P-Δ effects
C_b	Lateral-torsional buckling modification factor for nonuniform moment diagrams
C_d	Deflection amplification factor
C_L	Factor that accounts for the reduction in column sidesway stiffness due to the presence of axial load on the column
C_m	Coefficient accounting for nonuniform moment
C_w	Warping constant, in.^{6}
D	Nominal dead load, kips
E	Modulus of elasticity of steel = 29,000 ksi
F_{cr}	Critical stress, ksi
F_e	Elastic buckling stress, ksi
F_e'	Euler stress divided by a safety factor, ksi
F_u	Specified minimum tensile strength, ksi
F_y	Specified minimum yield strength, ksi
G	Shear modulus of elasticity of steel = 11,200 ksi
H	Story shear produced by the lateral forces used to compute Δ_H, kips
I	Moment of inertia in the plane of bending, in.^{4}
I_x, I_y	Moment of inertia about the principal axes, in.^{4}
J	Torsional constant, in.^{4}
K	Effective length factor
K_1	Effective length factor in the plane of bending, calculated based on the assumption of no lateral translation, set equal to 1.0 unless analysis indicates that a smaller value may be used.
K_2	Effective length factor in the plane of bending, calculated based on a sidesway buckling analysis.
K_{n2}	K value determined directly from alignment chart
L	Nominal live load, kips
L	Member length, in.
L	Story height, in.
L_b	Length between points that are braced against lateral displacement of compression flanges or braced against twist of the cross section, in.
L_c	Column length, in.

L'_g	Modified girder length for sidesway inhibited frames and girders with different boundary conditions, in.
L_p	Limiting laterally unbraced length for the limit state of yielding, in.
L_r	Limiting laterally unbraced length for the limit state of inelastic lateral-torsional buckling, in.
L_r	Nominal roof live load, kips
M_{cx}	Available flexural strength for strong-axis flexure, kip-in.
M_{lt}	First-order moment using LRFD or ASD load combinations caused by lateral translation of the frame only, kip-in.
M_n	Nominal flexural strength, kip-in.
M_{nt}	First-order moment using LRFD or ASD load combinations assuming there is no lateral translation of the frame only, kip-in.
M_r	Required second-order flexural strength using LRFD or ASD load combinations, kip-in.
M_u	Required flexural strength using LRFD load combinations, kip-in.
N	Minimum lateral load for gravity-only combinations, kips
N_i	Notional lateral load applied at level i, kips
N_x, N_y	Minimum lateral load for gravity-only combinations, kips
P	Applied vertical load, kips
P-Δ	Additional moment due to axial force acting through the relative transverse displacement of the member (or member segment) ends, kip-in.
P-δ	Additional moment due to axial force acting through the transverse displacement of the cross section centroid relative to a chord between the member (or member segment) ends, kip-in.
P_{br}	Required axial strength in the brace, kips
P_{br1}	Bracing force determined by first-order elastic analysis of the ideal geometrically perfect structure under the code required load combination considered in the calculation of P_{br}, kips
P_c	Available axial strength, kips
P_{co}	Available compressive strength out of the plane of bending, kips
P_e	Euler buckling load, kips
P_{e1}, P_{e2}	Elastic critical buckling load for braced and unbraced frame, respectively, kips
P_{eL}	Euler load based on actual length, kips
P_L	Story lateral load required to produce a unit first-order story drift, Δ_H/L, kips
P_{lt}	First-order axial force using LRFD or ASD load combinations, due to lateral translation of the structure only, kips
P_n	Nominal compressive strength, kips
P_{nt}	First-order axial force using LRFD and ASD load combinations, with the structure restrained against lateral translation, kips
P_r	Required axial compressive strength using LRFD or ASD load combinations, kips
P_u	Required axial strength in compression, kips
P_x	Total vertical design load at and above level x, computed with a load factor of not greater than 1.0 on all gravity loads, kips
P_y	Axial yield strength, kips
Q	Net reduction factor accounting for all slender compression elements

Q	Axial load in the girder at the buckling of the system, kips
Q	Stability index
Q_a	Reduction factor for slender stiffened elements
Q_{cr}	In-plane buckling load in the girder using K=1
Q_s	Reduction factor for slender unstiffened elements
R	Response modification factor
R_d	System ductility reduction factor
R_m	Coefficient to account for influence of P-δ on P-Δ
S	Nominal snow load, kips
S_x	Elastic section modulus taken about the x-axis, in.^{3}
V_x	Seismic story shear below level x, kips
Y_i	Gravity load from LRFD load combinations or 1.6 times ASD load combinations applied at level i, kips
W	Nominal wind load, kips
W_F	Nominal wind load at floor level, kips
W_R	Nominal wind load at roof level, kips
Z_x	Plastic section modulus about the x-axis, in.^{3}
b_f	Width of flange, in.
d	Nominal column depth, in.
f_a	Computed axial stress, ksi
h	Clear distance between flanges less the fillet or corner radius for rolled shapes, in.
h_o	Distance between the flange centroids, in.
h_{sx}	Story height below level x
r	Radius of gyration, in.
r_{ts}	Effective radius of gyration, in.
r_x	Radius of gyration about the x-axis, in.
r_y	Radius of gyration about the y-axis, in.
t_f	Thickness of flange, in.
t_w	Thickness of web, in.
w_{FD}	Uniformly distributed floor dead load, kip/ft
w_{FL}	Uniformly distributed floor live load, kip/ft
w_{RD}	Uniformly distributed roof dead load, kip/ft
w_{RL}	Uniformly distributed roof live load, kip/ft
α	ASD/LRFD force level adjustment factor
β	Ratio of shear demand to shear capacity for the story below level x
β_{act}	Actual brace stiffness provided, kip/in.
β_{br}	Required brace stiffness, kip/in.

Δ First-order interstory drift due to the design loads, in.

Δ Design story drift under earthquake loads E (including C_d and inelastic effects) occurring under V_x, in.

Δ_o Nominal initial out-of-plumbness, in.

Δ_{1st} First-order interstory drift due to the design loads, in.

Δ_{2nd} Second-order interstory drift due to the design loads, in.

Δ_H First-order interstory drift due to lateral forces, in.

$\overline{\Delta}_H$ First-order elastic sidesway deflection at the middle floor level due to ΣH, calculated using the DM reduced stiffnesses, in.

ϕ_b Resistance factor for flexure

ϕ_c Resistance factor for compression

Ω Safety factor

Ω_b Safety factor for flexure

Ω_c Safety factor for compression

Ω_o Structural overstrength factor

θ Stability coefficient

τ_a Column inelastic stiffness reduction factor

τ_b Stiffness reduction parameter

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